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Research of the Nonlinear System Identification Based on the Volterra RLS Adaptive Filter Algorithm

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Abstract

This paper mainly research a kind of nonlinear system modeling method based on the Volterra series RLS adaptive filter algorithm. Through the modeling second order Volterra nonlinear system, and adapting Volterra RLS algorithm to apply in the first order and second order item, under the gaussian noise condition, iterations times is not more than fifty times, the algorithm can realize convergence, which prove the algorithm accuracy. Volterra RLS adaptive filter algorithm can be effectively applied into the mechanical vibration damping and de-noising in active control, which has a broad application prospect.

Keywords*: Volterra series, adaptive filter, RLS, system identification*

1. Introduction

Adaptive filtering research began in the 1950's, Widrow and Hoff put forward the least mean square (LMS) algorithm. [1] Among the filter algorithms, recursive least square (RLS) algorithm has more robust character than the LMS algorithm [2], and is higher than the traditional adaptive linear filtering accuracy. So, along with the signal processing field development, nonlinear adaptive filter algorithm has gradually become a hot spot in the study. Among them, the Volterra adaptive filter algorithm has been successfully used in aerospace, military, etc. Signal processing or modeling has more and more reflect with its accuracy. This paper introduces a kind of Volterra RLS adaptive filter algorithm, and through the Volterra nonlinear system modeling to carry out the system identification, which verifying the Volterra RLS adaptive filtering algorithm accuracy and performance, as well as for the subsequent groundwork.

2. Nonlinear filter type

Linear system completely determined by the system impulse response function. But, nonlinear system does not have a unified score and description characteristics to the nonlinear model. Different researchers according to their study object, and put forward some nonlinear system model, but these models do not have a universal significance.

Nonlinear filter constructed by the nonlinear system model mainly have the following types [3].

(1) Homomorphic filter. It is the use of the nonlinear system making the nonlinear signal multiplicative or convolution combination transform into the additive signal combination, and then carries on the nonlinear filtering to use the nonlinear inverse system to carry out inverter process, and finally to get the homomorphic filter system output. (2) The order statistics filter. It is based on the sorting statistics, and includes a wide range of the categories nonlinear filter, one of the most famous is the median filter, including the cascade filter summarizes the types of very wide range of nonlinear filter, which has the threshold value decomposition characteristics and cascading combination characteristic, and can decompose the value signal into the binary sequence, which easily to carry out the parallel real-time processing. (3) The morphological filter. It is a new type of nonlinear filter based on the mathematical morphology. (4) The neural network. It is imitation and extension of the human brain intelligence, thinking and consciousness of functions, such as nonlinear adaptive system. The structure of the neurons in the neural network is closely contact with method is that the training network learning algorithm.

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Due to the nonlinear environment is usually a dynamic, the neural network must has the continuous learning characteristics. We must solve the time structure problem that neural network how to make its behavior adapt among the behavior space input signal changing. (5) The polynomial filter. It is very different from the linear filter structure that determined only by the input vector. The polynomial filter has many nonlinear coupling terms, and realization structure has uniqueness. Among the polynomial filter, Volterra filter is one of the most widely used nonlinear filter.

3. Nonlinear system Volterra series description

For a time invariant, cause-effect, degree of freedom for N nonlinear physical system, and the input is $x(t)=[x_1(t),\cdots,x_N(t)]^T$, output is $y(t)=[y_1(t),\cdots,y_N(t)]^T$. When the input signal $x(t)$ has a limited energy, so the system response $y(t)$ is the functional of $x(t)$, which can be expressed as: $y(t)=F[x(t)|_{-\infty}^{t}]$. Among the formula, $F[\cdot]$ is a nonlinear operator, $|_{-\infty}^{t}$ express the response of the t time, only response with the input before the t moment. [4]

According to the nonlinear dynamic system decomposition theorem, the nonlinear system can be represented by the continuous functional F. Hypothesis: (1) The input signal $x(t)$ energy is limited, that is $\|x(t)\|=[\int_{-\infty}^{+\infty}x^2(t)dt]^{1/2}<\infty$, among the formula, operator $\|\bullet\|$ expresses L_2 norm. (2) To any two meet (1) requirements, the signal $x_1(t)$ and $x_2(t)$, when $\|x_1(t)-x_2(t)\|\to 0$, $\|F[x_1(t)]-F[x_2(t)]\|\to 0$. The system can always arbitrary accurately is decomposed into a set of linear system and a nonlinear real-time system, which is shown as Figure 1. This is the nonlinear system decomposition theorem.

