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Application of Designing Economic Mechanisms to Power Market - Part 1 Generation Side Power Market Design

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Abstract

The paper studies on the core philosophy and algorithm of the designing economic mechanisms theory, a new algorithm of designing incentive compatible power market mechanisms is proposed, a generation side power market mechanism model which has features of inventive compatibility, informationally efficient and decentralized decision is constructed. The power market based on the designing economic mechanisms theory can lead to the Pareto Optimality of the resource allocation; meanwhile GENCOs are permitted to pursue profits maximization. The paper is in two parts. Part 1 focuses on the process of constructing a generation side power market competitive mechanism model based on the designing economic mechanisms theory. Part 2 presents the characteristic analysis of the generation side power market competitive mechanism.

Keywords*: power market, designing economic mechanisms, reflexive rectangle method, flagpoles method*

1. Introduction

The mechanism design of power market is the key issue about the successful implementation of power market and the market efficiency [1]. Thus, researchers always pay a close attention to how to design a power market mechanism with an efficient allocation of resources. In recent decades, the asymmetric information game theory, the social selection theory and the mechanism design theory are applied widely in economics, society and engineering field such as the electronic commerce, the auction, the pricing of public goods and the deregulation of utilities [2-9]. So, researchers begin to apply those theories to designing power market mechanisms and realize that incentive compatible mechanisms are able to restrain strategic game behaviors of GENCOs (Generation Companies) in essentially [10].

The designing economic mechanisms theory used in this paper is different from other mechanism design theories, which is able to achieve incentive compatibility and *informationally efficient* simultaneously. By applying the view of the information economics, the designing economic mechanisms theory researches on how to design a mechanism which meets expectations of designers in a system condition as the same as a power market with the freedom of choice, the voluntary exchange and the asymmetric information.

2. The Basic Theory and Algorithm of Designing Economic Mechanisms

The designing economic mechanisms theory which is presented by Professor Leonid Hurwicz who won the 2007 Nobel Prize in economics and Professor Stanley Reiter researches on how to design an incentive compatible, informationally efficient and decentralized mechanism [11, 12].

The designing economic mechanisms theory defines that a mechanism π consists of three parts: $M=\{m\}$ a message space, $g(m,\theta)$ the equilibrium relations and $h(m)$ the outcome function that translates equilibrium relations into the mechanism outcome. A mechanism models communication through messages, verification scenario by agents, and the outcomes associated with equilibrium messages. It can be written as

$$\pi=(M,g,h) \tag{1}$$

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The entire algorithm of the designing economic mechanisms can be divided into three stages: deriving the goal function, constructing the coverings of agents equilibrium message and generating the agents equilibrium message functions and outcome functions. The reflexive rectangles method and the flagpoles method are main algorithms in the process of designing.

(1) Reflexive Rectangles Method

An agent i is in equilibrium if and only if his equilibrium message function $g_i(m,\theta)=0$. The *coverings* C_v of agents equilibrium message functions are the set of points that makes every agent's $g_i(m,\theta)=0$ in the parameter space θ. The reflexive rectangles method is an important approach to constructing coverings C_v of agents equilibrium message function $g_i(m,\theta)$. Professor Hurwicz and his colleagues have proved that coverings C_v structured through the reflexive rectangles method are partitions and disjoint [13, 14], the mechanism is able to meet informationally efficient's requirement because of its low amount of information.

The coverings correspondence is written as $V(\theta)$, its domain is the parameter space θ and its range is the Cartesian product of θ. The correspondence $V(\theta)$ is given by

$$V(\theta)=G(\bar{\theta},\theta)=0 \tag{2}$$

In Equation (2), $V(\theta)$ is a self-belonging correspondence, it corresponds to a given factorization of the parameter space θ, which reflects the initial dispersion of information among the agents [11]. Also, $V(\theta)$ is compatible with the goal function in that all elements of $V(\theta)$ are in the same goal function's contour set.

