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Delay-Dependent Observers for Uncertain Nonlinear Time-Delay Systems

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Abstract

This paper is concerned with the observer design problem for a class of discrete-time uncertain nonlinear systems with time-varying delay. The nonlinearities are assumed to satisfy global Lipschitz conditions which appear in both the state and measurement equations. The uncertainties are assumed to be time-varying but norm-bounded. Two Luenberger-like observers are proposed. One is delay observer and the other is delay-free observer. The delay observer which has an internal time delay is applicable when the time delay is known. The delay-free observer which does not use delayed information is especially applicable when the time delay is not known explicitly. Delay-dependent conditions for the existences of these two observers are derived based on Lyapunpv functional approach. Based on these conditions, the observer gains are obtained using the cone complementarity linearization algorithm. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

Keywords*: robust observer, Lipchitz nonlinear systems, uncertain systems, delay-dependent, linear matrix inequality*

1. Introduction

The theory of state observers for linear systems has been receiving many researchers' interests in the past a few decades. Rich literature have been published and different kind of observers have been proposed [1-4]. When uncertainties appear in the system model, a robust observer should be considered. Many results on this topic have been reported (see, e.g. [5-6], and references therein). On the other hand, time-delay is often encountered in many practical systems. The design problem of observers for time-delay systems has also been studied for many years. For example, Linear functional observers for discrete-time systems with state delays were designed and delay-dependent stability conditions were derived in [7]. Moreover, increasing attention has been paid to the robust observer design problem of linear uncertain time-delay systems. For example, Robust H_∞ observers for linear time delay systems with parameter uncertainties were considered in term of the matrix Riccati-like equations in [8].

Recently, the design problem of observers for nonlinear systems, especially Lipschitz nonlinear systems, has received considerable attentions. A new observer design method based on a new Lyapunov-Krasovskii functional was proposed in [9]. An observer design method for discrete-time non-linear systems which have Lipschitz non-linearity and delayed output was proposed in [10]. However, most of the above literature assumed that the time delay is constant, which cannot be always the case in control systems [11-12]. To the best of the authors' knowledge, the problem of designing delay-dependent observers for discrete-time uncertain Lipschitz nonlinear systems with time varying delay has not been fully investigated, which motivates this study.

In this paper, the design problem of observers for a class of discrete time delay systems with Lipschitz nonlinear perturbations and norm-bounded uncertainties is considered. Two Luenberger-like observers are proposed and new delay-dependent existence conditions for these two observers are derived. Numerical examples are provided to illustrate the validity and less conservativeness of the proposed methods.

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2. Problem Formulation and Preliminaries

Consider a discrete-time uncertain nonlinear system with time-varying delay:

$$x(k+1) = (A + \Delta A(k))x(k) + (A_d + \Delta A_d(k))x(k - \tau(k)) + Gg(x(k)) \tag{1}$$

$$x(k) = \varphi(k), \qquad -\tau_{\max} \le k \le 0 \tag{2}$$

$$y(k) = (C + \Delta C(k))x(k) + Hh(x(k)) \tag{3}$$

where $x(k) \in \mathrm{R}^n$ is the state vector, $y(k) \in \mathrm{R}^m$ is the measurement output, $\varphi(k)$ is the initial condition sequence, $\tau(k)$ is an integer representing the time-varying bounded delay and satisfies $\tau_{\min} \le \tau(k) \le \tau_{\max}$, where $\tau_{\min} \ge 0$ and $\tau_{\max} > 0$ are constant scalars representing the minimum and maximum delays, respectively. $A,\ A_d,\ C,\ G$ and H are known real constant matrices with appropriate dimensions, $\Delta A(k),\ \Delta A_d(k)$ and $\Delta C(k)$ denote parameter uncertainties and satisfy the following conditions:

$$\left[\Delta A(k) \quad \Delta A_d(k)\right] = D_1 F(k)\left[E_1 \quad E_2\right],\ \Delta C(k) = D_2 F(k) E_1,\ F^{\mathrm{T}}(k)F(k) \le I, \qquad \forall k \tag{4}$$

$g(\cdot):\mathrm{R}^n \to \mathrm{R}^{n_g}$ and $h(\cdot):\mathrm{R}^n \to \mathrm{R}^{n_h}$ are known nonlinear functions. $g(x(k))$ and $h(x(k))$ meet $g(0) = 0$ and the following global Lipschitz conditions:

$$\left\|g(x_1(k)) - g(x_2(k)\right\| \le \left\|R_g(x_1(k) - x_2(k))\right\|, \left\|h(x_1(k)) - h(x_2(k)\right\| \le \left\|R_h(x_1(k) - x_2(k))\right\| \tag{5}$$

for all $x_1,\ x_2 \in R^n$, where R_g and R_h are known constant matrices with appropriate dimensions.