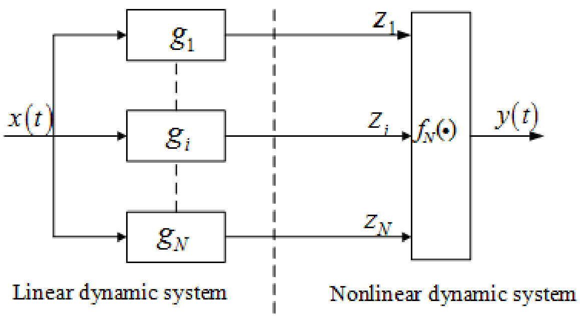

Figure 1. Nonlinear system decomposition diagram.

In the Figure 1, The function $g_1,g_2,\cdots,g_N$ make up of the linear system, and the function $f_N(z_1,z_2,\cdots,z_N)$ is the nonlinear "real-time system", to the input $\{z_i|i=1,2,\cdots,N\}$ only carry out the algebraic operation, but not carry out the "dynamic" operation of the derivation, integral, etc. The nonlinear system decomposition theorem description: it can always to meet certain conditions nonlinear system decomposition, decomposition results will attribute all memory factors to the linear system, and put all the nonlinear factors attribute to real-time system. This decomposition decomposes the complex nonlinear system into two simpler sub-system.

For discrete, we respectively set $x(n)$ and $y(n)$ is a Volterra nonlinear system input and output signal, then, the output $y(n)$ can use Volterra series represent. Second order Volterra with the memory length 3 filter frame diagram is shown as Figure 2.

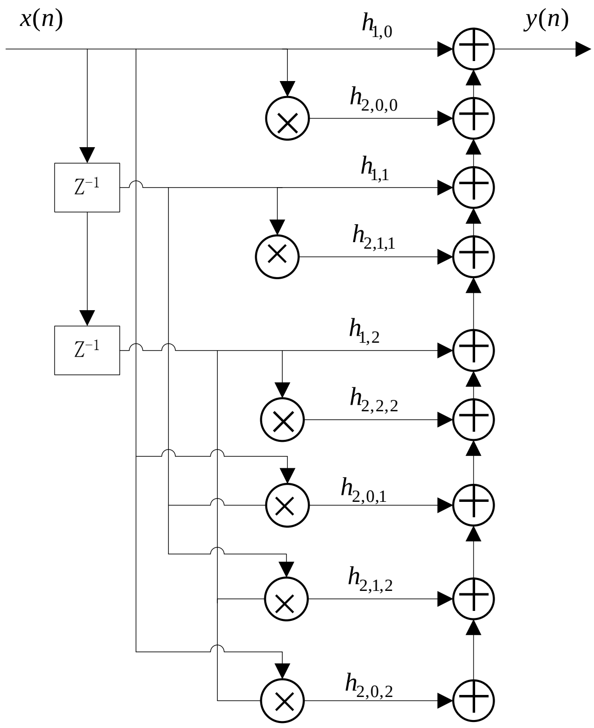

Figure 2. Second order Volterra filter with the memory length 3.

4. Volterra series RLS adaptive filter

A discrete causal nonlinear Volterra system relationship between the input signal $x(n)$ and its output $y(n)$ can be expressed as the Volterra series formula. [4]

$$\begin{aligned} y(n) &= h_0 + \sum_{m_1=0}^{\infty} h_1(m_1)x(n-m_1) + \sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty} h_2(m_1,m_2)x(n-m_1)x(n-m_2) + ... \\ &+ \sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}...\sum_{m_p=0}^{\infty} h_p(m_1,m_2,...,m_p)x(n-m_1)x(n-m_2)...x(n-m_p) + ... \end{aligned} \quad (1)$$

Among the formula (1), $h_p(m_1,m_2,...,m_p)$ is called as p orders Volterra kernel coefficient. It is called linear kernel when p=1. Volterra series can be seen as the Taylor series expansion with memory circumstance, which can approach to any continuous nonlinear system model. Formula (1) expresses there are infinity numbers Volterra kernel to the nonlinear system. In the fact application, we should carry out truncation process in the practical application. Truncation process contains two aspects of the order number p and memory depth N. How to truncate is relevant to the specific nonlinear system type and the performance of the requirements. Usually, only considering the second order truncation model, that is p=2, and hypothesis h_0 = 0, memory depth is N. System can be simplified:

$$y(n) = \sum^{\infty} h_1(m_1)x(n-m_1) + \sum^{N-1}\sum^{\infty} h_2(m_1,m_2)x(n-m_1)x(n-m_2) \quad (2)$$

Among the formula (2), we suggest the kernel of Volterra series is symmetrical. To any of p! numbers $m_1, m_2, ..., m_p$ transposition, $h_p(m_1,m_2,...,m_p)$ is equation. Thus, formula (2) has $N(N+3)/2$ numbers Volterra kernel. Considering the symmetrical character to the Volterra series.

