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Game Model on Income Distribution for Expressway Network Tolling

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Abstract

Reasonable income distribution for expressway network tolling plays a positive role in attracting investors and social funds to accelerate expressway construction and development. Influence factors of the income distribution problem for expressway network tolling were analyzed and the income distribution process of expressway network tolling was further viewed as a bargaining game in the paper. Then, an income distribution model based on bargaining game for expressway network tolling was proposed to increase the return rates for expressway investors and further mobilize their enthusiasm for expressway investment. Finally, we proposed one numerical example for the income distribution problem with the help of such game model. Also, The results demonstrated that the precedence investors with much more optimistic attitudes to expressway investment could share much more from expressway network tolling, and also the traffic volume distribution compensation from the government in an opened or hidden way further could enhance investors enthusiasm for the expressway investment with certain low discount rates.

Keywords: *expressway, network tolling, income distribution, bargaining game, model*

1. Introduction

Different investment entities for expressway investment have been getting closer than before since its network tolling. Because there is a long-term period for expressway construction and there are also rather heavy fund demands, how to design a fair and reasonable income distribution model for expressway network tolling, which plays a positive role in widely attracting these investors and social funds to speed up expressway construction and its development, is thus rather important for both the state and expressway business.

Income distribution are payed the most attention from expressway investment entities, and there are lots of existed researches on income distribution around the world. i.e. Dutta et al. (2001) proposed an income dynamic model taking account of mobility both within and between jobs the United Kingdom [1]. Ishikawa (2005) analyzed the database of high income companies in Japan to find a quantitative relation between the average capital of the companies and the Pareto index [2]. Also, Chattopadhyay and Mallick (2007) theoretically studied the changes in poverty with respect to the global mean and variance of the income distribution using Indian survey data and given modified deprivation function [3]. Moura and Riberro (2009) presented an empirical study of the evolution of the personal income distribution in Brazil [4], and Chami et al. (2011) further analyzed the Gompertz-Pareto distribution of personal income [5]. Besides, many scholars studied relative expressway network tolling problems which are helpful for more wide and general road construction [6]. For example, Palma et al. (2005) compared the second-best and third-best tolling schemes on a road network [7], and Szeto and Lo (2006) proposed several transportation network improvement and tolling strategies [8]. Also, Stewart (2007) analyzed the relationship between the number and price level of tolled links and optimal traffic flows with stochastic assignment for the tolling traffic links [9]. Besides, Shen and Zhang (2009) investigated the basic characteristics of the optimal dynamic traffic patterns and the corresponding tolling scheme and discussed the practical implications to corridor traffic management [10]. Furthermore, Xu et al. (2012) developed a reliable, objective and systematic model for determining a rational concession price for PPP highway projects based on pro forma financial statements developed during the feasibility study period [11]. Moreover, the existing

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income distribution for expressway network tolling is manily based on the traffic volume distribution with probabilistic, precise path identification method and so on. But existing researches scarely consider all above factors with different investment proportion for different investment entities partcipating in the expressway construction.

Bargaining is a special form of dynamic game theory, and also it is a game process in which common interests with conflict people want to achieve a consensus agreement. The income distribution for expressway network tolling is just the same case. Thus, adopting the dynamic game theory and giving the assumption that the income distribution is decided by the bargaining process among different investment entities (i.e. the investment companies participating in the distribution for expressway network tolling), also considering the traffic volume (also namely traffic flow [12]) distribution, investment proportion and its sequencing, negotiation rupture risk and also discount rate, the paper further proposes an income distribution model based on bargaining game for expressway network tolling.

2. Influence Factors about Income Distribution for Expressway Network Tolling

With the expressway network tolling mode, there are many influence factors on the income distribution problem, among which the most important factors are the traffic volume distribution in the expressway network and the investment proportion from different investors, also namely the expressway construction cost. For example, the road sections with more traffic volume should share more in their income distribution accordingly in order to achieve much more funds for expressway maintenance and management, while other road sections with higher construction cost should more and get much more return with fixed expressway network tolling because of its higher investment. Moreover, the traffic volume distribution is often associated with the society and economy in these districts where the expressway passes through, and thus these road sections with more traffic volume general lie in these districts with better economy situation. However, the expressways construction cost is often inconsistent with the economy situation, and at the underdevelopment districts such as the isolated mountains areas, much more investments are needed to construct an expressway, also its construction cost is much higher than others. Thus, the investors for these sections require more return or share more in the income distribution for the whole expressway network tolling.