The problem of interest is to design observers for the system (1)-(3) with norm-bounded uncertainties satisfying (4) and nonlinearities satisfying (5). The objectives are: (1) to design the following two different Luenberger-like observers to reconstruct the state $x(k)$ based on measurement output $y(k)$; (2) to provide an efficient procedure to compute the observer gains.

The two Luenberger-like observers have the following structures:

Delay observer: it has internal time delay, so it is applicable when the time delay is known.

$$\hat{x}(k+1) = A\hat{x}(k) + A_d\hat{x}(k - \tau(k)) + Gg(\hat{x}(k)) + L\left[y(k) - C\hat{x}(k) - Hh(\hat{x}(k))\right] \tag{6}$$

Delay-free observer: it does not have internal time delay; hence, it is especially applicable when the time delay is not known explicitly.

$$\hat{x}(k+1) = A\hat{x}(k) + Gg(\hat{x}(k)) + L\left[y(k) - C\hat{x}(k) - Hh(\hat{x}(k))\right] \tag{7}$$

3. Main results

In this section, two Luenberger-like observers are designed. Delay-dependent existence conditions for these two observers are derived.

3.1. Delay observer design

Before embarking on the delay observer design for uncertain Lipschitz nonlinear time-delay systems, results on the nominal observer design problem are presented.Consider the following nominal system:

$$x(k+1) = Ax(k) + A_d x(k - \tau(k)) + Gg(x(k)) \tag{8}$$

$$x(k) = \varphi(k), \qquad -\tau_{\max} \le k \le 0 \tag{9}$$

$$y(k) = Cx(k) + Hh(x(k)) \tag{10}$$

Let the error vector be $e(k)=x(k)-\hat{x}(k)$,From (6), 8) and (10), it can be obtained that

$$e(k+1)=(A-LC)e(k)+A_d e(k-\tau(k))+\begin{bmatrix} G & -LH \end{bmatrix}\begin{bmatrix} g(x(k))-g(\hat{x}(k)) \\ h(x(k))-h(\hat{x}(k)) \end{bmatrix} \tag{11}$$

The following theorem gives a sufficient delay-dependent stability criterion for system (11).

Theorem 1: For given $\tau_{\min}$ and $\tau_{\max}$, if there exist scalars $\varepsilon_i>0$, $\lambda_i>0$ $(i=1,\ 2)$, matrices $P>0$, $Q>0$, $S>0$, $Z>0$ and any matrices M_1, M_2, N_1 and N_2 such that

$$\Psi=\begin{bmatrix} \Psi_{11} & \Psi_{12} \\ * & \Psi_{22} \end{bmatrix}<0 \tag{12}$$

where

$$\Psi_{11}=\left[\begin{array}{c|c|c|cc} \Omega_1 & -M_1^{\mathrm{T}}+N_1 & 0 & 0 & 0 \\ \hline * & \Omega_2 & -M_2^{\mathrm{T}}+N_2 & 0 & 0 \\ \hline * & * & \Omega_3 & 0 & 0 \\ \hline * & * & * & -\mathrm{E} & 0 \\ * & * & * & * & -\tau_{\max}\Lambda \end{array}\right],\ \Psi_{12}=\begin{bmatrix} A_e^{\mathrm{T}}P & (A_e-I)^{\mathrm{T}}Z & \tau_{\max}M_1^{\mathrm{T}} & 0 \\ A_d^{\mathrm{T}}P & A_d^{\mathrm{T}}Z & \tau_{\max}N_1^{\mathrm{T}} & \tau_0 M_2^{\mathrm{T}} \\ 0 & 0 & 0 & \tau_0 N_2^{\mathrm{T}} \\ G_e^{\mathrm{T}}P & 0 & 0 & 0 \\ 0 & G_e^{\mathrm{T}}Z & 0 & 0 \end{bmatrix},$$

$\Psi_{22}=diag(-P,-Z/\tau_{\max},-\tau_{\max}Z,-\tau_0 Z)$,

$\Omega_1=-P+(\tau_0+1)Q+(\varepsilon_1+\tau_{\max}\lambda_1)R_g^{\mathrm{T}}R_g+(\varepsilon_2+\tau_{\max}\lambda_2)R_h^{\mathrm{T}}R_h+M_1+M_1^{\mathrm{T}}+S$,

$\Omega_2=-N_1^{\mathrm{T}}-N_1-Q+M_2^{\mathrm{T}}+M_2$,

$\Omega_3=-N_2^{\mathrm{T}}-N_2-S$,

$\mathrm{E}=\begin{bmatrix} \varepsilon_1 I & 0 \\ 0 & \varepsilon_2 I \end{bmatrix}$, $\Lambda=\begin{bmatrix} \lambda_1 I & 0 \\ 0 & \lambda_2 I \end{bmatrix}$, $\tau_0=\tau_{\max}-\tau_{\min}$.

then system (11) is asymptotically stable for any time-varying delay $\tau(k)$ satisfying $\tau_{\min}\le\tau(k)\le\tau_{\max}$.

