TELKOMNIKA, Vol. 11, No. 5, May 2013, pp. 2454 ~ 2459
ISSN: 2302-4046

Study on the Structrual Parameter Coupling of Articulated Arm Coordinate Measuring Machines

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Abstract

The articulated arm coordinate measuring machine (AACMM) is a kind of new coordinate measuring device based on non-Cartesian system. The measuring accuracy of the AACMM can be effectively improved by parameter identification. However, some of the structural parameters are coupling (linearly related), so the structural parameters usually cannot be identified correctly. The Jacobian matrix was obtained through differential transformation of the kinematic model of a 6-DOF AACMM. The rank calculation results show that the Jacobian matrix is not full rank, which means some structural parameters are linearly related. Singular value decomposition and the elementary row transform of the Jacobian matrix were carried out to determine the linearly related structural parameters. And two pairs of structural parameters were found to be linearly related, which can't be identified at the same time. Finally, the linear correlation was applied in the structural parameters identification, and the results show that the linear correlation can make the identification obtain correct structural parameters.

Keywords*: articulated arm coordinate measuring machine, structural parameter coupling, kinematic model, parameter identification, Jacobian matrix*

1. Introduction

The AACMM is a multi-DOF (typically 6-DOF) and non-Cartesian coordinate measuring machine (CMM), which is modeled according to the structure of human joints: a series of linkages connected by rotating joints. Comparing with traditional CMMs the AACMM has the features of small size, light weight, large measurement range, flexible and can be applied in industrial site [1, 2]. With these unique advantages the AACMM has been applied in the field of mold design, product quality online testing, equipment maintenance and assembly [3].

The accuracy of structural parameters is the main influencing factor to the measurement accuracy of AACMMs [4], the structural parameter identification is one of the main measures to improve the accuracy of AACMMs. Calibration is an integrated process of four steps including modeling, measurement, parameter identification and error compensation [5, 6]. Selection of the appropriate kinematics model and the calibration model is the premise of calibration of AACMM, and processing the data of calibration to realize structural parameter identification so as to achieve the calibration of AACMMs. In parameter identification linearly related structrual parameters can resulting Jacobian matrix to be singular, and the solution is not the required structrual parameters we need [7-9]. So the linearly related structrual parameters must be determined before identification.

2. The Kinematic Model

To study the relationship between the coordinate of probe and rotary angle of every joint Denavit-Hartenberg (D-H) method was used to model the measuring machine coordinate system, as shown in Figure 1 and Figure 2. According to D-H method the coordinate transformation matrix between the adjacent coordinate systems is shown in Eq. 1 and the nominal structrual parameters of the AACMM is shown in Table 1.

Received January 17, 2013; Revised March 9, 2013; Accepted March 18, 2013

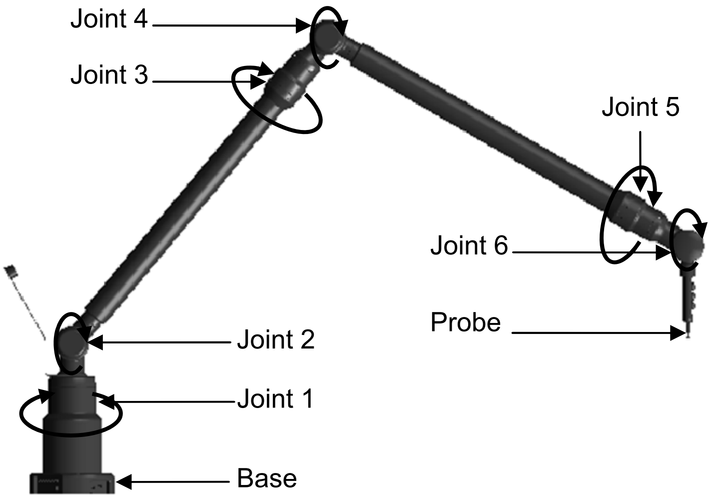

Figure. 1 The structure of the AACMM

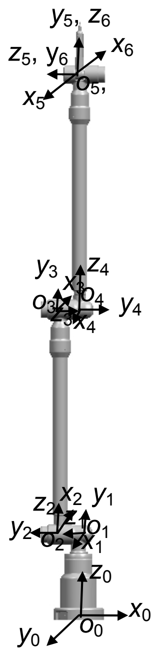

Figure. 2 The coordinate systems of the AACMM

$$T_{i-1,i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Where d_i is joint offset, a_i is link length, α_i is twist angle, θ_i is joint angle

Table 1. The nominal value of the structural parameters of the AACMM

Linkage No.	a_i[mm]	d_i[mm]	θ_i[°]	α_i[°]	l[mm]
1	0	300	θ_1	90	183.5
2	0	65.5	θ_2	90	
3	0	792.7	θ_3	90	
4	0	65.5	θ_4	90	
5	0	883.2	θ_5	90	
6	0	0	θ_6	90	

