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A fast spectral conjugate gradient method for solving nonlinear optimization problems

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ABSTRACT

This paper proposes a new spectral conjugate gradient (SCG) approach for solving unregulated nonlinear optimization problems. Our approach proposes Using Wolfe's rapid line scan to adjust the standard conjugate descent (CD) algorithm. A new spectral parameter is a mixture of new gradient and old search path. The path provided by the modified method provides a path of descent for the solution of objective functions. The updated method fits the traditional CD method if the line check is correct. The stability and global convergence properties of the current new SCG are technically obtained from applying certain well-known and recent mild assumptions. We test our approach with eight recently published CD and SCG methods on 55 optimization research issues from the CUTE library. The suggested and all other algorithms included in our experimental research were implemented in FORTRAN language with double precision arithmetic and all experiments were conducted on a PC with 8 GB ram Processor Intel Core i7. The results indicate that our proposed solution outperforms recently reported algorithms by processing and performing fewer iterations in a shorter time.

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1. INTRODUCTION

The mathematical model for the optimization algorithm can be used to find a solution to such problems. For some problems, an exact solution cannot be calculated directly. Instead, suitable algorithms must be chosen that will approximate the solution as closely as required to the optimal solution. We consider the problem of optimization:

$$\min\left\{f(x)\middle|x \in R^n\right\}, \tag{1}$$

where $f : R^n \to R$ Continuously separated feature. Nonlinear conjugate gradient (CG) algorithms are useful in solving nonlinear optimization problems formulated in (1). Such algorithmic improvements are demonstrated by:

$$x_k = x_{k-1} + \alpha_k d_{k-1}, \; k = 0,1,... \tag{2}$$

$$d_k = \begin{cases} -g_k, & if \;\; k = 0 \\ -g_k + \beta_k d_{k-1}, & if \;\; k \geq 1 \end{cases} \tag{3}$$

Where's the latest version x_k, $\alpha_k > 0$. The Wolfe line search method determines the next search path, denotes the gradient at, and is an appropriate parameter (also called the conjugacy function). CG was initially suggested by Hestenes and Stiefel in the 1950s [1] as an exact way to solve symmetric, strong, definite linear algebraic structures. In 1964, Fletcher and Reeves [2] expanded the scope of CG approaches to non-linear problems.

The main advantages of the CG methods are their low memory requirements, its convergence speed and its satisfaction of a quadratic termination property in which the method can locate the minimizer of a quadratic function in a finite number of iterations, see, for example, Hassan [3, 4] yet which can be applied iteratively to minimizing non-quadratic functions. This family of algorithms was proposed on the conjugacy parameter β_k in previous studies. Most well-known formulas were defined such as: Polak and Ribière (PR) [5], Fletcher (CD) [6]. Observe that in these algorithms the scalar α_k can be calculated by Wolfe [7] and:

$$\begin{aligned} & f(x_k + \alpha_k d_k) \leq f(x_k) + \delta \alpha_k g_k^T d_k \\ & \left| g(x_k + \alpha_k d_k)^T d_k \right| \leq -\sigma g_k^T d_k \end{aligned} \tag{4}$$

$$0 < \delta < \sigma < 1$$

The strong Wolfe conditions may not yield a direction of descent unless $\sigma \leq 1/2$. Inequality given in (4) is sometimes called the Armijo condition. Hager and Zhang [8] were proposed a CG method (HZ) That corresponds to the following upgrade parameter where $\|.\|$ denotes the Euclidean norm of vectors:

$$\overline{\beta}_k^{HZ} = \max\left\{\beta_k^{HZ}, \eta_k\right\}, \text{ where} \tag{5a}$$

$$\eta_k = \frac{-1}{\|d_k\| \min\left\{\eta, \|g_{k-1}\|\right\}}, \tag{5b}$$

$$\beta_k^{HZ} = \frac{1}{d_k^T y_{k-1}} \left(y_{k-1} - 2d_k \frac{\|y_{k-1}\|^2}{d_k^T y_{k-1}} \right)^T g_k, \tag{5c}$$

here, $\eta > 0$ is a constant. In this research, we proposed a fast-spectral CG approach to solve unconstrained nonlinear optimization issues. We plan to investigate the stability and global convergence properties of the proposed algorithm and perform some practical computational tests to demonstrate its efficiency suitable for solving certain nonlinear optimization problems with the well-known Powell restart criterion [9]. The remainder is organized into four sections. Section 2 explains earlier studies of spectral CG methods. Section 3 outlines our proposed method. Section 4 contains tests, results, and discussion. Section 5 contains the final remarks.