There are many entities participating in the bargaining to distribute the expressway network tolling income, and thus we view it as a bargaining game. As for a simple bargaining game, the final bargaining results are ensured by lots of factors, while the bids sequencing, participants' patience and also their attitudes to the negotiation rupture risk are the most vital among them. Thus, as for the income distribution for expressway network tolling based on such bargaining game, the expressway investment sequencing for these entities is viewed as the bid one in the game, and the precedence entities or investors can achieve much more first-mover advantage. Moreover, the more patience or optimistic attitudes to the bargaining for investment entities, the litter probability for its rupture risk. Thus, they may also share much more during the whole income distribution game. Furthermore, some influence factors of the income distribution for expressway network tolling mutual promote each other, while others restrict each other. In practice, the rupture risk always exists during the bargaining and occurs for unexpected chances or reasons to achieve a common consensus, which means the rupture risk and rupture point all both function well for it. Thus, to build or design a fair and reasonable income distribution model for expressway network tolling needs a comprehensive consideration all above factors. Next, we'll propose a bargaining game model in the next section.

3. Bargaining Game Model on Income Distribution for Expressway Network Tolling

As for such bargaining game about the income distribution for expressway network tolling, we'll construct such models with two participants joining such game or more than three participants joining such game which are deduced from the former one.

3.1. Assumptions

For accurately, we further propose two assumptions for such game.

(1) Participants taking part in the income distribution for expressway network tolling are all just investment entities or investors for expressway;

(2) During each stage of the bargaining game on the income distribution for expressway network tolling, the rupture risk probability P ($P \in [0,1]$) is a Poisson distribution.

3.2. Notations

General subscripts and parameters used in the proposed model are listed in Table 1.

Table 1. General Subscripts and Parameters Used in The Proposed Model

Symbol	Description
U_i	Distributed income for the *i*-th participant, also namely the decision variable.
N	Total number of participants joining in the bargaining game on the income distribution
v_i	Deadlock point: the distributed income for the *i*-th participant when bargaining deadlock occurs
θ_i	Rupture point: the distributed share proportion for the *i*-th participant when bargaining rupture occurs
ε	Total income for given expressway network tolling
τ_i	Discount rate for the *i*-th participant, also namely profit rate with other opportunities
λ_i	Bargaining rupture risk probability for the *i*-th participant
δ	Discount rate for given expressway investment project
α	Weight of investment
β	Weight of traffic volume
c_i	Share proportion in the total investment for the *i*-th participant
ω_i	Road proportion invested by the *i*-th participant in the total traffic volume

3.3. Bargaining Game Model Construction

Without considering the risk of bargaining rupture, for only two participants joining the game on income distribution for expressway network tolling, the accepted distribution proportion for both of them, meaning that the negotiation rupture occurs if the proportion is lower than the accepted one, thus namely the rupture points for them are calculated by

$$\theta_1 = \frac{1}{1+\delta}\ ,\ \theta_2 = \frac{\delta}{1+\delta}\ . \tag{1}$$

Furthermore, for more than participants joining the game with the same situation, the rupture point for the *i*-th participant is rewritten by

$$\theta_i = \frac{\delta^{i-1}}{1+\delta+\delta^2+\cdots+\delta^{N-1}} = \frac{\delta^{i-1}}{\sum_{n=1}^{N-1}\delta^n} \qquad \forall i \in \{1,2,\cdots,N\} \tag{2}$$

With considering the risk of bargaining rupture and from the assumption (2), the bargaining rupture risk probability obeys the Poisson distribution with the parameter λ, also namely the average rupture times within given unit period. Thus, the lasting time for the negotiation obeys the exponential distribution with the same parameter. From the characteristics of the exponential distribution, at the random λ_Δ the bargaining rupture risk probability is calculated by $1-e^{-\lambda_\Delta \Delta}$, of which Δ denotes the time interval of given bargaining stage. If the bargaining game breaks up at the stage without a common consensus for some reasons, the rupture point is expressed by $(\theta_1,\theta_2,\cdots,\theta_N)$, while there is also a critical point for deadlock to be denoted by $(v_1,v_2,\cdots,v_N)$. The rupture and deadlock points are the same without considering their discount rates. However, in actual fact, it's often another different thing. That's to say they are consistent with each other because of their different patience and attitudes to the game. The share ratio is thus decided by there subjective attitudes for each participant when the deadlock occurs. For example, the income for the first participant who thinks the rupture will occur at the probability of $1-e^{-\lambda_1\Delta}$ is calculated by $P_1\theta_1\varepsilon\sum((1-P_1)^t\delta_1^t)$, in which t denotes the stage of the bargaining game.

