

## Study on the Algorithm for Real-time Interpolation of NURBS Curve and Simulation

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### Abstract

*In order to meet the needs of high-speed and high- accuracy computerized numerical control machining and guarantee the smooth running in the interpolation processing, A NURBS curve algorithm based on adaptive acceleration and deceleration control of look-ahead s-shaped for the real-time interpolation is presented. The algorithm has merits such as higher position accuracy, short processing time, feed speed without variation and so on. Through dynamic path simulation and interpolation results analyses with MATLAB, the interpolation algorithm is proved to be correct and rational. Through actual machining by OPEN platform with VC++, the algorithm is proved to be real-time and practicable.*

**Key Words:** Real-time interpolation, Adaptive feed rate, Acceleration and deceleration control, Simulation

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### 1. Introduction

Non-Uniform Rational B-Spline (NURBS) has been widely adopted in curve and surface modeling since it not only offers a common mathematical form for representing free-form and analytical geometry, but also it has the capability of flexible shape control.

In view of this, the International Organization for Standardization (ISO) 1991 standard issued by the STEP (Standard for The Exchange of Product model data, the industrial product model data exchange standards) made NURBS as the only way to define free curves and surfaces [1]. In order to achieve the direct interpolation of parametric curves in numerical control system, many scholars have proposed a variety of real-time interpolation algorithm. for example, Shanghai Jiaotong University researchers [2] proposed an adaptive NURBS interpolation algorithm which based on real-time and forward-looking. The algorithm is able to ensure the smooth progress of the forward-looking in theory, while there is a large amount of computing in the actual interpolation process. Also the computing amount will rapid expansion with the selected acceleration, maximum feed rate and interpolation cycle change. Tsinghua University researchers [3] reported a NURBS interpolation algorithm is suitable for the PC numerical control system. Hang Houjun[4] proposed a NURBS interpolation algorithm based on cubic rational conic sections. However, the shortage is that it didn't in-depth study of the curve interpolation method. Shpitaln used the first-order Taylor expansion to calculate interpolation points [5]. This method has more excellent characteristics than the constant parameter increment interpolation, since there is no corresponding error control mechanism. Yang and Kong [6] extended the first-order Taylor formula to the second-order, in order to improve the calculation accuracy. Yeh [7, 8] designed an adaptive parameter curve interpolation algorithm, in which can ensure the stability of the machining process. However, the approach ignored the limitations of the machine acceleration. Yong [9] detected a smaller region of curvature radius with offline. It was accordance with the requirements of machine acceleration and jerk limited to plan pre-interpolation speed. At last, it achieved interpolation acceleration and deceleration control. However, a disadvantage in these algorithms lies in that they are ring-opening interpolation algorithm. Therefore, in this paper, an algorithm about NURBS real-time adaptive interpolation which based on S-shaped and forward-looking acceleration and deceleration was studied. Furthermore, the feasibility of the algorithm through theoretical simulation and the actual processing of the experimental are discussed.

## 2. NURBS Curve Geometric Model

A NURBS curve was a rational polynomial defined by the following form [10]:

$$p(u) = \frac{\sum_{i=0}^n \omega_i d_i N_{i,k}(u)}{\sum_{i=0}^n \omega_i N_{i,k}(u)} \quad u \in [0, 1] \quad (1)$$

Where  $u$  is a scalar parameter which varies from 0 to 1,  $d_i (i = 0, 1, \dots, n)$  is an ordered list of de Boor points (also called control points),  $\omega_i (i = 0, 1, \dots, n)$  is an list of weight factors, and  $N_{i,k}(u)$  is the normalized B-spline basis function of order  $k$  calculated by the following recurrence formula:

$$\left. \begin{aligned} N_{i,0} &= \begin{cases} 1, & \text{when } u_i \leq u \leq u_{i+1} \\ 0, & \text{others} \end{cases} \\ N_{i,k}(u) &= \frac{u - u_i}{u_{i+k} - u_i} N_{i,k-1}(u) + \frac{u_{i+k+1} - u}{u_{i+k+1} - u_{i+1}} N_{i+1,k-1}(u) \\ \text{define } \frac{0}{0} &= 0 \end{aligned} \right\} \quad (2)$$