2. LITERATURE REVIEW ON SPECTRAL CONJUGATE GRADIENT METHODS

Spectral CG (SCG) methods are a common CG approach for problem-solving(1). It was originally developed by Barzilai and Borwein and later studied by several authors [10-14]. Below is the general SCG process system.

Algorithm (SCG)

Step1: Choose $x_0 \in R^n$ and the parameter $0 < \mu \leq \sigma < 1$. Compute $f(x_0)$ and set $d_0 = -g_0$

Step2: Compute α_k from Wolfe conditions using (5).

Step3: If $\|g_k\| \leq \varepsilon$, then stop; otherwise continue.

Step4: If $\left|g_k^T g_{k-1}\right| \geq 0.2\|g_k\|^2$ is met, restart step by step by $-g_0$ direction; otherwise proceed.

Step5: Compute the parameter θ_k , corresponding to different studies in this field.

Step6: Compute the parameter β_k from the standard CG method

Step7: Calculate the new search path $d_k = -\theta_k g_k + \beta_k d_{k-1}$.

Step8: Set the iteration *k=k+1* and go to **Step2**.

During the last decade, much effort has been devoted to developing new SCG methods. One of the old studies in this field is proposed by Raydan (R) [15]. Al-Bayati and Abdullah (BA) [16] introduced a new class of SCG methods of unconstrained large-scale optimization problems using both spectral and scaling properties for their search paths. i.e.

$$\beta_k^{CD} = -\frac{g_k^T g_k}{g_{k-1}^T d_{k-1}} \quad \theta_k^{BA} = \frac{s_{k-1}{}^T s_{k-1}}{s_{k-1}^T s_{k-1} + \varepsilon \quad s_{k-1}^T y_{k-1}}, \qquad \varepsilon = 0.0001 \tag{6}$$

Al-Bayati and Hassan (BH) [17] investigated another SCG method to solve unconstrained optimization problems.

$$\beta_k^{CD} = -\frac{g_k^T g_k}{g_{k-1}^T d_{k-1}} \quad \theta_k^{BH} = \frac{\mu\left|g_k^T d_{k-1}\right| + y_{k-1}^T d_{k-1}}{\left|g_{k-1}^T d_{k-1}\right|} \quad . \tag{7}$$

Liu and Jiang (LJ) [18] proposed minor modification of the CD method so that the search directions produced are always downward. A mixed spectral method (LDW) is proposed by Liu, et al. [19] for solving some nonlinear optimization problems.

$$\beta_k^{LDW} = \begin{cases} \beta_k^{CD} + \min\,(0, \psi_k \beta_k^{CD}\ ), & if\ g_k^T d_{k-1} \leq 0, \\ 0, & else, \end{cases} \qquad \theta_k^{LDW} = 1 - \frac{g_k^T d_{k-1}}{g_{k-1}^T d_{k-1}} \tag{8}$$

$$\psi_k^{LDW} = -(1 - \frac{g_k^T d_{k-1}}{d_{k-1}(g_k - g_{k-1})}) \tag{9}$$

Livieris and Pintelas (LP) [20] suggested another type of SCG method providing sufficient descent directions, regardless of quality line search and global convergence property for general functions, given the line search technique meets Wolfe requirements. Another novel SCG method was proposed by Al-Bayati and Al-Khayat (BK) [21] in this field. They tried to construct a descent direction.

$$\beta_k^{CD} = \frac{g_k^T g_k}{-d_{k-1}^T g_{k-1}}, \quad \theta_k^{KB} = -\frac{d_{k-1}^T y_{k-1}}{d_{k-1}^T g_{k-1}} - \frac{d_{k-1}^T g_k g_k^T g_{k-1}}{\|g_k\|^2 d_{k-1}^T g_{k-1}} \tag{10}$$

Ghanbari (GAAA) suggested an important nonlinear SCG approach for solving optimization problems [22]. Their system is based on a hybrid spectral HS-CG system combining the advantages of spectral HS process with CD method. Recently, a new SCG method (LZX) was proposed by Hang et al. [23].