(2) Flagpoles Method

The function $H(\theta)=0$ is defined as a *flagpole*. When coverings C_v of the parameter space θ are partitions and the Jacobian matrix J which is made up of the partial derivative of each agent's equilibrium message function $\partial G/\partial\theta$ and *flagpoles* functions partial derivative $\partial H/\partial\theta$ is nonsingular, the *flagpoles* are called the system of distinct representatives (SDR). Then, the outcome function $h(m)$ is to be generated from the SDR [15], and the entire designing process will be accomplished in the end.

$$J=\begin{bmatrix} \partial G_1/\partial\theta_1 & \cdots & \partial G_1/\partial\theta_i \\ \vdots & \ddots & \vdots \\ \partial G_i/\partial\theta_1 & \cdots & \partial G_i/\partial\theta_i \\ \partial H_1/\partial\theta_1 & \cdots & \partial H_1/\partial\theta_i \\ \vdots & \ddots & \vdots \\ \partial H_i/\partial\theta_1 & \cdots & \partial H_i/\partial\theta_i \end{bmatrix} \tag{3}$$

3. The Competitive Mechanism Designing of Generation Side Power Market based on the Designing Economic Mechanisms Theory

The following uses the competitive mechanism design of generation side power market as an example to describe how to design an incentive compatible, informationally efficient and decentralized power market by the designing economic mechanisms theory.

3.1. The Derivation of Goal Function

In the generation side power market, we assume that each GENCO is equivalent to a generator unit. According to the unit's equivalent generation cost with quadratic function:

$$C_i(P_i)=\alpha_i+\beta_i P_i+\gamma_i P_i^2 \tag{4}$$

where P_i is the output of GENCO i, $C_i(P_i)$ is the generation cost under the output, α_i, β_i, γ_i are GENCO i economic parameters related to the generation cost.

A competitive generation side power market model with three GENCOs is to be established. For focusing on designing power market mechanism, the power system model is to be simplified by ignoring transmission losses, flows of power and some detailed technical parameters. GENCO i marginal cost MC_i is given by

$$MC_i = \frac{\mathrm{d}C_i}{\mathrm{d}P_i} = \beta_i + 2\gamma_i P_i \tag{5}$$

We assume that the total output of three GENCOs is greater than the next period loads P_L. The constraint (6) must be kept for the balance between outputs and the load.

$$P_1 + P_2 + P_3 = P_L \tag{6}$$

According to the principle of profit maximization in economics, the GENCO i is in equilibrium and gets the maximum profits when its marginal cost MC_i is equal to marginal revenue MR_i. At the same time, the resource allocation of power market is Pareto Optimality and the social benefit achieves the maximization.

From the view of the designing economic mechanism theory, inputs and outputs of the mechanism should be known. The next period requirement of load P_L and the generation cost function's parameters α_i, β_i, γ_i of GENCO i belong to inputs. Assuming that the settlement mode is the uniform clearing price, we regard GENCO i's power P_i which wins the bidding and the uniform clearing price of market p as outputs. Those information parameters decide the feasible configuration set, the available exchange set and the Pareto Optimality's exchange set of the whole generation side power market.

Considering the equilibrium condition of $MR_i = MC_i$, GENCO i's output P_i is given by

$$P_i = (p - \beta_i)/(2\gamma_i) \tag{7}$$

Then, the Equation (8) is to be derived by the market clearing condition (6) and the market equilibrium condition (7).

$$2\gamma_i P_i + \beta_i = p \qquad i = 1,2,3 \tag{8}$$

Define $2\gamma_1 = a_1$, $\beta_1 = a_2$, $2\gamma_2 = b_1$, $\beta_2 = b_2$, $2\gamma_3 = c_1$, $\beta_3 = c_2$, solving the Equation (8) leads to

$$P_1 = \frac{b_1c_2 - a_2c_1 - a_2b_1 + b_2c_1 + P_L b_1 c_1}{a_1b_1 + a_1c_1 + b_1c_1} \tag{9}$$