Where the knot vector belong to  $U = [u_i, \dots, u_{i+k+1}]$ . Based on Eq. (1) and (2), a NURBS curve can be defined when  $d_i$ ,  $\omega_i$ ,  $k$  and knot vector are given certain values. A NURBS curve is defined by three types of parameters: Locus of control, Weighted factor and Knot vector. The FANUC CNC system has a large share in the current CNC system market, and they make these parameters as part of the NC program command parameters, Interpolation G code shown in Figure 1:

```
G06.2K_U_X_Y_Z_W_F
      U_X_Y_Z_W_
      .....
      U_X_Y_Z_W_
      U_
      U_
```

Figure 1. NURBS Interpolation instruction format

Where, G06.2 expressed NURBS interpolation; K indicated the number of NURBS curve; U was for curve knot vector; X, Y, Z represented the three coordinate of the NURBS curve locus of control; W showed NURBS curve of the weighted factor; F was for the setting of NURBS curve processing feed speed.

The difference between a large number of traditional G01 forms of NC program is that the new NURBS interpolation will bring CNC programming qualitative change, of course, the application of NURBS interpolation can not be separated the CAM software with the function of NURBS, more common with UG □, Master CAM, Cimatron, etc. [11]. At present, G06.2 is the foreign typical NURBS interpolation way for G code, such as FANUC is G06.2.

### 3. NURBS Curve Real-Time Interpolation Algorithm

#### 3.1. NURBS Curve Interpolation Principle

NURBS curve interpolation is that using interpolation cycle as the unit of time to divided the whole process into many feed process. In each interpolation cycle, the straight line segments  $\Delta L_i (i = 1, 2, \dots, n)$  which approach the interpolated curve in the trajectory space can be calculated, and then this paper put the small straight line segment to be mapped to parameter variable space to obtain its corresponding parametric increment. Through parametric recursive, current point coordinates in the parametric space could be obtained. Finally, the coordinates of the interpolation points  $p_i (i = 1, 2, \dots, n)$  in each interpolation cycle in the trajectory space can be obtained. For NURBS curve, the classic iteration method of interpolation parameters is:

$$u_{i+1} = u_i + \Delta u_i$$

Where,  $u_i$  is the current interpolation point corresponding to parameter value,  $u_{i+1}$  is the parameter value of next interpolation point,  $\Delta u_i$  is parameter increment. the location of current interpolation point can be gained by taking the  $u_{i+1}$  into expression of parameter curve .

#### 3.2. Real - Time Interpolation Algorithm

NURBS curve direct interpolation process will be completed by two major steps: one is the shadow mapped from trajectory space to the parametric space, which is called parameter densification; and the other one is mapped from parametric space to trajectory space, also known as trajectory calculation.

##### 3.2.1. Parameter Densification

In this paper, it used improved Adams differential equations to complete parameter densification[12]. The formula can be described as follows:

$$\tilde{u}_{i+1} = \frac{1}{4} (9 \cdot u_i - 6u_{i-1} + u_{i-2}) \quad (3)$$

By substituting Eq. (3) into Eq. (1) , corresponding forecast interpolation points can be obtained:

$$\tilde{p}_{i+1} = p(\tilde{u}_{i+1})$$

The corresponding estimated feed step can be represented as follows:

$$\Delta \tilde{L}_i = |\tilde{p}_{i+1} - p_i| = \sqrt{(\tilde{x}_{i+1} - x_i)^2 + (\tilde{y}_{i+1} - y_i)^2 + (\tilde{z}_{i+1} - z_i)^2} \quad (4)$$

Correct iterative precision of Eq. (3) according to the relative deviation between estimated step and expected step, as the following formula:

$$\delta_i = \frac{|\Delta L_i - \Delta \tilde{L}_i|}{\Delta L_i} \times 100\%$$

Where  $\Delta L_i$  is estimated step which can be derived from Eq. (8).