We can define system kernel quantity in n times.

$$\boldsymbol{H}(n)=[h_1(0;n),h_1(1;n),\cdots,h_1(N-1;n),h_2(0,0;n),h_2(0,1;n),\cdots,$$
$$h_2(0,N-1;n),h_2(1,1;n),..,h_2(N-1,N-1;n)]^T \tag{3}$$

The same we can define system input vector in n time.

$$\boldsymbol{X}(n)=[x(n),x(n-1),\cdots,x(n-N+1),x^2(n),x(n)x(n-1),\cdots,x(n)x(n-N-1),$$
$$x^2(n-1),..,x^2(n-N+1)]^T \tag{4}$$

Thus the output can be expressed as formula (5) in n times.

$$y(n)=\boldsymbol{H}^T(n)\boldsymbol{X}(n) \tag{5}$$

Formula (2) and (5) state that a nonlinear system can be state extended to express as linear combination of the input vector $X(n)$ each component, which is the advantages of the nonlinear system Volterra series expressed. If the known system has the form style as the formula (5), but its kernel vector $\boldsymbol{H}(n)$ is unknown, so, we can use Figure 1 to identify the system kernel vector $\boldsymbol{H}(n)$ similar to the linear style.

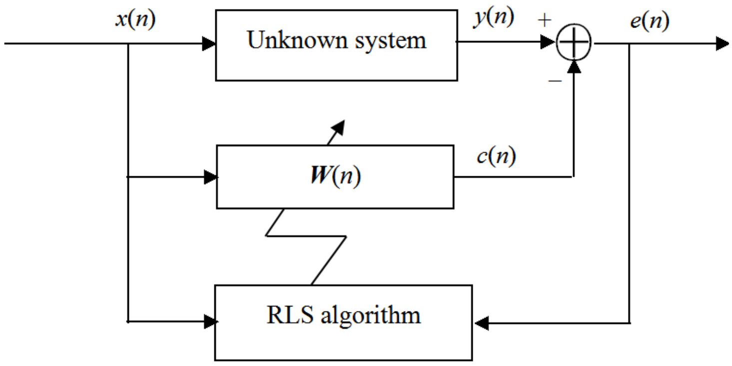

Figure 3. Identification method based on adaptive filter.

In the Figure 1, $\boldsymbol{W}(n)$ is Volterra filter coefficient vector with the length of $M=N(N+3)/2$. If we define the Volterra filter coefficient vector $\boldsymbol{W}(n)$ is : $w(n)=[w_0(n),w_1(n),\ldots,w_{M-1}(n)]^T$, then the output of Volterra filter $c(n)$ is : $c(n)=\boldsymbol{W}^T(n)\boldsymbol{X}(n)$, among the formula, $\boldsymbol{X}(n)$ is the input vector of the formula (3). The purpose of the system identification is changing filter coefficient vector $\boldsymbol{W}(n)$ through a adaptive algorithm, which to make error information $e(n)$ into minimum in a sense. That is to say, it will make some a cost function $J(n)$ of $e(n)$ into minimum. When the cost function $J(n)$ approaching to minimum, we can think $H(n)\approx \boldsymbol{W}(n)$. If we define the cost function $J(n)$ as formula (6).

$$J(n)=\sum_{k=0}^{n}\lambda^{n-k}(d(k)-H^T(n)X(k))^2 \tag{6}$$

Among the formula (6), $H(n)$ and $X(n)$ respectively is the kernel vector and the input signal vector, λ is known as the forgetting factor on behalf of the adaptive system memory range. Its introduction mainly to follow the input signal non-stationary component. In each iteration type, make $J(n)$ differentiate to $H(n)$, and we can obtain the best solution shown as formula (7).

$$H(n) = C^{-1}(n)P(n) \tag{7}$$

Among the formula (7).

$$C(n) = \sum_{k=0}^{n} \lambda^{n-k} X(n) X^{T}(n)$$

$$P(n) = \sum_{k=0}^{n} \lambda^{n-k} d(k) X(k)$$

Iterative formula for.

$$C(n) = \lambda C(n-1) + X(n)X^{T}(n) \tag{8}$$

$$P(n) = \lambda P(n-1) + d(n)X(n) \tag{9}$$

Kernel vector $H(n)$ can be renewed by the formula (8) and (9). From the above analysis and iterative formula we can see, for the changing of the input signal statistical properties, the RLS algorithm has more robust than the LMS algorithm.

