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By using a new iterative method to the generalized system Zakharov-Kuznetsov and estimate the best parameters via applied the pso algorithm

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ABSTRACT

In this paper, a new iterative method was applied to the Zakharov-Kuznetsov system to obtain the approximate solution and the results were close to the exact solution, A new technique has been proposed to reach the lowest possible error, and the closest accurate solution to the numerical method is to link the numerical method with the pso algorithm which is denoted by the symbol (NIM-PSO). The results of the proposed Technique showed that they are highly efficient and very close to the exact solution, and they are also of excellent effectiveness for treating partial differential equation systems.

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1. INTRODUCTION

She showed many scientific cases that occurred in the various mathematical and physical sciences, dynamic processes, etc. It can be described by taking several dimensions of the Korteweg-de Vries KdV equation. Zakharov Kuznetsov successfully proposed one of these models. The Zakharov-Kuznetsov equation given by (1) [1].

$$\frac{\partial u}{\partial t} + \alpha\, u\, \frac{\partial u}{\partial x} + \beta\, \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)_x = 0 \qquad (1)$$

It is considered one of the most popular 2D-dimensional generalizations of the (kdv) equation that has been studied in the literature. The generalized Zakharov-Kuznetsov system of the Zakharov-Kuznetsov equation is a hegemony system with a specific magnetic field in controlling the conduct of nonlinear ion waves in the plasma which is /in turn contain cold ions and electrons of equal temperatures. In addition, Since the Zakharov-Kuznetsov system supports solitary stabilized ground-swell and all this made it. The Zakharov-Kuznetsov system is a wonderful model for studying swirls in geophysical flows [2-5].

$$\left\{\begin{aligned} \frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} + \frac{\partial^3 u}{\partial y^2 \partial x} - 6u\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} = 0 \\ \frac{\partial v}{\partial t} + \delta\frac{\partial^3 v}{\partial x^3} + \lambda\frac{\partial^3 v}{\partial y^2 \partial x} + \eta\,\frac{\partial v}{\partial x} - 6\mu v\frac{\partial v}{\partial x} - \alpha\frac{\partial u}{\partial x} = 0 \end{aligned}\right\} \qquad (2)$$

The coupled Zakharov-Kuznetsov system are the model Associated two interacting weakened nonlinear ground-swell in anisotropic background apply followed flows. Here, x and y are the diffusion and transverse coordinates, η is a group velocity shift among the coupled models, δ and λ are the prorated longitudinal and transverse dispersion coefficient, and μ and α are the prorated nonlinear and coupled coefficients, they are optional constants [6].

The Zakharov-Kuznetsov system was solved by proposing a new technology based on the Taylor series [6]. Using the lie symmetric analysis method, the characteristics of the generalized fractional Zakharov-Kuznetsov system were studied [7]. The homogeneous balance method was used with the Riccati equation to solve the Zakharov-Kuznetsov system [8]. Rowlands and infield method the Zakharov-Kuznetsov system was examined for wave length disorders [9]. Via using the auxiliary equation method was solved Zakharov-Kuznetsov [10]. The solution of Zakharov-Kuznetsov system via variational iteration method with the description of partial derivatives using caputo [11]. Solve the Zakharov-Kuznetsov system using the Homotopy analysis method [12]. Find the approximate analytical solution of Zakharov-Kuznetsov system using the fractional iteration method [13]. In order to deal with normal and partial linear and non-linear differential equations we have used the new iterative method (NIM) because of its flexibility and ability to solve non-linear differential equations with high quality and smoothly and here lies its strong characteristic. It has also been used in the analytical processing of fractional partial differential equations in general physics and fluid mechanics [14]. The new iterative method was proposed by Daftardar-Gejji and Jafari in order to overcome the problem of solving partial and ordinary nonlinear differential equations, systems of differential and algebraic equations [15-18]. One of the most prominent problems we face in mathematical sciences, especially in numerical analysis, is solving non-linear equations. The new iterative method is one of the best modern methods for solving non-linear equations in addition to the Homotopy perturbation method, and decomposition method and others [19-21].