3. A FAST SPECTRAL CONJUGATE GRADIENT METHOD

We are focusing on a modern SCG-method to address unconstrained nonlinear optimization problems. Our approach reduces line search to conventional CD-method. We have shown that while the objective function is non-convex, the proposed Wolfe line search approach is globally convergent. To evaluate the descent directions for the new SCG process, let the current iterate be defined by:

$$d_k^{New} = \begin{cases} -g_k, & if \quad k=0 \\ -\theta_k^{New} g_k + \beta_k^{CD} d_{k-1}, & if \quad k \geq 1 \end{cases} \tag{11}$$

where β_k^{CD} is specified by (4d) with the following fast spectral parameter defined in (11) and the unknown parameter θ_k is defined by:

$$\theta_k^{New} = 1 - \left(\frac{g_k^T g_k}{d_{k-1}^T g_{k-1}}\right)\left(\frac{d_{k-1}^T g_k}{\|g_k\|^2}\right) - \frac{d_{k-1}^T g_k}{2 g_{k-1}^T g_{k-1}} \tag{12}$$

This new approach was designed to solve a variety of complicated nonlinear unconstrained optimization issues that are reduced to the classical CD approach if the line search is successful. For better results, we use Wolfe's inaccurate line search. In this algorithm, we must first prove it's an appropriate downward path.

3.1. Lemma

Suppose that the new search direction d_k^{New} which is defined by (11) and (12) and assume that α_k satisfies the condition (5) with $\sigma_k < 0.5$. Then:

$$g_k^T d_k \leq -c_1 \|g_k\|^2 \tag{13}$$

holds for any $k \geq 0$.

Proof. For initial k; k=0, we have

$$d_0^T g_0 = -\|g_0\|^2 \tag{14}$$

We presume condition (13) refers to all k-1 values; i.e.

$$d_{k-1}^T g_{k-1} = -\|g_{k-1}\|^2 \tag{15}$$

Then we can show the condition (13) is true for all k values, i.e.

$$g_k^T d_k = -\theta_k^{New} \|g_k\|^2 + \beta_k^{CD} g_k^T d_{k-1} \tag{16}$$

From the (4d); (11) and (12):

$$g_k^T d_k = -\left[1 - \left(\frac{g_k^T g_k}{d_{k-1}^T g_{k-1}}\right)\left(\frac{d_{k-1}^T g_k}{\|g_k\|^2}\right) - \frac{d_{k-1}^T g_k}{2 g_{k-1}^T g_{k-1}} + \frac{g_k^T d_{k-1}}{d_{k-1}^T g_{k-1}}\right] \|g_k\|^2 \tag{17}$$

$$g_k^T d_k = -\left[1 - \frac{d_{k-1}^T g_k}{d_{k-1}^T g_{k-1}} - \frac{d_{k-1}^T g_k}{2g_{k-1}^T g_{k-1}} + \frac{g_k^T d_{k-1}}{d_{k-1}^T g_{k-1}}\right]\|g_k\|^2$$

$$g_k^T d_k = -\left[1 - \frac{d_{k-1}^T g_k}{2g_{k-1}^T g_{k-1}}\right]\|g_k\|^2 \tag{18}$$

From second Wolfe condition defined in (5), we have:

$$d_{k-1}^T g_k \le -\sigma d_{k-1}^T g_{k-1};\ \sigma \quad is \quad a \quad positive \quad parameter \tag{19}$$

Therefore, using (19) and (18) becomes:

$$g_k^T d_k \le -\left[1 + \frac{\sigma d_{k-1}^T g_{k-1}}{2g_{k-1}^T g_{k-1}}\right]\|g_k\|^2 \tag{20}$$

Hence:

$$g_k^T d_k \le -\left[1 + \frac{\sigma}{2}\right]\|g_k\|^2 \tag{21}$$

This implies:

$$g_k^T d_k \le -c\|g_k\|^2 \text{ with } c = [1 + \sigma/2] > 0 \tag{22}$$

Thus, (13) is satisfied, the Lemma is true.