The coordinates of the probe in the coordinate of $\{x_6y_6z_6\}$ are (l_x,l_y,l). Because the probe is equivalent to the axis of z_6 of translational transformation in $\{x_6y_6z_6\}$, its coordinates in $\{x_6y_6z_6\}$ should be (0,0,l) and its coordinates in $\{x_0y_0z_0\}$ can be calculated through Eq. 2.

$$P = T_{0,1}\cdot T_{1,2}\cdot T_{2,3}\cdot T_{3,4}\cdot T_{4,5}\cdot T_{5,6}\cdot[0,0,l,1]^T \quad (2)$$

3. Analysis of Structrual Parameter Coupling

3.1. Error Model of the AACMM

In practical applications, due to the presence of manufacturing and assembling error, the change of environmental temperature and force-deformation the structural parameters will generate errors. Δa_i and Δd_i are the linkage length errors generated by manufacturing and assembly. $\Delta\alpha_j$ is the angle error produced by adjacent axis parallelism and perpendicularity. $\Delta\theta_j$ is the zero offset error generated by the angle of encoder zero and nominal model joint rotation zero misalignment in the assembly process. Considering each coordinate system has four structural parameters and the probe length there are totally 25 structural parameter errors should be calibrated.

When we specify the probe of AACMM moves to the position *P*, in fact because of these structural parameter errors the probe will move to the actual position *R*, the position errors will be:

$$\Delta P = R - P \tag{3}$$

If these structural parameters are small enough we can use differential kinematics model approximation instead of error equation [9, 10]. The probe position error of the AACMM by total differential of the kinematics can be written as Eq. 4.

$$\Delta P = \sum_{i=1}^{6} \frac{\partial P}{\partial \theta_i}\Delta\theta_i + \sum_{i=0}^{5} \frac{\partial P}{\partial \alpha_i}\Delta\alpha_i + \sum_{i=0}^{5} \frac{\partial P}{\partial a_i}\Delta a_i + \sum_{i=1}^{6} \frac{\partial P}{\partial d_i}\Delta d_i + \frac{\partial P}{\partial l}\Delta l \tag{4}$$

Eq. 4 is the error model of the AACMM.

3.2. Analysis of Structrual Parameter Coupling

Eq. 4 is written in matrix form as shown in Eq. 5.

$$\mathbf{\Delta P} = \mathbf{J}\mathbf{\Delta Q} \tag{5}$$

Where Δ**P** is a matrix of 3×1, Δ**Q** is a matrix of 25×1, the Jacobin matrix **J** is a matrix of 3×25. To solve the value of the 25 structural parameters' error in Δ**Q**, N (≥ 9) measuring points must be selected to constitute over-determined equations. However, in practical operation there are linearly related structural parameters which can make Jacobian matrix **J** to be singular. And the solution obtained from the equations is not the required value we need. Therefore the linearly related structural parameters must be found out and eliminated from the identification list. In matrix theory linearly related row can be found by singular value decomposition elementary row transformation of the orthogonal matrix decomposition [11, 12]. Premultiplying $\mathbf{J}^T$ for both sides of Eq.5 can obtain Eq. 6.

$$[\mathbf{J}^T\mathbf{J}]\mathbf{\Delta Q} = \mathbf{J}^T\mathbf{\Delta P} \tag{6}$$

Let **H**= [$\mathbf{J}^T \cdot \mathbf{J}$], then:

$$\mathbf{H} = \mathbf{U}\begin{bmatrix} \mathbf{S} & 0 \\ 0 & 0 \end{bmatrix}\mathbf{V}^T \tag{7}$$

Where **U** and **V** are orthogonal matrix of 25×25, **S**= diag ($\sigma_1, \sigma_2, \ldots, \sigma_r$)($r \le 25$), *r* is the rank of the matrix **H** and the Jacobian matrix **J**. So the number of linearly related parameters is 25-*r*. From Eq. 7 and Eq. 6 we can obtain Eq. 8.

$$\mathbf{V}^T\mathbf{\Delta Q} = \begin{bmatrix} \mathbf{S}^{-1} & 0 \\ 0 & 0 \end{bmatrix}\mathbf{U}^{-1}\mathbf{J}^T\Delta P \tag{8}$$

H is a symmetric matrix, $\mathbf{V}^T = \mathbf{U}^{-1}$ and **V** is rotation matrixes, so $\mathbf{V}^T \cdot \Delta\mathbf{Q}$ is equivalent to Δ**Q**. The linearly related structural parameters in Δ**Q** can be found out by elementary row transformation of the last 4 lines, as shown in Eq. 9.

$$\mathbf{\Delta P} = \mathbf{J}1\mathbf{\Delta Q}1 \tag{9}$$

Where **J**1 is a matrix of (3×*N*)×*r*, Δ**Q**1 is a matrix of *r*×1, Δ**P** is the matrix of (3×*N*)×1.