Proof: Define $d(k)=e(k+1)-e(k)$ and choose a Lyapunov functional candidate as follows:

$$V(k)=V_1(k)+V_2(k)+V_3(k)+V_4(k)+V_5(k)$$

where $V_1(k)=e^{\mathrm{T}}(k)Pe(k)$, $V_2(k)=\sum_{i=k-\tau_{\max}}^{k-1}e^{\mathrm{T}}(i)Se(i)$, $V_3(k)=\sum_{i=k-\tau(k)}^{k-1}e^{\mathrm{T}}(i)Qe(i)$,

$V_4(k)=\sum_{j=-\tau_{\max}+2}^{-\tau_{\min}+1}\sum_{i=k+j-1}^{k-1}e^{\mathrm{T}}(i)Qe(i)$, $V_5(k)=\sum_{i=k-\tau_{\max}}^{k-1}(i-k+\tau_{\max}+1)d^{\mathrm{T}}(i)Zd(i)$,

then, one can obtain that

$$\begin{aligned}\Delta V_1(k)&=e^{\mathrm{T}}(k)(A_e^{\mathrm{T}}PA_e-P)e(k)+2e^{\mathrm{T}}(k)A_e^{\mathrm{T}}PA_d e(K)+e^{\mathrm{T}}(K)A_d^{\mathrm{T}}PA_d e(K))\\&\quad+2\left[A_e e(k)+A_d e(K)\right]^{\mathrm{T}}PG_e\mathrm{B}-\mathrm{B}^{\mathrm{T}}T\mathrm{B}+\mathrm{B}^{\mathrm{T}}\mathrm{E}\mathrm{B}\end{aligned} \tag{13}$$

where $T=\mathrm{E}-G_e^{\mathrm{T}}PG_e$, $\mathrm{E}=diag(\varepsilon_1 I,\varepsilon_2 I)$, $K=k-\tau(k)$, $\mathrm{B}=\xi(x(k),\hat{x}(k))$.
From (5), (13), it is easy to obtain

$$\Delta V_1(k)\le e^{\mathrm{T}}(k)\Xi e(k)+2e^{\mathrm{T}}(k)\Pi e(K)+e^{\mathrm{T}}(K)\Omega e(K) \tag{14}$$

where

$\Xi=A_e^{\mathrm{T}}PA_e-P+A_e^{\mathrm{T}}PG_eT^{-1}G_e^{\mathrm{T}}PA_e+\varepsilon_1R_g^{\mathrm{T}}R_g+\varepsilon_2R_h^{\mathrm{T}}R_h$, $K=k-\tau(k)$,

$\Pi=A_e^{\mathrm{T}}PA_d+A_e^{\mathrm{T}}PG_eT^{-1}G_e^{\mathrm{T}}PA_d$,

$\Omega=A_d^{\mathrm{T}}PA_d+A_d^{\mathrm{T}}PG_eT^{-1}G_e^{\mathrm{T}}PA_d$,

and

$$\Delta V_2(k)=e^{\mathrm{T}}(k)Se(k)-e^{\mathrm{T}}(k-\tau_{\max})Se(k-\tau_{\max}) \tag{15}$$

$$\Delta V_3(k)\le e^{\mathrm{T}}(k)Qe(k)-e^{\mathrm{T}}(k-\tau(k))Qe(k-\tau(k))+\sum_{i=k+1-\tau_{\max}}^{k-\tau_{\min}} e^{\mathrm{T}}(i)Qe(i) \tag{16}$$

$$\Delta V_4(k)=(\tau_{\max}-\tau_{\min})e^{\mathrm{T}}(k)Qe(k)-\sum_{i=k+1-\tau_{\max}}^{k-\tau_{\min}} e^{\mathrm{T}}(i)Qe(i) \tag{17}$$

$$\Delta V_5(k)=\tau_{\max}d^{\mathrm{T}}(k)Zd(k)-\sum_{i=k-\tau(k)}^{k-1} d^{\mathrm{T}}(i)Zd(i)-\sum_{i=k-\tau_{\max}}^{k-\tau(k)-1} d^{\mathrm{T}}(i)Zd(i) \tag{18}$$

