

DMC Based on Weighting Correction of Predictive Model Errors

Liu Yumin^{*1}, Sun Yonghe¹, XU Fengming², Wang Tao²

¹School of Electrical Engineering & Information, Northeast Petroleum University,

²Daqing Oilfield Company, Development Street 199#, Gao xin District, 163318

*Corresponding author, e-mail: liuyumin330@163.com

Abstract

Ordinary DMC correct predictive value only using current error, so the correction is not enough. This paper proposes an algorithm. In error correction, it introduces predictive model error and predictive value are weighting corrected to improve the capability of resisting disturbance and regulating speed of the control system. By theoretical analysis and simulation, it has been proven that the algorithm is effective.

Keywords: DMC; predictive model error; weighted correction

Copyright © 2013 Universitas Ahmad Dahlan. All rights reserved.

1. Introduction

Dynamic Matrix Predictive Control (DMC) is a kind of predictive control [1]. In 1979, Culter first proposed the Dynamic Matrix Control algorithm in annual meeting of American Chemical Industry. DMC algorithm include predictive model, feedback correction and rolling optimization. DMC has many advantages, such as simple algorithm, less computation and strong robustness, etc. It is very suitable for non-minimum phase system with dead time and open-loop gradually stabilization characters. So dynamic matrix predictive control algorithm is widely applied in the fields of industry.

However, DMC can't solve the contradiction between the effect of the disturbance on the stable value and the regulating speediness. Aiming at this problem, the method of calculating prediction error feedback coefficient μ based on steady-state error and its derivative was proposed in literature [2], this method can decrease static error and speed up disturbance adjusting. An error variation-corrected predictive control algorithm was proposed in literature [3]. A prediction-error filter was proposed in literature [4], it design weighted coefficient using internal model principle. Because ordinary DMC correct predictive value only use current error, so the correction is not enough. This paper proposes an algorithm. In error correction, it introduce predictive model error and predictive value are weighting corrected to improve the capability of resisting disturbance and regulating speed of the control system. By theoretical analysis and simulation, this method can improve noise immunity and rapidity.

2. DMC based on Weighted Correction of the Predicted Errors

2.1. Ordinary DMC

DMC's predictive model is described by unit step response discrete sampling datas $a_1, a_2 \dots a_N$. Where N is the length of the model. Generally, under the action of M continuous control increment $\Delta u(k), \dots, \Delta u(k+M-1)$, predictive model output in the future of P moments is:

$$\mathbf{Y}_m(k+1) = \mathbf{Y}_0(k+1) + \mathbf{A}\Delta\mathbf{U}(k) \quad (1)$$

where, $\mathbf{Y}_m(k+1) = [y_m(k+1|k) \quad y_m(k+2|k) \quad \dots \quad y_m(k+P|k)]^T$
 $\mathbf{Y}_0(k+1) = [y_0(k+1|k) \quad y_0(k+2|k) \quad \dots \quad y_0(k+P|k)]^T$

$$\mathbf{A} = \begin{bmatrix} a_1 & 0 & \cdots & 0 \\ a_2 & a_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_p & a_{p-1} & \cdots & a_{p-M+1} \end{bmatrix}_{P \times M}$$

$$\Delta \mathbf{U}(k) = [\Delta u(k) \quad \Delta u(k+1) \quad \cdots \quad \Delta u(k+M-1)]^T$$

Matrix A is called dynamic matrix, matrix elements are step response coefficients that can describe system dynamic characteristic. M is the length of control horizon, P is the length of optimization horizon.