Moreover, Grosan and Abraham [6], also debate the applicability of the iterative methods for solving nonlinear systems in various sciences like: neurophysiology, the kinematics synthesis problem, the burning problem, and the economics modeling problem [22-24]. One of the best improvements from nature is PSO, which was developed by James Kennedy and Russell Eberhart in 1995 [1, 2]. PSO has recently emerged as an algorithm in solving multiple improvement problems and in various science and engineering. [25-27]. The particle swarm optimization algorithm is a population-based random search algorithm that is a good solution to the problem of nonlinear optimization, the basis of the PSO algorithm is inspired by the social behavior of animals such as the flow of fish, the education of birds, etc. As everyone knows, when a group of birds or fish want to find food, they search in an area to find the best place to eat. The nature of social behavior enables any member of this squadron to find a way that is desirable (the best way) then the process of following the members of the squadron begins to follow the optimal path depending on the location of the particle and its speed as the squadrons communicate this information to each other while adjusting its location and speed Dynamically exploited best For each site particles then the next step begins a transition to the rest of the members of the squadron to the ideal location, The PSO method has becoming known and widely used because of its features, including ease and simplicity in implementation, its ability to reach convergence quickly to an optimal solution, The use of rudimentary mathematical operators [28, 29]. The PSO algorithm also features the following:

The pso algorithm does not contain derivatives, there are a finite number of parameters and their impact is small compared to other optimization techniques, you do not need a long period of time in order to guarantee the closeness and optimal value of the problem, It's easy in theory [30, 31], Work to reduce the error in the numerical method used and the proposed strategy and thus to reach the slightest error we can get an approximate and analytical solution closer to the accurate solution of the Zakharov-Kuznetsov system.

In this article it is tidy as follows: 1. In the first paragraph the mathematical model and reference review are presented and a simple profile of the pso algorithm 2. In the second paragraph, he presented the basic idea of the numerical method used 3. In the third paragraph, we solve the non-linear Zakharov-Kuznetsov system with the numerical method used. 4. In the fourth paragraph is dedicated to clarifying the proposed method of (NIM-PSO) 5. In the fifth paragraph, discuss the final observations on the results of the proposed strategy, its effectiveness and its proximity to the precise solution of Zakharov-Kuznetsov system.

2. THE NUMERICAL UTILIZED

Daftardar-Gejji and Dr. Jafari studied the next practical equation [8].

$$G = f + L(G) + N(G) \tag{3}$$

L: is a nonlinear factor, N a nonlinear factor and (f) a famous function. (u) is supposed to be a solution of (3) owning the chain shape:

$$G=\sum_{i=0}^{\infty}G_i \tag{4}$$

That is:

$$H_0=N(G_0) \tag{5}$$

$$H_m=N(\sum_{i=0}^{m}G_i)-N(\sum_{i=0}^{m-1}G_i) \tag{6}$$

The nonlinear limit N can be decomposed as:

$$N(\sum_{i=0}^{\infty}G_i)=\underbrace{N(G_0)}_{H_0}+\underbrace{N(G_0+G_1)-N(G_0)}_{H_1}+\underbrace{N(G_0+G_1+G_2)-N(G_0+G_1)}_{H_2}+\underbrace{N(G_0+G_1+G_2+G_3)-N(G_0+G_1+G_2)}_{H_3}+\cdots\cdots \tag{7}$$

moreover, the recurrence relation is acquainting as:

$$G_0=f\ ; \tag{8}$$

$$G_1=L(G_0)+H_0\ ; \tag{9}$$

$$G_{m+1}=L(G_m)+H_m\ ;m=1,2,3,\dots\dots\dots \tag{10}$$

$$Since\ (L)\ is\ a\ linear\ factor\ \sum_{i=0}^{m}L(G_i)=L(\sum_{i=0}^{m}G_i),\ \ then \tag{11}$$

$$\begin{Bmatrix}\sum_{i=1}^{m+1}G_i & =\sum_{i=0}^{m}L(G_i)+N(\sum_{i=0}^{m}G_i)\\ & =L(\sum_{i=0}^{m}G_i)+N(\sum_{i=0}^{m}G_i)\end{Bmatrix} \tag{12}$$

$$\sum_{i=0}^{\infty}G_i=f+L(\sum_{i=0}^{\infty}G_i)+N(\sum_{i=0}^{\infty}G_i) \tag{13}$$

As shown from (13), it is an equivalent solution $(\sum_{i=0}^{\infty}G_i)$ to (3), where i = 0,1,2,3 .... are given by (8), (10). The R-term Approximate solution of (13) is given by $u=\sum_{i=0}^{R-1}G_i$. The n-border solutions of (1), it is as follows $j\approx j_0+j_1+j_2+j_3+\cdots\cdots+j_{n-1}$. As the numerical method is Convergence [16, 32].