It can obtained that

$$\Delta V_5(k)\le\tau_{\max}d^{\mathrm{T}}(k)Zd(k)+E_0{}^{\mathrm{T}}\Delta_1E_0+\tau_{\max}E_0{}^{\mathrm{T}}\mathrm{O}_1E_0+E_m{}^{\mathrm{T}}\Delta_2E_m+(\tau_{\max}-\tau_{\min})E_m{}^{\mathrm{T}}\mathrm{O}_2E_m \tag{19}$$

where

$\mathrm{E}_0=\begin{bmatrix} e(k) \\ e(k-\tau(k))\end{bmatrix}$, $\mathrm{E}_m=\begin{bmatrix} e(k-\tau(k)) \\ e(k-\tau_{\max})\end{bmatrix}$, $\Delta_i=\begin{bmatrix} M_i^{\mathrm{T}}+M_i & -M_i^{\mathrm{T}}+N_i \\ * & -N_i^{\mathrm{T}}-N_i\end{bmatrix}$, $\mathrm{O}_i=\begin{bmatrix} M_i^{\mathrm{T}} \\ N_i^{\mathrm{T}}\end{bmatrix}Z^{-1}\begin{bmatrix} M_i & N_i\end{bmatrix}$ $(i=1,2)$.

From (14)-(19), using Schur complements, (12) can be obtained. (12) implies $\Delta V(k)<0$, which guarantees the error state system (14) is asymptotically stable. □

Define the error vector as $e(k)=x(k)-\hat{x}(k)$. From (1), (3) and (6), it is easy to obtain

$$e(k+1)=(A-LC)e(k)+A_de(k-\tau(k))+(\Delta A(k)-L\Delta C(k))x(k)+\Delta A_d(k)x(k-\tau(k))+G_e\xi(x(k),\hat{x}(k)) \tag{20}$$

where

$G_e=\begin{bmatrix} G & -LH\end{bmatrix}$, $\xi(x(k),\hat{x}(k))=\begin{bmatrix} g(x(k))-g(\hat{x}(k)) \\ h(x(k))-h(\hat{x}(k))\end{bmatrix}$.

Let the augmented state vector be $z(k)=\begin{bmatrix} x(k) \\ e(k)\end{bmatrix}$,then

$$z(k+1)=(A_z+\Delta A_z(k))z(k)+(A_{dz}+\Delta A_{dz}(k))z(k-\tau(k))+G_z\xi_z(x(k),\hat{x}(k)) \tag{21}$$

where

$$A_z=\begin{bmatrix} A & 0 \\ 0 & A-LC\end{bmatrix},\ G_z=\begin{bmatrix} G & 0 \\ 0 & G_e\end{bmatrix},\ \xi_z(x(k),\hat{x}(k))=\begin{bmatrix} g(x(k)) \\ g(x(k))-g(\hat{x}(k)) \\ h(x(k))-h(\hat{x}(k))\end{bmatrix},\ A_{dz}=\begin{bmatrix} A_d & 0 \\ 0 & A_d\end{bmatrix},$$

$\Delta A_z(k)=\hat{D}_1F(k)\hat{E}_1$, $\Delta A_{dz}(k)=\hat{D}_2F(k)\hat{E}_2$.

The following theorem gives a sufficient delay-dependent stability condition for augmented system (21).

Theorem 2: For given $\tau_{\min}$ and $\tau_{\max}$, if there exist scalars $\varepsilon_i>0$, $\lambda_i>0$ $(i=1,2,3)$, $\delta_1>0$ and $\delta_2>0$, matrices $P>0$, $Q>0$, $S>0$, $Z>0$ and any matrices M_1, M_2, N_1 and N_2 such that

$$\Sigma=\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \Sigma_{22}\end{bmatrix}<0 \tag{22}$$

where

$$\Sigma_{11}=\begin{bmatrix} \Omega_1^{'} & -M_1^{\mathrm{T}}+N_1 & 0 & 0 \\ * & \Omega_2^{'} & -M_2^{\mathrm{T}}+N_2 & 0 \\ * & * & \Omega_3^{'} & 0 \\ * & * & * & \Omega_4^{'}\end{bmatrix},\quad \Sigma_{12}=\begin{bmatrix} A_z^{\mathrm{T}}P & (A_z-I)^{\mathrm{T}}Z & \tau_{\max}M_1^{\mathrm{T}} & 0 & 0 & 0 \\ A_{dz}^{\mathrm{T}}P & A_{dz}^{\mathrm{T}}Z & \tau_{\max}N_1^{\mathrm{T}} & \tau_0M_2^{\mathrm{T}} & 0 & 0 \\ 0 & 0 & 0 & \tau_0N_2^{\mathrm{T}} & 0 & 0 \\ G_z^{\mathrm{T}}P & 0 & 0 & 0 & 0 & 0 \\ 0 & G_z^{\mathrm{T}}Z & 0 & 0 & 0 & 0\end{bmatrix},$$