$$P_2 = \frac{a_1c_2 - a_1b_2 + a_2c_1 - b_2c_1 + P_L a_1 c_1}{a_1b_1 + a_1c_1 + b_1c_1} \tag{10}$$

$$P_3 = \frac{a_1b_2 + a_2b_1 - a_1c_2 - b_1c_2 + P_L a_1 b_1}{a_1b_1 + a_1c_1 + b_1c_1} \tag{11}$$

$$p = \frac{a_1b_1c_2 + a_1b_2c_1 + a_2b_1c_1 + P_L a_1 b_1 c_1}{a_1b_1 + a_1c_1 + b_1c_1} \tag{12}$$

In parameter space θ, we define that the goal function $F(\theta)$ is GENCO 1 output P_1, and hence, the goal function given to the designer is

$$F(\theta)=\frac{b_1c_2-a_2c_1-a_2b_1+b_2c_1+P_Lb_1c_1}{a_1b_1+a_1c_1+b_1c_1} \tag{13}$$

In the Equation (13), $\theta=(a,b,c)$, $a=(a_1,a_2)$, $b=(b_1,b_2)$, $c=(c_1,c_2)$.

3.2. The Construction of Equilibrium Messages Coverings

The coverings C_v of the parameter space θ are constructed by the reflexive rectangles method. The equilibrium relation of GENCO 1 is defined by

$$G_1(\overline{\theta},a)=F(\overline{\theta},a)-F(\overline{\theta})=0 \tag{14}$$

We obtain the GENCO 1 equilibrium function (15) by solving the Equation (14).

$$G_1(\overline{\theta},a)=a_1\overline{F}+a_2-\overline{p}=0 \tag{15}$$

where, $\overline{F}=\dfrac{\overline{b}_1\overline{c}_2-\overline{a}_2\overline{c}_1-\overline{a}_2\overline{b}_1+\overline{b}_2\overline{c}_1+P_L\overline{b}_1\overline{c}_1}{\overline{a}_1\overline{b}_1+\overline{a}_1\overline{c}_1+\overline{b}_1\overline{c}_1}$, $\overline{p}=\dfrac{\overline{a}_1\overline{b}_1\overline{c}_2+\overline{a}_1\overline{b}_2\overline{c}_1+\overline{a}_2\overline{b}_1\overline{c}_1+P_L\overline{a}_1\overline{b}_1\overline{c}_1}{\overline{a}_1\overline{b}_1+\overline{a}_1\overline{c}_1+\overline{b}_1\overline{c}_1}$

Solving the Equation (15) for a_2 and according to the reflexive rectangles method, the equilibrium function of GENCO 2 is given by

$$G_2(\overline{\theta},b)=b_1(P_L-\overline{F}-\overline{R})+b_2-\overline{p}=0 \tag{16}$$

Similarly, the equilibrium function of GENCO 3 is given by

$$G_3(\overline{\theta},c)=c_1\overline{R}+c_2-\overline{p}=0 \tag{17}$$

where, $\overline{R}=(\overline{p}-\overline{c}_2)/\overline{c}_1$.

Now, according to the reflexive rectangles method, coverings C_v of GENCO's equilibrium messages in the parameter space $\theta=(a,b,c)$ have been constructed completely.

$$C_v=\begin{cases}G_1(\overline{\theta},a)=0\\ G_2(\overline{\theta},b)=0\\ G_3(\overline{\theta},c)=0\end{cases} \tag{18}$$

3.3. Generating Equilibrium Message Functions and Outcome Functions

For using the *flagpoles method*, we assume $a_1=b_1=c_1=1$, *flagpole* functions are defined by $H_1=a_1-1=0$, $H_2=b_1-1=0$ and $H_3=c_1-1=0$.