In Lemma 3.1, by using an exact line search, dk is f at xk and:

$$\theta_k^{New} = 1 - (\frac{g_k^T g_k}{d_{k-1}^T g_{k-1}})(\frac{d_{k-1}^T g_k}{\|g_k\|^2}) - \frac{d_{k-1}^T g_k}{2g_{k-1}^T g_{k-1}} = 1 \tag{23}$$

and the proposed new form is reduced to the CD.

3.2. Wolfe's accelerated line search

In this section, we find an acceleration scheme in [4]. The latest calculation of the minimum point is estimated as follows:

$$x_{k+1} = x_k + \lambda_k \alpha_k d_k \tag{24}$$

Let $g_z = \nabla f(z)\ and\ \ z = x_k + \alpha_k d_k$

Compute spectral parameters $a_k = \alpha_k g_k^T d_k\ , b_k = -\alpha_k (g_k - g_z)^T d_k$ (25)

If $b_k \ne 0$, then compute $\lambda_k = -a_k / b_k$ and (24),

Otherwise, update $x_{k+1} = x_k + \alpha_k d_k$

3.3. Outline of the new proposed algorithm

Step 1: Take $x_0 \in R^n$; set the parameters $0 < \delta \le \sigma < 1$; *eps* are the small positive number.
Compute $f(x_0)$ and $g_0 = \nabla f(x_0)$; set $d_0 = -g_0$ for $k = 0$.

Step 2: Compute Wolfe conditions parameter α_k using (5).

Compute, $y_k = g_k - g_z$, $g_z = \nabla f(z)$.

Acceleration scheme: compute, $a_k = \alpha_k g_k^T d_k$, $b_k = -\alpha_k y_k^T d_k$,

If $b_k \neq 0$, then

Calculate, $\lambda_k = -(a_k / b_k)$ and

Find the new factor as $x_{k+1} = x_k + \lambda_k \alpha_k d_k$

Else

Find the new factor as $x_{k+1} = x_k + \alpha_k d_k$

Step 3: If $\|g_k\|_\infty \le eps$ it is satisfied then stop; otherwise continue.

Step 4: If Powell restart requirement $\left|g_k^T g_{k-1}\right| \ge 0.2\|g_k\|^2$ is met, restart step by step by $-g_0$ direction; otherwise proceed.

Step 5: Calculate spectral parameters:

$$\beta_k^{CD} = \frac{g_k^T g_k}{-d_{k-1}^T g_{k-1}}$$

$$\theta_k^{New} = 1 - \left(\frac{g_k^T g_k}{d_{k-1}^T g_{k-1}}\right)\left(\frac{d_{k-1}^T g_k}{\|g_k\|^2}\right) - \frac{d_{k-1}^T g_k}{2 g_{k-1}^T g_{k-1}}$$

Step 6: Calculate the current spectral path

$$d_k^{New} = -\theta_k^{New} g_k + \beta_k^{CD} d_{k-1}$$

Step 7: Let k = k+1 to move to Step 2.
Conditions (5) and Powell restart criterion are sufficient to show the Fast-SCG method's global convergence.

3.4. Proposed convergence algorithm

Consistency testing of the proposed Quick SCG method mentioned in (11) and (12), the following common and general assumptions can be used to prove convergence of any CG process:

3.5. Assumption

a) For the starting point x_1, the level set $S = \{x : x \in R^n, f(x) \le f(x_1)\}$ is bounded.

b) Neighborhood function f is constantly differentiable Ω of S, and the gradient g satisfies:

c) $$\| g(x) - g(x_k) \| \le L \| x - x_k \|, \forall x, x_k \in \Omega; \quad L \ge 0 \tag{26}$$

Obviously:
from the Assumption (i), true constant D occurs such that:

$$D = \max\{\|x - x_k\|, \forall x, x_k \in S\} \text{, D is the diameter of } \Omega \tag{27}$$

Assumption (ii) involves a constant $\Gamma \geq 0$, such that:

$$\|g(x)\| \leq \Gamma, \ \forall x \in S \tag{28}$$

3.6. Convergence of newly developed algorithm

The new search directions are given by (11) and (12) satisfies:

$$\lim_{k \to \infty} \inf \|g_k\| = 0 \tag{29}$$

Under the above assumptions, with $\alpha_k > 0$ and satisfies (5); $\sigma_k \leq 0.5$

Proof.