3. SOLVE THE ZAKHAROV-KUZNETSOV IN ANUMERIACL METHOD SYSTEM ZAKHAROV-KUZNETSOV

Here, labor on for disbanding G (x, t) and T (x, t) the initial conditions which proper (2):

$$\begin{Bmatrix}G(x,0)=-\frac{\alpha+\eta+\omega}{6\mu}-\mathrm{sech}(kx+vy)+\frac{3k(\alpha+2\mu)}{\alpha(3k+\omega)}sech^2(kx+vy)\\ T(x,0)=-\frac{\alpha+\eta+\omega}{6\mu}-\mathrm{sech}(kx+vy)+\frac{3(k\alpha+8k\mu+2\mu\omega)}{\alpha(3k+\omega)}sech^2(kx+vy)\end{Bmatrix} \tag{14}$$

We construction in the Coupled System Zakharov-Kuznetsov (2) which satisfy:

$$\begin{Bmatrix}B(G,T)=-\frac{\partial^3u}{\partial x^3}-\frac{\partial^3u}{\partial y^2\partial x}+6u\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}\\ J(G,T)=-\delta\frac{\partial^3v}{\partial x^3}-\lambda\frac{\partial^3v}{\partial y^2\partial x}-\eta\frac{\partial v}{\partial x}+6\mu v\frac{\partial v}{\partial x}+\alpha\frac{\partial u}{\partial x}\end{Bmatrix} \tag{15}$$

Now the integration of the system Coupled System Zakharov-Kuznetsov produces us the following:

$$\begin{Bmatrix}G(\mathrm{x,t})=\int_0^t B(G(\mathrm{x,t}),T(\mathrm{x,t}))dt=\int_0^t\left(-\frac{\partial^3G}{\partial x^3}-\frac{\partial^3G}{\partial y^2\partial x}+6u\frac{\partial G}{\partial x}+\frac{\partial T}{\partial x}\right)dt\\ T(\mathrm{x,t})=\int_0^t J(G(\mathrm{x,t}),T(\mathrm{x,t}))dt=\int_0^t\left(-\delta\frac{\partial^3T}{\partial x^3}-\lambda\frac{\partial^3T}{\partial y^2\partial x}-\eta\frac{\partial T}{\partial x}+6\mu v\frac{\partial T}{\partial x}+\alpha\frac{\partial G}{\partial x}\right)dt\end{Bmatrix} \tag{16}$$

Using the initial conditions given, we get:

$$G_1(\mathrm{x,t}) = \int_0^t B(G_0,T_0)dt = \int_0^t \left(-\frac{\partial^3 G_0}{\partial x^3} - \frac{\partial^3 G_0}{\partial y^2 \partial x} + 6G_0\frac{\partial G_0}{\partial x} + \frac{\partial T_0}{\partial x}\right)dt \quad (17)$$

After the integration on the (17) we get

$$G_1(\mathrm{x,t}) = -\frac{1}{cosh(8x-3)^5}(0.01134215501\sinh(8x-3)\cdots\cdots\cdots + (3.474432 * 10^6)t)$$

$$T_1(\mathrm{x,t}) = \int_0^t J(G_0,T_0)dt = \int_0^t \left(-\delta\frac{\partial^3 T_0}{\partial x^3} - \lambda\frac{\partial^3 T_0}{\partial y^2 \partial x} - \eta\frac{\partial T_0}{\partial x} + 6\mu T_0\frac{\partial T_0}{\partial x} + \alpha\frac{\partial G_0}{\partial x}\right)dt \quad (18)$$

After the integration on the (18) we get

$$T_1(\mathrm{x,t}) = -\frac{1}{cosh(8x-3)^5}(0.01512287334\sinh(8x-3)\cdots\cdots + (7.9480800 * 10^6)t)$$

$$G_2(\mathrm{x,t}) = \int_0^t B(G_1,T_1)dt = \int_0^t \Bigg(\left(-\frac{\partial^3 G_0}{\partial x^3} - \frac{\partial^3 G_0}{\partial y^2 \partial x} + 6G_0\frac{\partial G_0}{\partial x} + \frac{\partial T_0}{\partial x}\right) + \left(-\frac{\partial^3 G_1}{\partial x^3} - \frac{\partial^3 G_1}{\partial y^2 \partial x} + 6G_1\frac{\partial G_1}{\partial x} + \frac{\partial T_1}{\partial x}\right) - \\ (-\frac{\partial^3 G_0}{\partial x^3} - \frac{\partial^3 G_0}{\partial y^2 \partial x} + 6G_0\frac{\partial G_0}{\partial x} + \frac{\partial T_0}{\partial x})\Bigg)dt \quad (19)$$