$\Omega_1^{'} = -P + \Upsilon_g + \Upsilon_h + S + (\tau_0 + 1)Q + M_1^{\mathrm{T}} + M_1 + \delta_1 \hat{E}_1^{\mathrm{T}} \hat{E}_1$,

$\Omega_2^{'} = -N_1^{\mathrm{T}} - N_1 - Q + M_2^{\mathrm{T}} + M_2 + \delta_2 \hat{E}_2^{\mathrm{T}} \hat{E}_2$, $\Omega_3^{'} = -N_2^{\mathrm{T}} - N_2 - S$,

$\Omega_4^{'} = diag(-E_z, -\tau_{\max}\lambda_1 I, -\tau_{\max}\lambda_2 I, -\tau_{\max}\lambda_3 I)$, $\mathrm{E}_z = diag(\varepsilon_1 I, \varepsilon_2 I, \varepsilon_3 I)$,

$\Sigma_{22} = \begin{bmatrix} \mathrm{X}_{11} & \mathrm{X}_{12} \\ \mathrm{X}_{21} & \mathrm{X}_{22} \end{bmatrix}$, $\mathrm{X}_{12} = \begin{bmatrix} 0 & P\hat{D}_1 & P\hat{D}_2 \\ 0 & Z\hat{D}_1 & Z\hat{D}_2 \\ 0 & 0 & 0 \end{bmatrix}$, $\mathrm{X}_{21} = \begin{bmatrix} 0 & 0 & 0 \\ Z\hat{D}_2 & Z\hat{D}_1 & 0 \\ P\hat{D}_2 & P\hat{D}_1 & 0 \end{bmatrix}$,

$\mathrm{X}_{11} = diag(-P, -Z/\tau_{\max}, -\tau_{\max} Z)$, $\mathrm{X}_{22} = diag(-\tau_0 Z, -\delta_1 I, -\delta_2 I)$, $\tau_0 = \tau_{\max} - \tau_{\min}$,

$\Upsilon_g = \begin{bmatrix} \varepsilon_1 R_g^{\mathrm{T}} R_g & 0 \\ 0 & (\varepsilon_2 + \tau_{\max}\lambda_2) R_g^{\mathrm{T}} R_g \end{bmatrix}$, $\Upsilon_h = \begin{bmatrix} \tau_{\max}\lambda_1 R_g^{\mathrm{T}} R_g & 0 \\ 0 & (\varepsilon_3 + \tau_{\max}\lambda_3) R_h^{\mathrm{T}} R_h \end{bmatrix}$.

then augmented system (21) is asymptotically stable for all time-varying delay $\tau(k)$ satisfying $\tau_{\min} \le \tau(k) \le \tau_{\max}$.

Proof: Following the same line as Theorem 1, and applying the method in [13], (22) can be obtained. □

Theorem 1 and 2 provide existence conditions of nominal and robust observers for a class of Lipschitz nonlinear time-delay systems. Considering $A_e = A - LC$ and $A_z = \begin{bmatrix} A & 0 \\ 0 & A - LC \end{bmatrix}$, condition (12) and (22) contain $L^{\mathrm{T}} P$. Therefore, these conditions are non-convex when they are used to compute the observer gains, and they cannot be solved using some convex programming tools. Therefore, a procedure to compute the observer gains will be introduced. Take Theorem 2 for example, and condition (22) can be rewritten as:

$$\hat{\Sigma} = \begin{bmatrix} \hat{\Sigma}_{11} & \hat{\Sigma}_{12} \\ * & \hat{\Sigma}_{22} \end{bmatrix} < 0 \tag{23}$$

where

$\hat{\Sigma}_{12} = \begin{bmatrix} A_z^{\mathrm{T}} & (A_z - I)^{\mathrm{T}} & \tau_{\max} M_1^{\mathrm{T}} & 0 & 0 & 0 \\ A_{dz}^{\mathrm{T}} & A_{dz}^{\mathrm{T}} & \tau_{\max} N_1^{\mathrm{T}} & \tau_0 M_2^{\mathrm{T}} & 0 & 0 \\ 0 & 0 & 0 & \tau_0 N_2^{\mathrm{T}} & 0 & 0 \\ G_z^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ 0 & G_z^{\mathrm{T}} & 0 & 0 & 0 & 0 \end{bmatrix}$, $\hat{\Sigma}_{22} = \begin{bmatrix} Y_{11} & Y_{12} \\ * & Y_{22} \end{bmatrix}$, $Y_{12} = \begin{bmatrix} 0 & 0 & 0 \\ \hat{D}_2 & \hat{D}_1 & 0 \\ \hat{D}_2 & \hat{D}_1 & 0 \end{bmatrix}$,

$\hat{\Sigma}_{11} = \Sigma_{11}$, $Y_{11} = diag(-P^{-1}, -Z^{-1}/\tau_{\max}, -\tau_{\max} Z)$, $Y_{22} = diag(-(\tau_{\max} - \tau_{\min})Z, -\delta_1 I, -\delta_2 I)$, $\tau_0 = \tau_{\max} - \tau_{\min}$.