Generally speaking, when the time interval Δ closes to the infinitesimal, there is $P_i=\lambda_i\Delta$ and thus $\delta_i=e^{-\tau_i\Delta}=1-\tau_i\Delta$. The distribution limitation is expressed by $\lambda_i\theta_i\varepsilon/(\tau_i+\lambda_i)$, and the deadlock point vector for two participants are further expressed by

$$(v_1,v_2)=(\frac{\lambda_1\theta_1\varepsilon}{\tau_1+\lambda_1},\frac{\lambda_2\theta_2\varepsilon}{\tau_2+\lambda_2}). \quad (3)$$

Thus, the distributed income for the sub-game perfect equilibrium (SPE) with two participants joining in the income distribution for expressway network tolling is calculated by

$$U_i=v_i+\frac{\tau_j+\lambda_j}{\tau_1+\tau_2+\lambda_1+\lambda_2}\left(\varepsilon-v_1-v_2\right) \qquad \forall i,j\in\{1,2\},i\neq j. \quad (4)$$

For formula (4), the perfect equilibrium distributed income is composed of two parts, including the share from its deadlock point associated with the participant's patience and attitudes and the proportion from the rest income.

Similarly, we deduce the bargaining game with more than three participants from that with two ones, and its distribution limitation is also expressed by $\lambda_i\theta_i\varepsilon/(\tau_i+\lambda_i)$. Then, its deadlock point vector and SPE are thus rewritten by formula (5) and formula (6) respectively,

$$(v_1,v_2,\cdots,v_N)=(\frac{\lambda_1\theta_1\varepsilon}{\tau_1+\lambda_1},\frac{\lambda_2\theta_2\varepsilon}{\tau_2+\lambda_2},\cdots,\frac{\lambda_N\theta_N\varepsilon}{\tau_N+\lambda_N}), \quad (5)$$

$$U_i=\frac{\lambda_i v_i}{\tau_i+\lambda_i}+\frac{\sum_{j=1,j\neq i}^{N}(\tau_j+\lambda_j)}{\sum_{j=1}^{N}(\tau_j+\lambda_j)}(\varepsilon-\sum_{j=1}^{N}v_j) \qquad \forall i,j\in\{1,2,\cdots,N\},i\neq j. \quad (6)$$

In Equation (6), if $\forall\tau_i\to 0$, the limitation income proportion for the *i*-th participant rises up with the declining of its bargaining rupture risk probability λ_i, but the increasing of others rupture risk probability $\lambda_j\ (i\neq j)$.

Furthermore, considering the traffic volume distribution and the total construction cost or investment for given expressway, and assigning certain weights for both of them for their deadlock, the distributed share proportion for the i-th participant when the bargaining deadlock occurs is calculated by

$$v_i=\frac{\lambda_i N(\alpha c_i+\beta\omega_i)\theta_i\varepsilon}{\tau_i+\lambda_i}, \quad (7)$$

where there are $\sum c_i=1,\sum\omega_i=1,\alpha+\beta=1$.

Thus, the bargaining game on income distribution for expressway network tolling is constructed by Equation (6), Equation (7) and also Equation (2). To analyze the function of traffic volume distribution compensation from the government, we further analyze the derivative for the distributed income about the total traffic volume. Thus, there is

$$\frac{\partial U_i}{\partial\omega_i}=\frac{\lambda_i}{\tau_i+\lambda_i}\cdot\frac{\lambda_i\theta_i N\beta\varepsilon}{\tau_i+\lambda_i}-\frac{\sum_{j=1,j\neq i}^{N}(\tau_j+\lambda_j)}{\sum_{j=1}^{N}(\tau_j+\lambda_j)}\cdot\frac{\lambda_i\theta_i N\beta\varepsilon}{\tau_i+\lambda_i}$$

$$= \frac{\lambda_i \theta_i N \beta \varepsilon}{\tau_i + \lambda_i} \cdot (\frac{\lambda_i}{\tau_i + \lambda_i} + \frac{\tau_i + \lambda_i}{\sum_{j=1}^{N} (\tau_j + \lambda_j)} - 1) . \quad (8)$$

From Equation (8), if $\forall \tau_i \to 0$, then $\frac{\partial U_i}{\partial \omega_i} > 0$,meaning the traffic volume distribution compensation enhances investors' enthusiasm for expressway investment. Otherwise, there is no obvious intimated relationship between them with higher discount rates for the participants joining in the expressway investment because of their other opportunities.