Suppose a positive constant exists $\varphi > 0$ such that:

$$\|g_k\| \geq \varphi \tag{30}$$

therefore, for all k (23) yields,

$$\|d_k\|^2 = d_k^T d_k$$

$$= (-\theta_k^{New} g_k + \beta_k^{CD} d_{k-1})(-\theta_k^{New} g_k + \beta_k^{CD} d_{k-1})$$

$$= (\theta_k^{New})^2 \|g_k\|^2 - 2\theta_k^{New} \beta_k^{CD} d_{k-1}^T g_k + (\beta_k^{CD})^2 \|d_{k-1}\|^2$$

Since $d_k = -\theta_k^{New} g_k + \beta_k^{CD} d_{k-1}$ then:

$$= (\theta_k^{New})^2 \|g_k\|^2 - 2\theta_k^{New} (d_k^T + \theta_k^{New} g_k^T) g_k + (\beta_k^{CD})^2 \|d_{k-1}\|^2$$

$$= (\theta_k^{New})^2 \|g_k\|^2 - 2\theta_k^{New} g_k^T d_k - 2(\theta_k^{New})^2 \|g_k\|^2 + (\beta_k^{CD})^2 \|d_{k-1}\|^2$$

$$= (\theta_k^{New})^2 \|d_{k-1}\|^2 - 2\theta_k^{New} g_k^T d_k - (\theta_k^{New})^2 \|g_k\|^2 \tag{31}$$

if we divide both sides of the above equality by $(g_k^T d_k)^2$, then from (13), (26) and (31) we obtain:

$$\|d_k\|^2 /(g_k^T d_k)^2 = [(\beta_k^{CD})^2 \|d_{k-1}\|^2 - 2\theta_k^{New} g_k^T d_k - (\theta_k^{New})^2 \|g_k\|^2]/(g_k^T d_k)^2 \tag{32}$$

$$= \left[\frac{\|g_k\|^2}{d_{k-1}^T g_{k-1}} \right]^2 \frac{\|d_{k-1}\|^2}{(g_k^T d_k)^2} - (\theta_k^{New})^2 \frac{\|g_k\|^2}{(g_k^T d_k)^2} - 2\theta_k^{New} \frac{1}{(g_k^T d_k)}$$

From (21) we have,

$$g_{k-1}^T d_{k-1} \leq -c\|g_{k-1}\|^2, \quad g_k^T d_k \leq -c\|g_k\|^2$$

In (32) becomes:

$$\leq \left[\frac{\|g_k\|^2}{c\|g_{k-1}\|^2} \right]^2 \frac{\|d_{k-1}\|^2}{c^2\|g_k\|^4} - (\theta_k^{New})^2 \frac{\|g_k\|^2}{c^2\|g_k\|^4} - 2\theta_k^{New} \frac{1}{c\|g_k\|^2} \tag{33}$$

Reformulate; add and subtract a positive number yields:

$$\leq \frac{\|d_{k-1}\|^2}{c^4\|g_{k-1}\|^4} \ -\{(\theta_k^{New} \frac{\|g_k\|}{c\ \|g_k\|^2})^2 + 2\theta_k^{New} \frac{1}{c\|g_k\|^2}\} \ + \frac{1}{\|g\|^2} - \frac{1}{\|g\|^2} \tag{34}$$

$$\leq \frac{\|d_{k-1}\|^2}{c^4\|g_{k-1}\|^4} - \left[\theta_k^{New} \frac{\|g_k\|}{c\|g_k\|^2} + \frac{1}{\|g_k\|} \right]^2 + \frac{1}{\|g_k\|^2} \tag{35}$$

$$\leq \frac{\|d_{k-1}\|^2}{c^4\|g_{k-1}\|^4} + \frac{1}{\|g_k\|^2} \quad \|d_1\|^2 = -d_1^T g_1 = \|g_1\|^2 \tag{36}$$