Condition (23) does not contain the product of L^{T} and P but contains P^{-1} and Z^{-1}, so it is still non-convex. However, it can be converted to a cone complementarity problem similar to [14] and using an iterative algorithm [15] a suboptimal feasible solution can be obtained.

3.2. Delay-free observer design

The above delay observer contains delayed states, so it cannot be applicable when the delay is not available. The delay-free observer (10) does not need the exact value of the delay, so it is applicable especially when the delay is not known explicitly.

Define the error vector as $e(k) = x(k) - \hat{x}(k)$. From (1), (3) and (7), the error dynamics is described by:

$$e(k+1) = (A - LC)e(k) + (A_d + \Delta A_d(k))e(k - \tau(k)) + (\Delta A(k) - L\Delta C(k))x(k) + G_e \xi(x(k), \hat{x}(k)) \tag{24}$$

Let the augmented vector be $\theta(k) = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}$. Combining (1), (3) with (24) yields

$$\theta(k+1)=(A_\theta+\Delta A_\theta(k))\theta(k)+(A_{d\theta}+\Delta A_{d\theta}(k))\theta(k-\tau(k))+G_\theta\xi_\theta(x(k),\hat{x}(k))$$

(25) where $A_\theta=\begin{bmatrix}A & 0\\ 0 & A-LC\end{bmatrix}$, $G_\theta=\begin{bmatrix}G & 0\\ 0 & G_e\end{bmatrix}$, $A_{d\theta}=\begin{bmatrix}A_d & 0\\ A_d & 0\end{bmatrix}$, $\xi_\theta(x(k),\hat{x}(k))=\begin{bmatrix}g(x(k))\\ g(x(k))-g(\hat{x}(k))\\ h(x(k))-h(\hat{x}(k))\end{bmatrix}$,

$\Delta A_\theta(k)=\bar{D}_1F(k)\bar{E}_1$, $\Delta A_{d\theta}(k)=\bar{D}_2F(k)\bar{E}_2$.

The following theorem gives a sufficient stability condition for augmented system (25).

Theorem 3: For given $\tau_{\min}$ and $\tau_{\max}$, if there exist scalars $\varepsilon_i>0$, $\lambda_i>0$ $(i=1,2,3)$, $\delta_1>0$ and $\delta_2>0$, matrices $P>0$, $Q>0$, $S>0$, $Z>0$ and any matrices M_1, M_2, N_1 and N_2 such that

$$\Phi=\begin{bmatrix}\Phi_{11} & \Phi_{12}\\ * & \Phi_{22}\end{bmatrix}<0 \tag{26}$$

where

$$\Phi_{11}=\left[\begin{array}{c|c|c|cc}\Omega_1^\Delta & -M_1^{\mathrm{T}}+N_1 & 0 & 0 & 0\\ \hline * & \Omega_2^\Delta & -M_2^{\mathrm{T}}+N_2 & 0 & 0\\ \hline * & * & \Omega_3^\Delta & 0 & 0\\ \hline * & * & * & -\mathrm{E}_\theta & 0\\ * & * & * & * & -\tau_{\max}\Lambda_\theta\end{array}\right],$$

$\Omega_1^\Delta=-P+\Upsilon_g+\Upsilon_h+S+\delta_1\bar{E}_1^{\mathrm{T}}\bar{E}_1+(\tau_0+1)Q+M_1^{\mathrm{T}}+M_1$,

$\Omega_2^\Delta=-N_1^{\mathrm{T}}-N_1-Q+M_2^{\mathrm{T}}+M_2+\delta_2\bar{E}_2^{\mathrm{T}}\bar{E}_2$,