When the error  $\delta_i$  is in the allowable range,  $\tilde{p}_{i+1}$  is the request interpolation point  $p_{i+1}$ . Once the error exceeds the allowable range, we correct the parameter which is obtained from Eq. (3) by using the following formula:

$$u_{i+1} = u_i + \frac{\Delta L}{\Delta \tilde{L}_i} \cdot (\tilde{u}_{i+1} - u_i) \quad (5)$$

### 3.2.2. Trajectory Calculation

By substituting the new parameters obtained from Eq. (3) into Eq. (1), the location of next interpolation points can be obtained. Through the process of parameter densification and trajectory calculation, the next position of the interpolation points can be gained within each interpolation cycle and control the movement of moving parts by inputting the feed command of each axis into the servo control system, at this point, a interpolation process is over. Repeating the above two real-time interpolation steps until end, the interpolation track of the whole NURBS curve can be obtained.

### 3.2.3. Adaptive Feed Step Determination

In order to meet the needs of processing accuracy and processing efficiency, the feed step is proposed with confined chord error and normal acceleration in each interpolation cycle.

(1) Feed step controlled by chord error  $\Delta L_{i1}$ .

In interpolation preprocessing, chord error is inevitable. When the machining error exceeds the confined chord error, the feed rate is modified in real-time to guarantee the machining accuracy by using the following formula:

$$\Delta L_{i1} = \sqrt{\frac{h_{\max}}{\tilde{h}_i}} \times \Delta \tilde{L}_i \quad (6)$$

Where  $\Delta \tilde{L}_{i1}$  is determined by Eq. (4).

(2) Feed step controlled by normal acceleration  $\Delta L_{i2}$

In order to ensure the normal acceleration could change within the permitted range which the mechanical characteristics of machine required, the feed step should be obtained as

$$\Delta L_{i2} = \frac{T}{2} \sqrt{\frac{a_{N \max}}{2h_{\max}}} \times \Delta L_{i1} \quad (7)$$

In the interpolation preprocessing, the maximum allowable feed step of a machine is due to feed step unconstrained  $\Delta L_{i0}$ , feed step controlled by both chord error  $\Delta L_{i1}$  and feed step controlled by normal acceleration  $\Delta L_{i2}$ . We take the minimum value of the three interpolation step as the current feed step[13], the instantaneous feed step  $\Delta L_i$  is represented as:

$$\Delta L_i = \min\{\Delta L_{i0}, \Delta L_{i1}, \Delta L_{i2}\} \quad (8)$$

By substituting Eq. (8) into Eq. (5), interpolation parameter values corrected can be taken.

The algorithm used above is adaptive interpolation one, the unconstrained feed step can be controlled by introducing chord error and the normal acceleration, the feed rate can be improved maximumly within the premise that it can meet the processing accuracy.

The algorithm realized the optimization of processing accuracy and feed to some extent, and adequately satisfied with the requirements of modern high-speed and high-precision machining CNC machining systems.

## 4. Look-Ahead S-shaped Pre-Acceleration and Deceleration Method

### 4.1. S-shaped Acceleration-Deceleration Control

In this paper, acceleration-deceleration mechanism adopted is s-type curve [14]. System in the deceleration phase of the feed speed curve is S type. According to its more flexible, this control mode could make the speed change smoothly at start and end stages. Meanwhile, the deceleration point can be predicted by the symmetry of NURBS curve [15], s-shaped speed curve is shown in Figure 1.

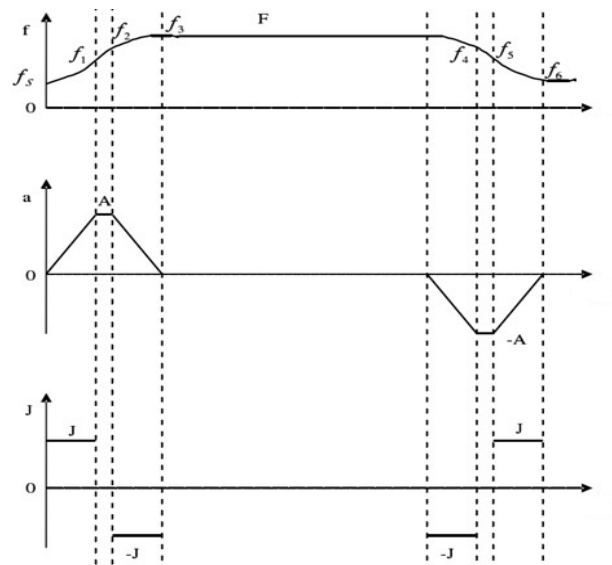


Figure 2. S-shaped speed curve

By integrating the acceleration and jerk shown in Figure 1, feed rate in each stage of S-shaped curve could be obtained as follows