$$\|d_k\|^2 /(d_k^T g_k)^2 \leq \sum_{i=1}^{k} 1/\|g_i\|^2 \tag{37}$$

Therefore, from (37) and (30) we have:

$$(d_k^T g_k)^2 /\|d_k\|^2 \geq \varphi / k \tag{38}$$

which indicates:

$$\sum_{k=1}^{\infty} (g_k^T d_k)^2 /\|d_k\|^2 \Rightarrow +\infty \tag{39}$$

It challenges our assumption (29). Therefore, this theorem's proof is complete and the proposed Fast-SCG has a global convergence property.

4. EXPERIMENTAL RESULTS

Here we analyze the reliability of the real modern CUTE library solution suggested by Bongartz, et al. [24-26] set of 55-complicated nonlinear test problems. All these assessment questions are posed (n=100,400,700,1000). Calculate optimum output dependent on computation time (CPU), the number of iterations (NOI) and the number of function measures (NOF). All methods stop before the following state is met.

$$\|g_k\|_\infty \leq 10^{-5} \tag{40}$$

They also require such routines to end if NOI crosses 1000 or NOF hits 2000 without the minimum. We report the findings of the newly proposed process, claim, against Fast-SCG (CD [6], FR [2], PR [5] and HZ [8], LDW [19], BA [16], BH [17] and BK [21]). We are in Table1.

To show the new method's efficiency against other methods on complete NOI, NOF, and CPU results for 55 test problems, the relative percentage improvement (RPI) is measured and described. RPI shows that our system finds better outcomes in approaches previously suggested. (See in (41)) for RPI of our tools and [25] for the details).

RPINOI (MethodX)= (MethodXNOI– MinNOI)/MinNOI (41)

Table 2 indicates that CD is the worst and HZ is the best method in terms of NOI and NOF among the previous studies. Although the PR evaluates much more candidate solutions than HZ to reach an optimum solution, it is the fastest method among the previous studies. When we compare the results of Fast-SCG with the results of previous studies, it is clear that Fast-SCG improves the HZ results (NOI and NOF) by more than 25% and it completes its search process in half-time of the PR. In other words, Fast-SCG finds the optimum results by evaluating fewer candidate solutions in a shorter CPU time.

The results in Table 3 are obtained by four SCG methods indicate that BK is the best method among the previous studies in terms of all three criteria. BA is the worst method in terms of NOI and NOF. LDW is the slowest method in terms of CPU. Furthermore, the Fast-SCG method outperforms all four previous methods in terms of all three criteria. It reduces the NOI and NOF by more than 50% and it speeds up by more than 25% when compared against BK.

The following figures demonstrate the efficiency of the fast-SCG algorithm compared to regular and spectral CG algorithms according to the following points:

a) The Figure 1 indicates the similarity of 9 NOI algorithms.
b) The Figure 2 indicates 9 NOF algorithms equivalent.
c) The Figu3 re reveals 9 CPU-related algorithms.

Table 1. Total NOI; NOF and CPU for 55 test problems

Total 55 Funct.	Conjugate Gradient Methods				Spectral Conjugate Gradient Methods				
	CD	FR	PR	HZ	LDW	BK	BA	BH	Fast SCG
TOTAL NOI	3671	3382	3626	1911	3001	2952	3416	3311	1432
TOTAL NOF	7053	6779	6747	3994	6465	6377	6835	6784	2887
TOTAL CPU	1.96	1.08	0.69	0.72	2.14	0.52	0.57	2.08	0.38

Table 2. RPI results for (Cd, Fr, PR, Hz, and Fast SCG)

	CD	FR	PR	HZ	Fast-SCG
RPI_{NOI}	0.92	0.77	0.90	0.00	-0.25
RPI_{NOF}	0.77	0.70	0.69	0.00	-0.28
RPI_{CPU}	1.84	0.57	0.00	0.04	-0.45