The rank of **J** is 23, thus there are two linearly related structural parameters. We can obtain linearly related parameters by carrying out singular value decomposition and the elementary row transform.

$$\Delta a_6 = l\Delta\theta_6 \quad (10)$$

$$\Delta d_6 = l\Delta\alpha_6 \quad (11)$$

As shown in Eq. 10 and 11 a_6 is linearly related with θ_6 and d_6 is linearly related with α_6.

4. Test of Structrual Parameter Coupling in Identification

We assume there are some errors in the structural parameters, as shown in Table 2. These errors can result in measuring error of AACMMs.

Table 2. The set errors of the structural parameters

Linkage No.	Δa_i[mm]	Δd_i[mm]	$\Delta\theta_i$[°]	$\Delta\alpha_i$[°]	Δl[mm]
1	0.10	-0.05	0.0014	0.0010	-0.03mm
2	-0.08	0.11	-0.0023	0.0014	
3	-0.12	0.06	0.0017	-0.0012	
4	0.03	0.13	0.0010	0.0017	
5	0.07	-0.04	-0.0014	0.0010	
6	0	0	0.0010	-0.0010	

To identify the errors of the structural parameters by the method proposed in this paper, we need ten groups of joint angles which were given in Table 3.

Table 3. The joint angles used in identification

No.	1	2	3	4	5	6	7	8	9	10
θ_1[°]	0	180	180	90	90	90	0	90	120	0
θ_2[°]	180	120	90	180	180	180	150	-90	180	-90
θ_3[°]	0	0	90	90	90	-90	-90	0	-90	-180
θ_4[°]	180	180	90	90	180	90	90	-90	120	-90
θ_5[°]	0	0	0	90	90	-90	90	0	90	0
θ_6[°]	180	180	180	0	90	90	-90	-90	60	-90

As a_6 is linearly related with θ_6 and d_6 is linearly related with α_6, the errors of a_6 and d_6 can be compensated by θ_6 and α_6. In the parameter identification the errors of a_6 and d_6 are set to be 0. With the data in Table 3 the least squares solution of Eq. 9 was reached, as shown in Table 4.

Table 4. The identified errors of the structural parameters

Linkage No.	Δa_i[mm]	Δd_i[mm]	$\Delta\theta_i$[°]	$\Delta\alpha_i$[°]	Δl[mm]
1	0.100	-0.052	0.0014	0.0010	-0.035mm
2	-0.078	0.098	-0.0023	0.0014	
3	-0.126	0.060	0.0017	-0.0012	
4	0.045	0.131	0.0010	0.0017	
5	0.068	-0.046	-0.0014	0.0010	
6	0	0	0.0010	-0.0010	

Table 4 and Table 2 shows that the identified errors are all nearly equal to the set errors, the mean and the maximum difference of Δa_i, Δd_i and Δl is 0.005mm and 0.015mm, the difference of $\Delta\theta_i$ and $\Delta\alpha_i$ is 0°.

Structrual parameter identification can improve the measument accuracy of AACMMs. Table 5 shows the measurement errors before and after identification of the AACMM.

Table 5. Measurement errors before and after identification of the AACMM

No.	Error Before Identification[mm]	Error After Identification[mm]
1	(5.457, -2.687, -0.086)	(-0.006, -0.011, 0.014)
2	(-2.751, -0.498, -4.704)	(-0.007, 0.010, 0.013)
3	(-4.434, -2.144, -6.955)	(-0.016, 0.022, 0.005)
4	(-0.144, 5.075, -5.824)	(0.013, -0.007, -0.03)
5	(-1.276, 6.517, 1.256)	(0.012, 0.002, 0.011)
6	(0.455, -2.055, 0.196)	(-0.007, -0.001, -0.002)
7	(-1.900, -0.198, 1.025)	(-0.002, 0.006, -0.001)
8	(1.008, 3.103, 1.239)	(0.002, -0.008, -0.006)
9	(-0.284, -0.946, 0.610)	(-0.007, -0.002, -0.001)
10	(-0.962, 0.783, 2.178)	(-0.003, 0.007, -0.002)

Table 5 shows that before structrual parameter identification the maximum error is 6.955mm while ater identification the maximum error is 0.022mm. So the the measument accuracy of the AACMM was improved greatly.

4. Conclusion

The kinematic model and coordinate systems based on D-H model were established. The rank calculation results show that the Jacobian matrix is not full rank, which means some structural parameters are linearly related. Singular value decomposition and the elementary row transform of the Jacobian matrix were carried out to determine the linearly related structural parameters. And that a_6 is linearly related with θ_6 and d_6 is linearly related with α_6 were found to be linearly related, which can't be identified at the same time. Finally, the linear correlation was applied in the structural parameters identification, and the results show that after identificaiton the position error of the AACMM was improved greatly.

Acknowledgements

This work was financially supported by the Yunnan Provincial Natural Science Foundation of China (Grant No.2011FB028).

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