The Jacobian matrix J like the Equation (3) can be deriving from the equation set of coverings (18) and *flagpole* functions.

$$J=\begin{bmatrix}1&0&0&\overline{F}&0&0\\0&1&0&0&P_L-\overline{F}-\overline{R}&0\\0&0&1&0&0&\overline{R}\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix} \tag{19}$$

The Jacobian matrix (19) is nonsingular, which means the *flagpoles* decided by the solution θ^* of the equation set (18) and flagpole functions exist in the parameter space $\theta=(a,b,c)$. Substituting $a_1^* = b_1^* = c_1^* = 1$ into the equation set (18) leads to:

$$a_2^* = \overline{p} - \overline{F} \tag{20}$$

$$b_2^* = \overline{p} - P_L + \overline{F} + \overline{R} \tag{21}$$

$$c_2^* = \overline{p} - \overline{R} \tag{22}$$

Define the message space M as a point set $\theta^* = (a_1^*, a_2^*, b_1^*, b_2^*, c_1^*, c_2^*)$ which belongs to *flagpoles* in parameter space $\theta=(a,b,c)$. The basic elements of message space M are $m=(m_1,m_2,m_3)$, where $m_1 = a_2^*$, $m_2 = b_2^*$, $m_3 = c_2^*$. Substituting $m=(m_1,m_2,m_3)$ into the Equation (20), (21) and (22), we obtain

$$\overline{F} = \frac{-2m_1 + m_2 + m_3 + P_L}{3} \tag{23}$$

$$\overline{R} = \frac{m_1 + m_2 - 2m_3 + P_L}{3} \tag{24}$$

$$\overline{p} = \frac{m_1 + m_2 + m_3 + P_L}{3} \tag{25}$$

GENCO i's equilibrium message function $g_i(m,\theta)$ is obtained from substituting the Equation (23), (24) and (25) into the Equation (18).

$$g_1(m,a) = a_1(-2m_1 + m_2 + m_3 + P_L) + 3a_2 - (m_1 + m_2 + m_3 + P_L) \tag{26}$$

$$g_2(m,b) = b_1(-2m_2 + m_1 + m_3 + P_L) + 3b_2 - (m_1 + m_2 + m_3 + P_L) \tag{27}$$

$$g_3(m,c) = c_1(-2m_3 + m_1 + m_2 + P_L) + 3c_2 - (m_1 + m_2 + m_3 + P_L) \tag{28}$$

Outcome functions $h_i(m)$ of the generation side power market are given by flagpoles in parameter space.

$$h_1(m) = \frac{-2m_1 + m_2 + m_3 + P_L}{3} = \overline{F} \tag{29}$$

$$h_2(m) = \frac{m_1 + m_2 - 2m_3 + P_L}{3} = \overline{R} \tag{30}$$

$$h_3(m) = \frac{m_1 + m_3 + m_2 + P_L}{3} = \overline{p} \tag{31}$$

$h_1(m)$, $h_2(m)$ and $h_3(m)$ correspond to P_1 the output of GENCO 1, P_3 the output of GENCO 3 and p the uniform clearing price. The output of GENCO 2 is to be solved by

$$P_2 = P_L - P_1 - P_3 \tag{32}$$

Given by the Equation (33), the final mechanism is made up of message space M, equilibrium message functions $g_i(m,\theta)$ and outcome functions $h(m)$.

$$\pi = [M,\, g_i(m,\theta),\, h(m)\,] \tag{33}$$

Now, we have constructed the competitive generation side power market mechanism which follows the principle of profit maximization. Meanwhile, the mechanism with incentive compatibility and *inforamtionally efficient* is able to achieve the maximum social benefit.

4. Conclusion

The paper studies on the designing economic mechanisms theory's core algorithms such as the *reflexive rectangles method* and the *flagpoles method* are also researched.

A new approach based on the designing economic mechanisms theory to designing an incentive compatible power market mechanism is proposed. The paper constructs the competitive mechanism model of generation side power market by using the *reflexive rectangles method* and the *flagpoles method*. The power market designed by the designing economic mechanisms theory has key features of the incentive compatibility, the *informationally efficient* and the decentralized decision. Thus, it is able to realize the Pareto Optimality and the social benefit achieves the maximization when GENCOs also achieve profit maximization.

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