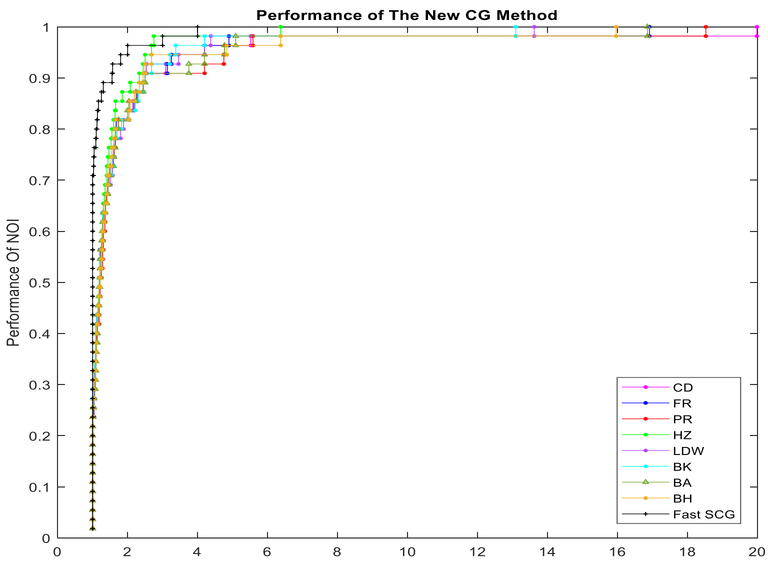

Figure 1. Comparisons w.r.t. NOI

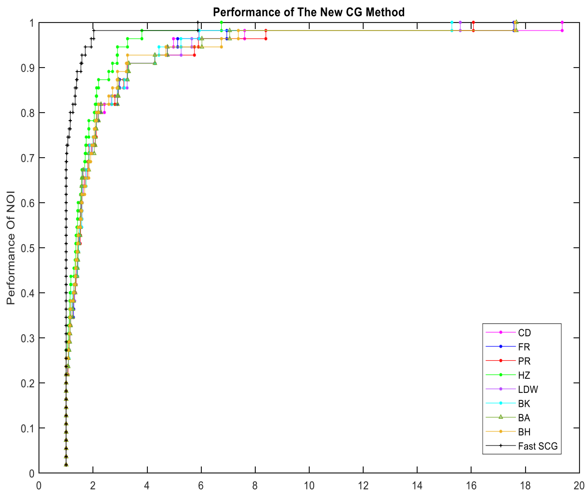

Figure 2. Comparisons w.r.t. NOF

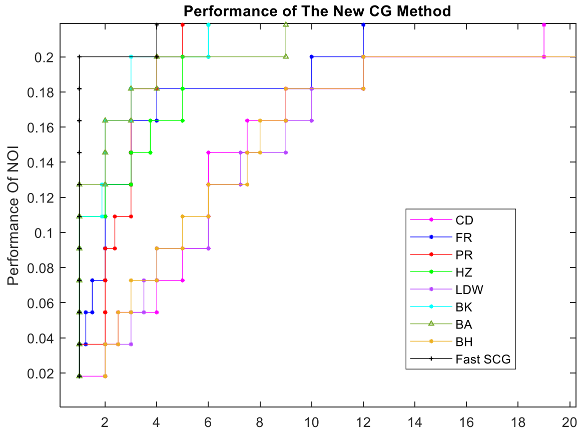

Figure 3. Comparisons w.r.t. CPU

Table 3. RPI results For (LDW, BK, BA, BH, and Fast-SCG)

	LDW	BK	BA	BH	Fast-SCG
RPI_{NOI}	0.02	0.00	0.16	0.12	-0.51
RPI_{NOF}	0.01	0.00	0.07	0.06	-0.55
RPI_{CPU}	3.12	0.00	0.10	3.00	-0.27

5. CONCLUSION

In this review, by introducing a new spectral parameter and changing the Wolfe line search algorithm, we propose a new SCG (Fast-SCG). It doesn't require some matrix data, so it's quickly applied both technically and experimentally. Global convergence property for the proposed new SCG approach is technically obtained. The suggested approach is contrasted with traditional and spectral CG models, i.e. (FR, PR, CD, and HZ) and (BK; BH; BA and LDW) using 55 well-known non-linear experiments. Our proposed solution incorporates the following eight performance mechanisms (NOI, NOF, and CPU).

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