$$G_2(\mathrm{x,t}) = -\frac{1}{cosh(8x-3)^{11}}(0.0032\, t^2\cdots\cdots + 3.1593126 * 10^7\cosh(8x-3)^{10})$$

$$T_2(\mathrm{x,t}) = \int_0^t J(G_1,T_1)dt = \int_0^t \Bigg(\left(-\delta\frac{\partial^3 T_0}{\partial x^3} - \lambda\frac{\partial^3 T_0}{\partial y^2 \partial x} - \eta\frac{\partial T_0}{\partial x} + 6\mu T_0\frac{\partial T_0}{\partial x} + \alpha\frac{\partial G_0}{\partial x}\right) + \left(-\delta\frac{\partial^3 T_0}{\partial x^3} - \lambda\frac{\partial^3 T_1}{\partial y^2 \partial x} - \\ \eta\frac{\partial T_1}{\partial x} + 6\mu T_1\frac{\partial T_1}{\partial x} + \alpha\frac{\partial T_1}{\partial x}\right) - (-\delta\frac{\partial^3 T_0}{\partial x^3} - \lambda\frac{\partial^3 T_0}{\partial y^2 \partial x} - \eta\frac{\partial T_0}{\partial x} + 6\mu T_0\frac{\partial T_0}{\partial x} + \alpha\frac{\partial G_0}{\partial x})\Bigg)dt \quad (20)$$

$$T_2(\mathrm{x,t}) = -\frac{1}{cosh(8x-3)^{11}}(0.00032 * t^2\cdots\cdots + 3.72574961 * 10^8\, cosh(8x-3)^{10})$$

And finally:

$$G_3(\mathrm{x,t}) = \int_0^t B(G_2,T_2)dt = \int_0^t \Bigg(\left(-\frac{\partial^3 G_0}{\partial x^3} - \frac{\partial^3 G_0}{\partial y^2 \partial x} + 6G_0\frac{\partial G_0}{\partial x} + \frac{\partial T_0}{\partial x}\right) + \left(-\frac{\partial^3 G_1}{\partial x^3} - \frac{\partial^3 G_1}{\partial y^2 \partial x} + 6G_1\frac{\partial G_1}{\partial x} + \\ \frac{\partial T_1}{\partial x}\right) + \left(-\frac{\partial^3 G_2}{\partial x^3} - \frac{\partial^3 G_2}{\partial y^2 \partial x} + 6G_2\frac{\partial G_2}{\partial x} + \frac{\partial T}{\partial x}\right) - \left(-\frac{\partial^3 G_0}{\partial x^3} - \frac{\partial^3 G_0}{\partial y^2 \partial x} + 6G_0\frac{\partial G_0}{\partial x} + \frac{\partial T_0}{\partial x}\right) + (-\frac{\partial^3 G_1}{\partial x^3} - \frac{\partial^3 G_1}{\partial y^2 \partial x} + \\ 6G_1\frac{\partial G_1}{\partial x} + \frac{\partial T_1}{\partial x})\Bigg)dt \quad (21)$$

$$G_3(\mathrm{x,t}) = -\frac{1}{cosh(8x-3)^{23}}(0.0032 * t^2\cdots\cdots - 3.638173925 * 10^{26}\sinh(8x-3)\, t^2)$$

$$T_3(\mathrm{x,t}) = \int_0^t J(G_2,T_2)dt = \int_0^t \Bigg(\left(-\delta\frac{\partial^3 T_0}{\partial x^3} - \lambda\frac{\partial^3 T_0}{\partial y^2 \partial x} - \eta\frac{\partial T_0}{\partial x} + 6\mu T_0\frac{\partial T_0}{\partial x} + \alpha\frac{\partial G_0}{\partial x}\right) + \left(-\delta\frac{\partial^3 T_1}{\partial x^3} - \\ \lambda\frac{\partial^3 T_1}{\partial y^2 \partial x} - \eta\frac{\partial T}{\partial x} + 6\mu T_1\frac{\partial T_1}{\partial x} + \alpha\frac{\partial G_1}{\partial x}\right) + \left(-\delta\frac{\partial^3 T_0}{\partial x^3} - \lambda\frac{\partial^3 T_1}{\partial y^2 \partial x} - \eta\frac{\partial T_1}{\partial x} + 6\mu T_1\frac{\partial T_1}{\partial x} + \alpha\frac{\partial G_1}{\partial x}\right) - \\ \left(-\delta\frac{\partial^3 T_0}{\partial x^3} - \lambda\frac{\partial^3 T_0}{\partial y^2 \partial x} - \eta\frac{\partial T_0}{\partial x} + 6\mu T_0\frac{\partial T_0}{\partial x} + \alpha\frac{\partial G_0}{\partial x}\right) + (-\delta\frac{\partial^3 T_1}{\partial x^3} - \lambda\frac{\partial^3 T_1}{\partial y^2 \partial x} - \eta\frac{\partial T_1}{\partial x} + 6\mu T_1\frac{\partial T_1}{\partial x} + \\ \alpha\frac{\partial G_1}{\partial x})\Bigg)dt \quad (22)$$