$$f(t) = \begin{cases} f_s + \frac{1}{2}Jt^2 & 0 \leq t \leq T_1 \\ f_1 + At & T_1 \leq t \leq T_2 \\ f_2 + At - \frac{1}{2}Jt^2 & T_2 \leq t \leq T_3 \\ f_3 & T_3 \leq t \leq T_4 \\ f_4 - \frac{1}{2}Jt^2 & T_4 \leq t \leq T_5 \\ f_5 - At & T_5 \leq t \leq T_6 \\ f_6 - At + \frac{1}{2}Jt^2 & T_6 \leq t \leq T_7 \end{cases}$$

Where  $f_s$  is the initial speed.

$$f_1 = f_s + \frac{1}{2}JT_1^2, f_2 = f_1 + AT_2, f_3 = f_2 + AT_3 - \frac{1}{2}JT_3^2, f_4 = f_3, f_5 = f_4 - \frac{1}{2}JT_5^2, f_6 = f_5 - AT_6.$$

Where  $T_1, T_2, T_3$  are the corresponding length of time from the acceleration increasing phase, constant acceleration phase and acceleration decreasing phase, or the deceleration increasing phase, constant deceleration phase, the deceleration decreasing phase, respectively.

Based on the algorithm above, unconstrained feed step  $\Delta L_{i0}$  generated by the S-shaped curve can be calculated in each interpolation cycle, the S-shaped curve of the unconstrained feed step.

#### 4.2. Look-Ahead Method

Since the introduction of adaptive algorithm, the acceleration at the curve section where the curvature is very big would surpass the permitted value  $A_{\max}$ , and then, the shock and impact of the machine are occurred, the machining quality and accuracy are deteriorated and reduce the service life of the machine is reduced. Therefore, feed rate need to be processed with acceleration-deceleration control. In this paper, the feed rate calculated by the S type adaptive deceleration interpolation algorithm is handled with the Look-ahead method. These problems can be resolved by the following algorithm called look-ahead method. Through back and re-interpolation strategy, this paper predicts speed mutation points in advance, and then,

the federate which is derived from s-shaped control mode could be modified in real-time and to guarantee, and the acceleration would be changed within the permitted range[16].

Let the maximum granted acceleration be  $A_{max}$ , and the corresponding feed rate be  $V_{max}$  on condition actual deceleration be equal to  $A_{max}$ , then the decelerating distance  $S_{min}$  from  $V_{max}$  to  $V = 0$  is  $S_{min} = V_{max}^2 / 2A_{max}$ . The specific look-ahead process includes the following steps:

- a) Based on the premise of the length of the curve  $S \geq S_{min}$ , this paper interpolates the look-ahead curve section, and then, revises the speed of end point A of the curve section.
- b) Supposing the speed of point A is  $V_i$ , if  $|V_i - V_{i-1}| > T \times A_{max}$  and  $V_i - V_{i-1} > 0$ , the interpolation process is in acceleration phase, if  $V_i - V_{i-1} < 0$ , go to step c).
- c) Supposing the current speed  $V_i$  is the starting speed, go back with the speed increment  $T \times A_{max}$  until arrive the interpolation point B. The absolute value of difference speed between point B and its previous interpolation points does not exceed the value  $T \times A_{max}$ .
- d) Regarded point B as the start point, this paper re-interpolates the curve segment from point B to point A with the speed increment  $A_{max}$ . And it uses new interpolation speed to take the place of the previously stored speed, thus it completes the re-planning of the feed rate.
- e) Exit the look-ahead processing module.