$\Omega_3^\Delta=-N_2^{\mathrm{T}}-N_2-S$, $\Lambda_\theta=diag(\lambda_1I,\lambda_2I,\lambda_3I)$, $\mathrm{E}_\theta=diag(\varepsilon_1I,\varepsilon_2I,\varepsilon_3I)$,

$$\Phi_{12}=\begin{bmatrix}A_\theta^{\mathrm{T}}P & (A_\theta-I)^{\mathrm{T}}Z & \tau_{\max}M_1^{\mathrm{T}} & 0 & 0 & 0\\ A_{d\theta}^{\mathrm{T}}P & A_{d\theta}^{\mathrm{T}}Z & \tau_{\max}N_1^{\mathrm{T}} & \tau_0M_2^{\mathrm{T}} & 0 & 0\\ 0 & 0 & 0 & \tau_0N_2^{\mathrm{T}} & 0 & 0\\ G_\theta^{\mathrm{T}}P & 0 & 0 & 0 & 0 & 0\\ 0 & G_\theta^{\mathrm{T}}Z & 0 & 0 & 0 & 0\end{bmatrix},\ \Phi_{22}=\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix},$$

$Z_{11}=diag(-P,-Z/\tau_{\max},-\tau_{\max}Z)$, $Z_{22}=diag(-\tau_0Z,-\delta_1I,-\delta_2I)$, $\tau_0=\tau_{\max}-\tau_{\min}$,

$$Z_{12}=\begin{bmatrix}0 & P\bar{D}_1 & P\bar{D}_2\\ 0 & Z\bar{D}_1 & Z\bar{D}_2\\ 0 & 0 & 0\end{bmatrix},\ Z_{21}=\begin{bmatrix}0 & 0 & 0\\ Z\bar{D}_2 & Z\bar{D}_1 & 0\\ P\bar{D}_2 & P\bar{D}_1 & 0\end{bmatrix},$$

$$\Upsilon_g=\begin{bmatrix}\varepsilon_1R_g^{\mathrm{T}}R_g & 0\\ 0 & (\varepsilon_2+\tau_{\max}\lambda_2)R_g^{\mathrm{T}}R_g\end{bmatrix},\ \Upsilon_h=\begin{bmatrix}\tau_{\max}\lambda_1R_g^{\mathrm{T}}R_g & 0\\ 0 & (\varepsilon_3+\tau_{\max}\lambda_3)R_h^{\mathrm{T}}R_h\end{bmatrix}.$$

then augmented system (25) is asymptotically stable for any time-varying delay $\tau(k)$ satisfying $\tau_{\min}\le\tau(k)\le\tau_{\max}$.

Proof: The proof is similar to that of Theorem 2; hence, it is omitted here. □

4. Numerical examples

Consider the following uncertain discrete-time Lipschitz nonlinear system with

$$A=\begin{bmatrix}-0.08 & 0.3 & 0.1\\ 0.2 & -0.1 & 0.2\\ -0.2 & 0.2 & -0.1\end{bmatrix},\quad A_d=\begin{bmatrix}0.3 & 0.1 & 0\\ -0.1 & 0 & -0.05\\ 0.3 & 0.05 & 0.05\end{bmatrix},\quad G=\begin{bmatrix}0.2 & 0.1 & -0.2\\ 0.1 & 0.1 & 0\\ 0.05 & 0.2 & -0.1\end{bmatrix},\quad D_1=\begin{bmatrix}0.05 & 0\\ 0.2 & 0.1\\ 0.1 & 0.1\end{bmatrix},$$

$$C=\begin{bmatrix}0.1 & 0.2 & 0.1\\ 0.1 & 0.2 & 0.1\end{bmatrix},\ E_1=\begin{bmatrix}0.1 & 0.1 & 0.1\\ 0.1 & 0.1 & 0.2\end{bmatrix},\ E_2=\begin{bmatrix}0.1 & 0.2 & 0.2\\ 0 & 0.1 & 0.1\end{bmatrix},\ D_2=\begin{bmatrix}0 & 0.2\\ 0.1 & 0.1\end{bmatrix},\ H=\begin{bmatrix}0.1 & 0.1 & 0.1\\ 0.2 & 0.1 & 0.2\end{bmatrix},$$

$$F(k)=\begin{bmatrix}\sin(k) & 0\\ 0 & \sin(k)\end{bmatrix}.$$

Assume $0 \le \tau(k) \le 4$. Using results in this paper, the delay observer and the delay-free observer are respectively obtained as

$$L_1 = \begin{bmatrix} 0.0420 & 0.3674 \\ -0.1429 & 1.4533 \\ -1.6470 & 1.2910 \end{bmatrix},\ L_2 = \begin{bmatrix} -0.2497 & 0.0966 \\ 1.0011 & -0.4147 \\ -0.9737 & 0.5439 \end{bmatrix}.$$

Assume the initial value $x(0) = [1 \quad 2 \quad -1]^{\mathrm{T}}$ and denote the error state $e_i = x_i - \hat{x}_i \ \ (i = 1, 2, 3)$. For the above two observer gains, the initial responses of error dynamics are shown in Figure 1 and 2, respectively. The simulation results show that the methods proposed in this paper are valid and effective.