$\mathbf{U}(k-1) = [u(k-N+1), \dots, u(k-1)]^T$ is previous time control quantity, it can describe $\mathbf{Y}_0(k+1)$, where:

$$\mathbf{Y}_0(k+1) = \mathbf{A}_0 \mathbf{U}(k-1) \quad (2)$$

Then Equation (1) is:

$$\mathbf{Y}_m(k+1) = \mathbf{A} \Delta \mathbf{U}(k) + \mathbf{A}_0 \mathbf{U}(k-1) \quad (3)$$

Where \mathbf{A}_0 is:

$$\mathbf{A}_0 = \begin{bmatrix} a_N - a_{N-1} & a_{N-1} - a_{N-2} & a_{N-2} - a_{N-3} & \cdots & a_3 - a_2 & a_2 \\ & a_N - a_{N-1} & a_{N-1} - a_{N-2} & \cdots & a_4 - a_3 & a_3 \\ 0 & & & \vdots & \vdots & \vdots \\ & & a_N - a_{N-1} & \cdots & a_{p+2} - a_{p+1} & a_{p+1} \end{bmatrix}$$

$\mathbf{Y}_m(k+1)$ in Equation (3) is predictive value model output.

In Equation (3), $\mathbf{Y}_m(k+1)$ is predictive model output vectors of the future P moments with the effect of $\Delta u(k)$. $\mathbf{Y}_m(k+1)$ is forecast at the k time. $\mathbf{Y}_0(k+1)$ is initialization vector of the future P moments with not the effect of $\Delta u(k)$. $\mathbf{Y}_0(k+1)$ is also forecast at the k time.

Because the effect of model error and interference, predictive outputs of system need be modified by actual error on the basis of predictive model outputs. The formula is as follows.

$$\begin{aligned} \mathbf{Y}_p(k+1) &= \mathbf{Y}_m(k+1) + \mathbf{h}[y(k) - y_m(k)] \\ &= \mathbf{A} \Delta \mathbf{U}(k) + \mathbf{A}_0 \mathbf{U}(k-1) + \mathbf{h}e(k) \end{aligned} \quad (4)$$

Where,

$\mathbf{Y}_p(k+1)$ is predictive outputs of system.

$$\mathbf{Y}_p(k+1) = [y_p(k+1), y_p(k+2), \dots, y_p(k+P)]^T$$

$e(k)$ is error of predictive model output at k time, $e(k) = y(k) - y_m(k)$.

\mathbf{h} is error correction coefficient in current hour, $\mathbf{h} = [h_1, h_2, \dots, h_p]^T$

$(0 \leq h_i \leq 1, i = 1, \dots, P)$.

Optimal control regular is determined by quadratic performance index,

$$\begin{aligned} J_p &= [\mathbf{Y}_p(k+1) - \mathbf{Y}_r(k+1)]^T \mathbf{Q} [\mathbf{Y}_p(k+1) - \mathbf{Y}_r(k+1)] + \Delta \mathbf{U}^T(k) \lambda \Delta \mathbf{U}(k) \\ &= [\mathbf{A} \Delta \mathbf{U}(k) + \mathbf{A}_0 \mathbf{U}(k-1) + \mathbf{h}e(k) - \mathbf{Y}_r(k+1)]^T \mathbf{Q} [\mathbf{A} \Delta \mathbf{U}(k) + \mathbf{A}_0 \mathbf{U}(k-1) \\ &\quad + \mathbf{h}e(k) - \mathbf{Y}_r(k+1)] + \Delta \mathbf{U}^T(k) \lambda \Delta \mathbf{U}(k) \end{aligned} \quad (5)$$

Where, \mathbf{Y}_r is reference trajectory.

$\mathbf{Q} = \text{diag}(q_1, \dots, q_p)$ is error weight matrix.

$\boldsymbol{\lambda} = \text{diag}(\lambda_1, \dots, \lambda_M)$ is control weight matrix.

Because $\partial J_p / \partial \Delta \mathbf{U}(k) = 0$, So:

$$\Delta \mathbf{U}(k) = (\mathbf{A}^T \mathbf{Q} \mathbf{A} + \boldsymbol{\lambda})^{-1} \mathbf{A}^T \mathbf{Q} [\mathbf{Y}_r(k+1) - \mathbf{A}_0 \mathbf{U}(k-1) - \mathbf{h}e(k)] \quad (6)$$