**4.3. The Role of S-shaped Acceleration-Deceleration with the Look-Ahead Method**

In conclusion, handled with the Look-ahead method, the feed acceleration can be changed in the machine allowed range, and this can improve the inadequacies produced by the adaptive algorithm such as mutation of feed rate and the shock of machine, so as to improve the machining accuracy of parts. The figure below shows the flow char of the whole NURBS curve real-time interpolation:

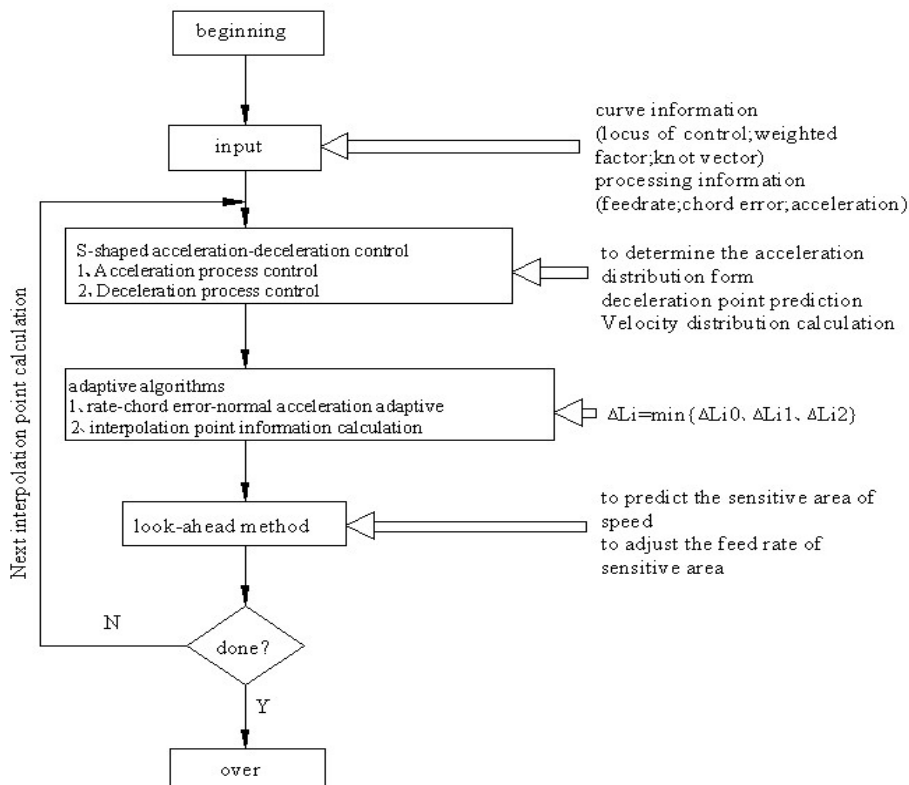


Figure 3. NURBS curve real-time interpolation of flow chart

## 5. Machining Simulation

### 5.1. MATLAB Simulation

In order to authenticate the proposed method, the whole interpolation process is completed by MATLAB software with the following parameters: the maximum feed rate is  $18m/min$ ; the starting feed rate is  $0m/min$ . The maximum normal acceleration is  $a_{N\max} = 0.5g$ . The maximum tangent acceleration is  $A_{\max} = 2400mm/s^2$ ; interpolation cycle is  $T = 1ms$ . The maximum contour error is  $h_{\max} = 1\mu m$ . The maximum interpolation deviation is  $\delta_{\max} = 0.001$ . The maximum jerk is  $J = 48000mm/s^3$ , respectively.

This paper takes the three times NURBS curves mentioned in Literature [17] as the analytical object to have a simulation study, the corresponding parameters are expressed as follows:

$$d_i: (10,10,0),(20,10,0),(12,8,0),(10,20,0),(8,8,0), (0,20,0),(10,0,0)$$

$$U_i: [0 \ 0 \ 0 \ 0 \ 0.25 \ 0.5 \ 0.75 \ 1 \ 1 \ 1 \ 1]$$

$$W: [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1]$$

The trident curve expressed by NURBS and created by using MATLAB software is shown in figure 4:

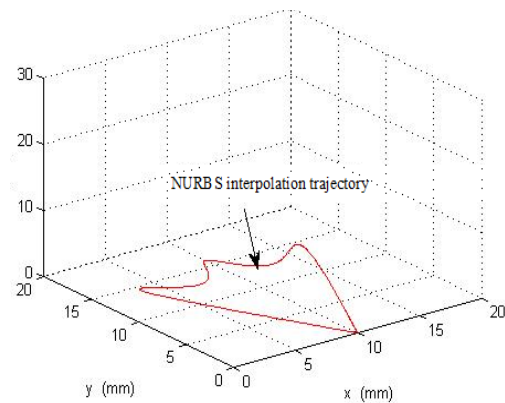
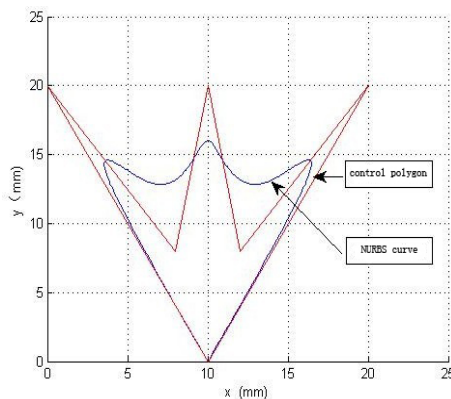


Figure 4. NURBS expresses the trident curve      Figure 5. NURBS curve interpolation trajectory

Using MATLAB software, the simulation interpolation trajectory can be obtained and shown in Figure 5:

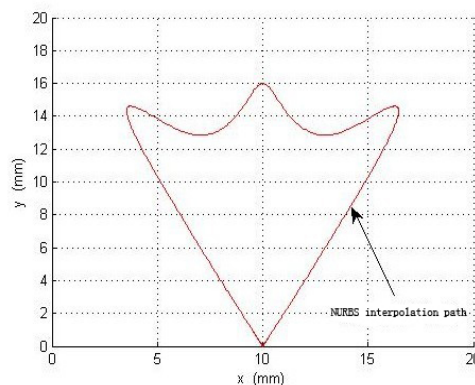


Figure 6. NURBS interpolation trajectory x-y to projection diagram

Through the interpolation simulation, interpolation feed speed curve, contour error curve, interpolation step deviation curve and feed acceleration curve are shown in Figure 7, Figure 8, Figure 9, Figure 10, respectively.

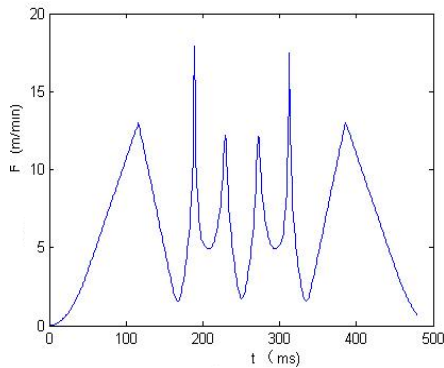


Figure 7. Interpolation feed speed

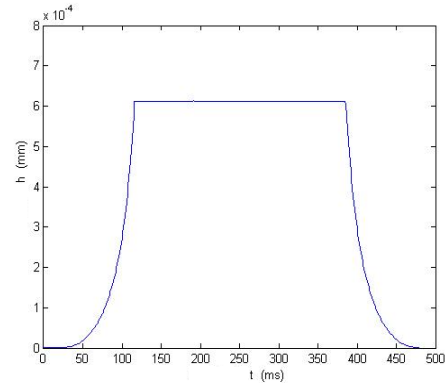


Figure 8. Chord error

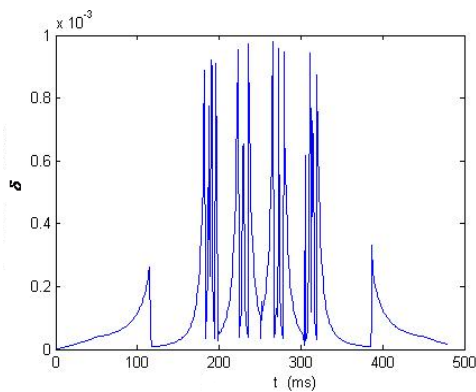


Figure 9. Interpolation step deviation

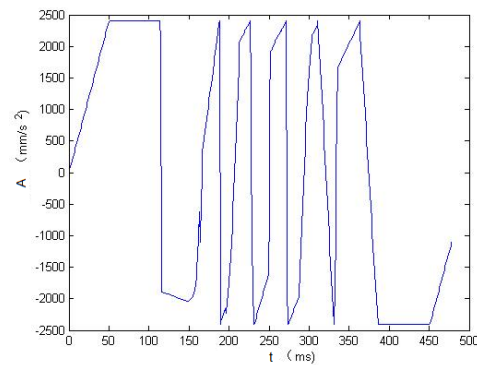


Figure 10. Feed acceleration

Through the figures above, the range of interpolation results are shown in Table 1.