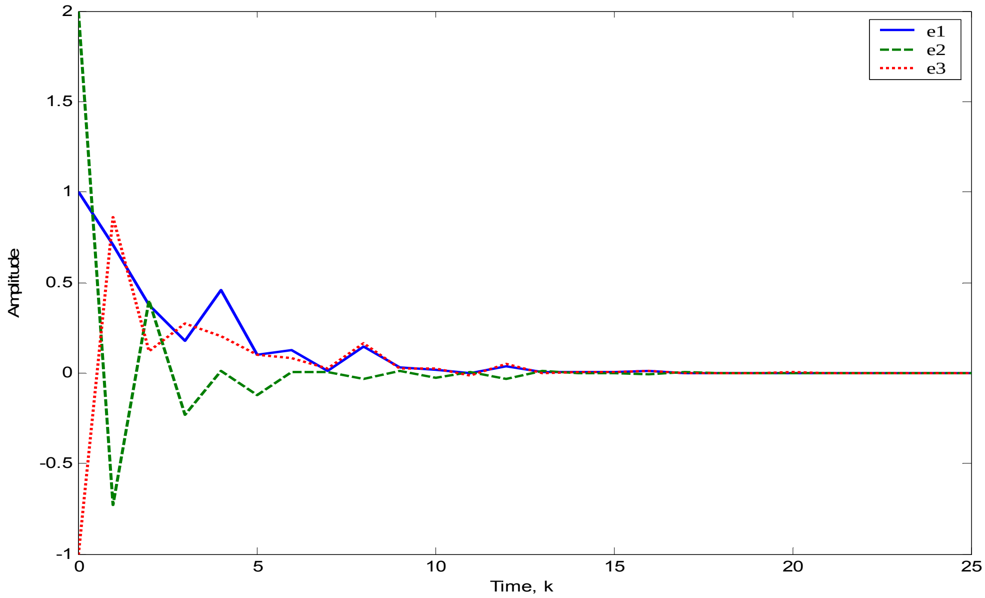

Figure 1. Estimation error of delay observer

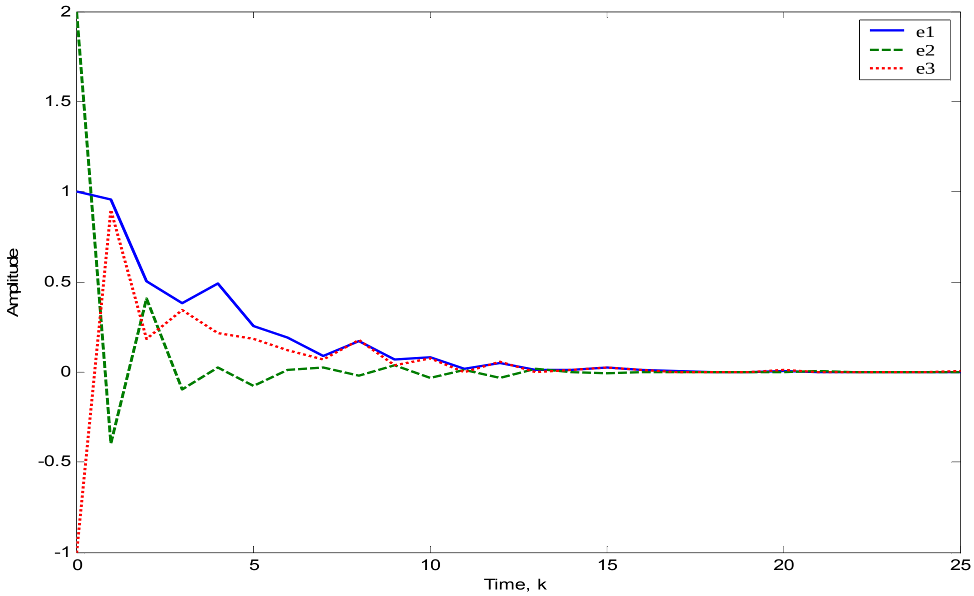

Figure 2. Estimation error of delay-free observer

5. Conclusion

In this paper, the design problem of observers for a class of nonlinear uncertain time-delay systems has been studied. The nonlinearities are assumed to satisfy the global Lipschitz condition and the uncertainties are assumed to be time-varying but norm-bounded. Delay and delay-free observers have been designed for this class of nonlinear systems. Delay-dependent existence conditions for these two observers have been derived. Since the obtained conditions are non-convex, a cone complementarity linearization algorithm has been presented to calculate the observer gains. A numerical example has illustrated the effectiveness of the proposed methods.

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