If only execute the $\Delta u(k)$ in current hour k and recalculate the control increment of $k+1$ and since $k+1$ time, then only need calculate the first row of $\mathbf{d}_i^T = (\mathbf{A}^T \mathbf{Q} \mathbf{A} + \boldsymbol{\lambda})^{-1} \mathbf{A}^T \mathbf{Q}$. So:

$$\Delta \mathbf{U}(k) = (1, 0, \dots, 0)(\mathbf{A}^T \mathbf{Q} \mathbf{A} + \boldsymbol{\lambda})^{-1} \mathbf{A}^T \mathbf{Q} [\mathbf{Y}_r(k+1) - \mathbf{A}_0 \mathbf{U}(k-1) - \mathbf{h}e(k)] \quad (7)$$

2.2. Algorithm Improvement

DMC can't solve the contradiction between the effect of the disturbance on the stable value and the regulating speediness. When system exist unmeasurable disturbance or model error, predictive model output is inconsistent with real output. Coefficient h in Equation (4) can play the role of adjusting. But it correct predictive value only using current error, the correction is not enough. So, a DMC algorithm with predictive error correction was proposed in this paper. On the basis of DMC, new algorithm consider not only the current error but also the predictive model error of previous time. Algorithm realize predictive errors weighted correction. This method improve the capability of resisting disturbance and the regulating speed of the control system. The detailed methods is as follows.

First, introduce the predictive model error \mathbf{E}' .

$$\mathbf{E}'(k+1) = \mathbf{Y}'_p(k+1) - \mathbf{Y}'_m(k+1) = [e'(k+1), e'(k+2), \dots, e'(k+P)]^T \quad (8)$$

$\mathbf{Y}'_p(k+1)$ is predictive output of the future P moments, it is forecast at the $k-1$ time. Where, $\mathbf{Y}'_p(k+1) = [y_p(k+1|k-1), y_p(k+2|k-1), \dots, y_p(k+P|k-1)]^T$

Because can not compute the $k+P$ time's predictive output in $k-1$ time, so $y_p(k+P|k-1)$ is approximate to $y_p(k+P-1|k-1)$.

$\mathbf{Y}'_m(k+1)$ is predictive model output of the future P moments, it is forecast at the $k-1$ time. Where $y_p(k+P|k-1)$ is approximate to $y_p(k+P-1|k-1)$.

The formula is as follows.

$$\mathbf{Y}'_m(k+1) = [y_m(k+1|k-1), y_m(k+2|k-1), \dots, y_m(k+P|k-1)]^T$$

Second, the errors was weighted.

$$e(k+i) = \frac{h_i[y(k) - y_m(k)] + f_i e'(k+i)}{h_i + f_i} \quad i = (1, \dots, P) \quad (9)$$

h_i is the error correction coefficient in current hour.

Vector representation is $\mathbf{h} = [h_1, h_2, \dots, h_p]^T$ ($0 \leq h_i \leq 1$, $i = 1, \dots, P$).

f_i is the predictive error correction coefficient.

Vector representation is $\mathbf{f} = [f_1, f_2, \dots, f_p]^T$ ($0 \leq f_i \leq 1$, $i = 1, \dots, P$).

The Equation (4) can be written in another way:

$$\mathbf{Y}_p(k+1) = \mathbf{Y}_m(k+1) + \mathbf{E}(k) = \mathbf{A} \Delta \mathbf{U}(k) + \mathbf{A}_0 \mathbf{U}(k-1) + \mathbf{E}(k) \quad (10)$$

Where, $Y_p(k+1)$ is predictive output of the future P moments, it is forecast at the k time. Where,

$$Y_p(k+1) = [y_p(k+1|k), y_p(k+2|k), \dots, y_p(k+P|k)]^T ;$$

$Y_m(k+1)$ is predictive model output of the future P moments, it is forecast at the k time. Where,

$$Y_m(k+1) = [y_m(k+1|k), y_m(k+2|k), \dots, y_m(k+P|k)]^T ;$$

$E(k)$ is weighted value of predictive errors and actual errors. Where,

$$E(k) = [e(k+1), e(k+2), \dots, e(k+P)]^T$$

This error correction method adopts not only the actual errors in current hour but also the future errors that forecast in previous time. This method can correct errors more completely and can overcome the effect of interference and model mismatch.