Table 1. NURBS interpolation results analysis

	$h$ (mm)	$\delta$	$F$ (m/min)	$a$ (mm/s <sup>2</sup> )
max	$3.1493 \times 10^{-13}$	$1.574 \times 10^{-7}$	$4.8000 \times 10^{-5}$	2400
min	$6.1359 \times 10^{-4}$	$9.7969 \times 10^{-4}$	17.916	-2400

From Table 1 and Figure 7, it can be seen that the results are consistent with the theoretical method completely. Meanwhile, the feed rate is adaptively adjusted at curve section where the variation ratio of curvature is very big to guarantee high accuracy and fine surface quality. From Figure 8, the contour error changes with the variation of curvature, but the error is always within the permitted range.

In summary, through the simulation by using MATLAB software, the whole interpolation trajectory can be obtained, and the machining accuracy and federate can be guaranteed effectively and the validity of the algorithm is demonstrated.



## 5.2. Actual Machining

Considering the real-time nature of the actual processing of the algorithm, this paper machines the NURBS curve above by using OPEN CNC platform and VC to verify the real-time and feasibility of the algorithm.

Figure 11 and Figure 12 show the interface map and the actual processing of the development of the VC curve.

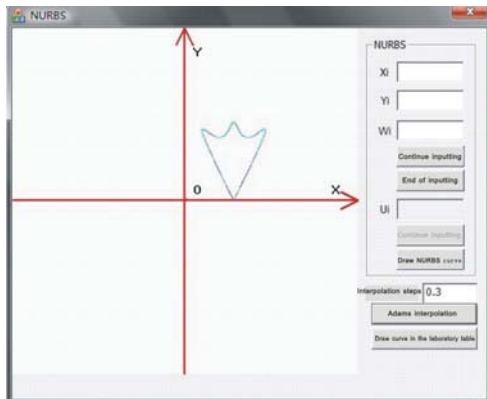


Figure 11. VC ++ interface

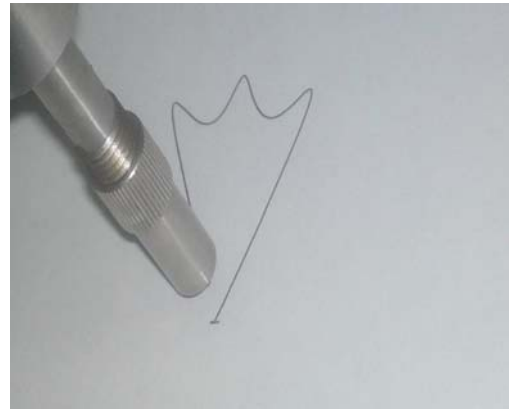


Figure 12. The actual processing path

From the actual processing above Figure 11, the feasibility of the algorithm can be proved. Testing the interpolation procedure can be found that the maximum interpolation time through which one interpolation process could be completed is 0.057ms and the minimum interpolation time is 0.005ms. This time is far less than the interpolation cycle specified by the system. The data sampling period of the current mainstream servo system is usually 1 to 2ms, the time used in every CNC system interpolation can be exceed 30% to 40% of the interpolation sampling period, thus the interpolation algorithm adopted in this paper can meet the requirements of real-time. That should be noted here is the running time of interpolation algorithm is related to the frequency of CPU, therefore, different frequency may affect the instantaneity of the system. So, in the actual processing, it should be chosen according to the circumstances.

In a word, the interpolation algorithm presented in this paper could satisfy the correctness, rationality and real-time requirements which can be used for actual processing.

## 6. Conclusion

With the proposed algorithm, the machining accuracy and speed are guaranteed, and the feed rate is adaptively adjusted according to the curvature to makes machining motion quite smooth. Through simulation analysis and examples of processing, the interpolation algorithm can not only meet the requirements of speed and accuracy, and also meet the real-time processing requirements.

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