RLS algorithm process can be summarized as [5] :

$$K(n) = \frac{\lambda^{-1}P(n-1)X(n)}{1+\lambda^{-1}X^{T}(n)P(n-1)X(n)} \tag{10}$$

$$\varepsilon(n) = y(n) - W^{T}(n-1)X(n) \tag{11}$$

$$W(n) = W(n-1) + K(n)\varepsilon(n) \tag{12}$$

$$P(n) = \lambda^{-1}P(n-1) - \lambda^{-1}K(n)X^{T}(n)P(n-1) \tag{13}$$

Among the formula, the initial value of *W*(*n*) can also set $W(0) = [0, 0, ..., 0]^{T}$, and the initial valule of *P*(*n*) can be made as $P(0) = \delta^{-1}I$, δ is a small positive number, *I* is a $M \times M$ d unit matrix. RLS has the advantages of fast convergence speed and most calculation amount. [6]

5. Algorithm simulation and performance analysis

Considering the following nonlinear system identification problem. Input *x*(*n*) is the normal white noise with the zero mean unit variance, the unknown system is expressed as the following nonlinear model.

$$\begin{aligned} d(n) &= 0.2342x(n) + 0.5674x(n)^{2} - 0.3284x(n)x(n-1) \\ &+0.5674x(n)x(n-2) - 0.4325x(n-1) + 0.6751 * x(n-1)^{2} \\ &+1.4322x(n-1)x(n-2) + 0.6549x(n-2) + 2.4562x(n-2)^{2} + noise(n) \end{aligned}$$

Simulation results uses the Volterra RLS method to carry out the nonlinear system identification. If we adopt the second order Volterra filter with the memory length of m=3. *noise*(*n*) is zero mean with the gaussian white noise in variance of 1 and the memory length *N* is 3, through the independent simulation results of the fifty times taking average results. When the input signal is a weak correlation signal, namely *a*=0.3, Each weights value convergence curve based on the Volterra RLS algorithm adaptive filtering is shown as Figure 4.

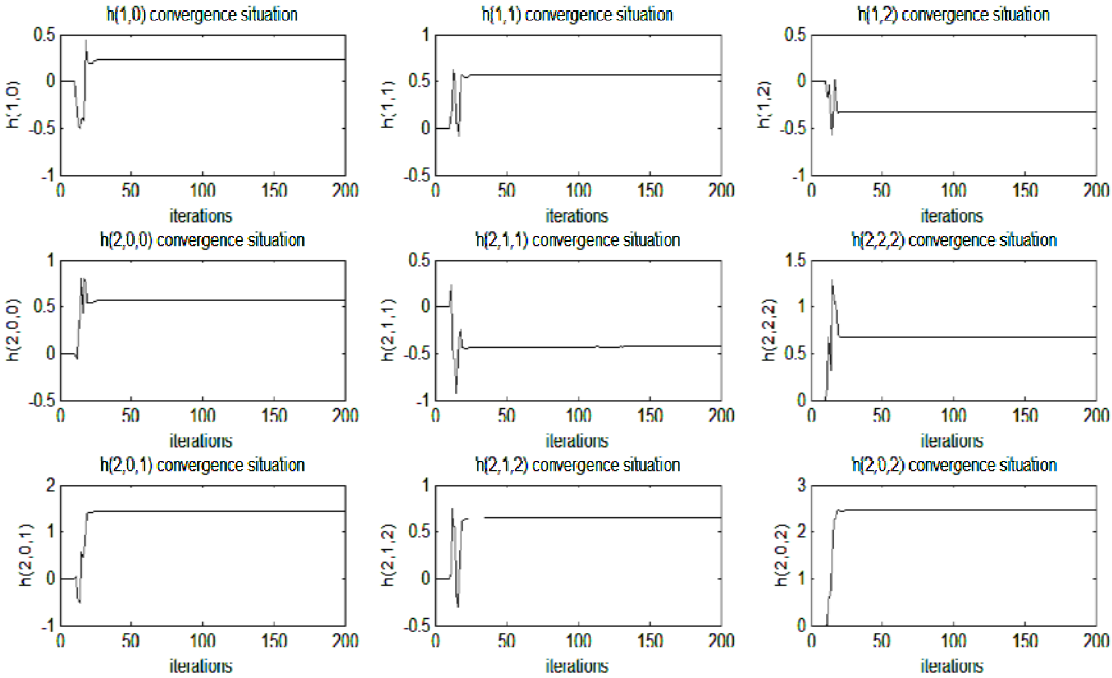

Figure 4. The convergence condition of each weights value of the input signal.

From the Figure 4 we can see, Volterra RLS algorithm can achieve rapidly convergence, and the steady state disorder quantity is low. RLS algorithm convergence speed is faster than the LMS algorithm, but the amount of calculation is larger. Because the Volterra series can be described as the nonlinear system with the memory effects, the use of adaptive Volterra filter can be used to solve a widely class of nonlinear system identification problems [8] [9]

6. Conclusions

This paper studied the Volterra model RLS adaptive filter algorithm and realize the first order and second order terms Volterra kernel identification. Volterra RLS adaptive filter algorithm has better convergence performance and steady state performance. The RLS adaptive filter algorithm can better able to used in the mechanical model damping and de-noise, which has a broad application prospect. Above although is for second order Volterra system discussion, but it is easy to spread to higher order Volterra system.

Acknowledgments

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