3. Algorithm Simulation

This paper select transfer function $G(s) = \frac{s+1}{s^2+3s+6}$ to make a study and analysis. This model is simulated by MATLAB in order to test the validity of the algorithm. In order to study the immunity, after the system stabilization, step disturbance was added in 5s. The result is as shown in Figure 1. Curve a is simulation curve of ordinary DMC, curve b is simulation curve of improved DMC. By comparison, adjusting time of ordinary DMC is 0.3s quicker than improved DMC. When disturbs appears, adjusting time of ordinary DMC is 1.2s quicker than improved DMC.

In the actual control, often appear the model mismatch. In this paper, the algorithm in the model mismatch is also simulated. Transfer function is $G(s) = \frac{s+1}{s^2+3s+6}$, model

mismatch transfer function is $G(s) = \frac{s+2}{s^2+3s+7}$. After the system stabilization, step

disturbance was added in 5s. The result is as shown in Figure 2. Curve a is simulation curve of ordinary DMC, curve b is simulation curve of improved DMC. By comparison, when model mismatch, overshoot of ordinary DMC is 27.3%, overshoot of improved DMC is 17.3%. When disturbs appears, adjusting time of ordinary DMC is 0.6s quicker than improved DMC.

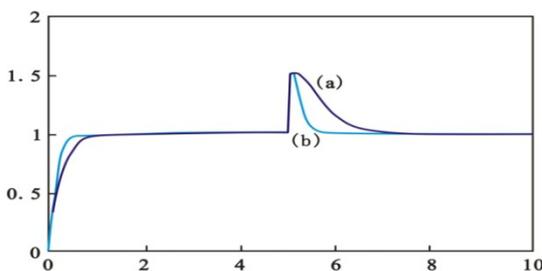


Figure 1. Simulation Results without Model Mismatch

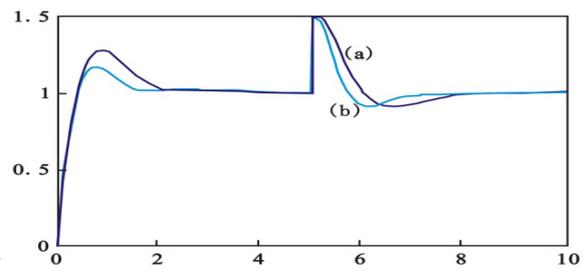


Figure 2. Simulation Results with Model Mismatch

4 Conclusion

This algorithm considers the future predictive error value, and the errors are weighting corrected. This method allow error correction to be more comprehensive, and enhance the robustness of the algorithm. The simulation results show that the improved DMC algorithm has small overshoot, fast response speed. When disturbs appears, it can quickly return to the set value. When model mismatch, it can well maintain the stability of the system.

References

- [1] Wang Cai-xia, Zhu Qiu-qin. An Improved Dynamic Matrix Control Algorithm. *Journal of Northwest University for Nationalities*. 2005; 04.
- [2] Zhu Xuefeng, Huang Daoping, Liu Yongqing. Modified Dynamic Matrix Control. *Control Theory and Applications*. 1988; 5(2): 38-46.
- [3] Chen Guanming. An Error Variation-corrected Predictive Control Algorithm. *Automation And Instrumentation*. 1999; 2.
- [4] Sui Xiao-mei, Li Ping, Zhang Bin. Dynamic Matrix Control Based on the Error Feedback-Weighting Correction. *Journal of Fushun Petroleum Institute*. 2003; 1.
- [5] FAN Liping, ZHANG Jun, HUANG Xing, HUANG Dong. The design of the MISO Model Predictive Controller for Bioreactor. 2012; 10(6).
- [6] Jiejia LI, Rui QU, Yang CHEN. Construction Equipment Control Research Based on Predictive Technology. *TELKOMNIKA Indonesia Journal of Electrical Engineering*. 2012; 10(5): 960-967.