4. Numerical Example and Discussions

There are 5 expressways composed an expressway network and invested by 5 different investment companies respectively in certain province in China. Since the implementation of network tolling for them, there are ¥ 300 million to be distributed to them till June last year, namely ε=300 (¥ one million). Let the discount rate for the whole project δ=0.98 (Generally, its value scale is from 0.97 to 1). Also, let the weights for their investment and traffic volume α=0.3 and β=0.7 respectively. The investment sequencing for the five companies is as follows: A→B→C →D→E, and their other parameters values are shown in Table 2.

Table 2. Parameters Values For Such Example

Investment company	Investment (¥ 1million)	Distribution with traffic volume (¥ 1million)	Discount rate (τ_i)	Bargaining rupture risk probability (λ_i)	Share proportion (c_i)	Road proportion (ω_i)
Company A	5 000	30	0.006	0.04	0.2042	0.1000
Company B	2 500	45	0.004	0.05	0.1020	0.1500
Company C	12 000	75	0.005	0.04	0.4898	0.2500
Company D	3 000	60	0.004	0.06	0.1224	0.2000
Company E	2 000	90	0.005	0.06	0.0816	0.3000
In total	24 500	300	0.024	0.25	1.0000	1.0000

With the bargaining game on income distribution for expressway network tolling, the final results for the income distribution are shown in Table 3.

Table 3. Results for the Income Distribution with the Proposed Bargaining Game Model

Investment company	Rupture points (θ_i)	Deadlock points (v_i/¥ 1million)	Distributed income (U_i/¥ 1million)
Company A	0.20816	35.6 291	41.68 195
Company B	0.20400	38.4 234	44.26 386
Company C	0.19992	85.816	91.89 536
Company D	0.19592	48.69 263	54.26 759
Company E	0.19200	62.34 286	67.89 128
In total	1.00000	270.9 040	300

From Table 2 and Table 3, as for company A and C with the same bargaining rupture risk probability, the investment from C is 2.4 times that from C while the final distributed income for C is about 2.2 times that for A, and thus less than 2.4 times for the reason that A is priority to C in their investments. Moreover, from company D and E, it's clear that the D's investment is more than that of E and D is priority to E in their investments, but the final distributed income for E is higher than that for D. That's because the road proportion of the total traffic volume for E is 1.5 times and thus higher than that for D. Also, from company A and B, although the investment from A is twice that form B, their final distributed incomes are nearly the same even the latter one is slightly higher than the former one owning to the lower discount rate, higher road proportion and deadlock point for A. Furthermore, the rupture points become lower litter by litter with the investment sequencing and there aren't clear other relationships between different influence factors and their final distributed incomes.

As for the implementations for the traffic volume in the expressway network provided by the government, we can calculated the derivative for the distributed income about the total traffic volume by formula (8) and according to Table 2 and their values are as follows: A 7.1174; B 24.3962; C 9.9122; D 32.9936; E 29.8392. Thus, they are all above zero meaning the more traffic volume implementations from the government, the more distributed income, and further the more enthusiasm for investment. But if the discount rates are 0.012, 0.016, 0.005, 0.1, 0.02, the derivative values are -6.8913, -0.2513, 9.9122, -3.1674, 6.3460 with the same bargaining rupture risks probability for the five companied. We can know that some are below zero while others are above zero. Thus, the discount rate is so high enough that the implementations don't function well, and the reason is that there are much better opportunities for the company to make more profits. So, the traffic volume distribution compensation functions well only with certain low discount rates for the investors.

5. Conclusions

In this paper, we analyze the influence factors about the income distribution problem for expressway network tolling and further propose a bargaining game model for the income distribution. Also, a numerical example is put forward to test the proposed model, and we further get the following conclusions: the precedence investors can share more from the distributed income; the more distributed traffic volume, the more share from the income; thus their patience and optimistic attitudes to the bargaining game seems to have positive function in the income distribution and the rupture points become lower litter by litter with the investment sequencing; the traffic volume distribution compensation from the government in an opened or hidden way may enhance investors enthusiasm for the expressway investment only with a relative low discount rate, while it doesn't function well because of others opportunities for the investors beyond certain scope. Thus, our future research aims to find the specific scope of discount rate to provide an effective decision support for both the policy from the government and the decision from investment companies.

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