$$T_3(\mathrm{x,t}) = -\frac{1}{cosh(8x-3)^{23}}(1.6 * 10^{-7} * t\cdots - 2.375860822 * 10^{-7}\cosh(8x-3)^9 * t^3)$$

$$G(\mathrm{x,t}) = -\frac{1}{cosh(8x-3)^{23}}\ (1.00 * 10^{-9}(-2.021960064 * 10^{14}\cosh(8x-3)^{22} * t^2 \\ + \cdots\cosh(8x-3) \quad (23)$$

$$T(\mathrm{x},\mathrm{t}) = -\frac{1}{cosh(8x-3)^{23}}(1.00 * 10^{-9}(1.192239875 * 10^{14}\cosh(8x-3)^{22} * t^2 \cdots \sinh(8x-3)\cosh(8x-3)) \quad (24)$$

4. THE SUGGESTED STRATEGY

The basic idea of the Suggested method is to look for the best parameters of the nonlinear (ZK) system which have a direct and strong effect on improving the results, where a series of solutions $G_{(1,2,3)}$ was also used as $T_{(1,2,3)}$ (23), (24) Using the PSO algorithm with the nim method:

$$\begin{Bmatrix} G(k,\lambda,\eta) = \sum_{a=1}^{P}\sum_{b=1}^{L}\left(G(x_i,t_j) - \hat{G}(e_i,i_j)\mathrm{e},\mathrm{i}\right)^2 \\ T(k,\lambda,\eta) = \sum_{a=1}^{n}\sum_{b=1}^{m}\left(T(x_i,t_j) - \hat{T}(e_i,i_j)\right)^2 \\ F = \frac{1}{2}|G(k,\lambda,\eta) + T(k,\lambda,\eta)| \end{Bmatrix}$$

Where (P) and (L) represent the total number of steps used in the solution field and (G&T) respectively, nonlinear system solutions (ZK) (23), (24), (G, T̂) are the careful solutions of the system used. (F) symbolize the fitness function (means square fault) is solved by algorithm (PSO) by the proposed technique, the optimal worths of the system (23), (24) were obtained as follows:

$$\begin{Bmatrix} k = -8 \\ \lambda = -3.35987 \\ \eta = -8.75 \end{Bmatrix}$$

By comparing the results of the numerical method used and the proposed strategy with the numerical solution, it turns out that the fault is decreasing to 10^{-12} at its lowest level and to 10^{-15}. At its better as described in Table 1, Figure 1, and Figure 2.

Table 1. Comparison between utter fault between the careful solution and numerical solution

t	$\lvert G_{exact} - G_{nim}\rvert$	$\lvert G_{exact} - G_{pso}\rvert$
0.1	$6.702415*10^{-5}$	$6.414042*10^{-15}$
0.2	$4.990430*10^{-5}$	$5.119540*10^{-14}$
0.3	$3.081272*10^{-4}$	$1.727122*10^{-13}$
0.4	$6.291093*10^{-4}$	$4.093323*10^{-13}$
0.5	$9.433738*10^{-4}$	$7.994233*10^{-13}$
0.6	$1.814400*10^{-3}$	$1.381353*10^{-12}$
0.7	$1.273757*10^{-3}$	$2.193489*10^{-12}$
0.8	$1.150644*10^{-3}$	$3.274199*10^{-12}$
0.9	$7.422191*10^{-4}$	$4.661851*10^{-12}$
1	$2.165810*10^{-5}$	$6.394813*10^{-12}$
MSE	$6.282541862*10^{-7}$	$7.8508321*10^{-24}$

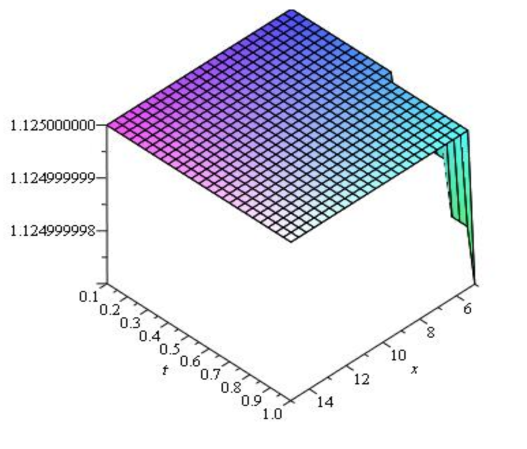

Figure 1. The numeral solution G-3 using the proposed technique between the numerical method and the algorithm used

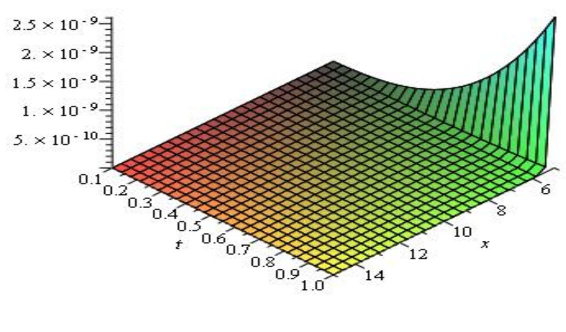

Figure 2. Utter fault G-3 using the proposed technique between the numerical method and the algorithm used

By comparing the results of the numerical method used and the proposed strategy with the numerical solution, it turns out that the fault is decreasing to 10^{-12} at its lowest level and to 10^{-15} at its better as described in Table 2, Figure 3 and Figure 4.

Table 2. Comparison between utter fault between the careful solution and numerical solution

t	$\lvert T_{exact} - T_{nim} \rvert$	$\lvert T_{exact} - T_{nim(pso)} \rvert$
0.1	1.115099*10^{-5}	8.245542*10^{-15}
0.2	4.158531*10^{-5}	6.583776*10^{-14}
0.3	7.263316*10^{-5}	2.221242*10^{-13}
0.4	8.570244*10^{-5}	5.264519*10^{-13}
0.5	6.228989*10^{-5}	1.028168*10^{-12}
0.6	1.600810*10^{-5}	1.776620*10^{-12}
0.7	1.674838*10^{-4}	2.821154*10^{-12}
0.8	4.103056*10^{-4}	4.211118*10^{-12}
0.9	7.625057*10-4	5.995858*10^{-12}
1	1.241967*10^{-3}	8.224723*10^{-12}
MSE	6.28254187410*10^{-7}	1.320076037*10^{-23}

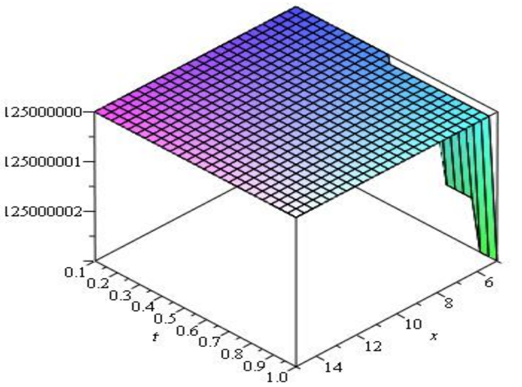

Figure 3. The numerical solution T-3 using the proposed technique between the numerical method and the algorithm used

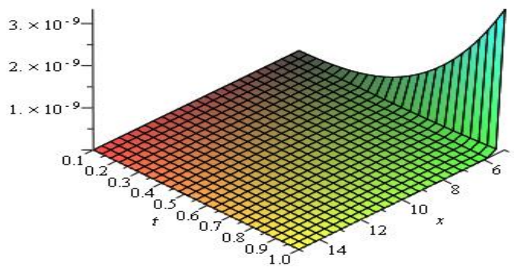

Figure 4. Utter fault T-3 using the proposed technique between the numerical method and the algorithm used

5. CONCLUSIONS

In this work, the numerical method was utilized, then a modern technique It has been suggested that the NIM-PSO algorithm be combined. to a particle swarm optimization (PSO-NIM) algorithm, to obtain approximate and analytical solutions to partial differential equation systems that are effective and easy to use In the Zakharov-Kuznetsov system, the parameters k, λ, Ƞ have a direct effect on the results despite the complexity of the system, and the results after comparison with the exact solution of the system used showed a highly efficient convergence, and the results demonstrated the correctness and methodology of the proposed strategy.

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