TELKOMNIKA, Vol. 11, No. 4, April 2013, pp. 1883~1888
ISSN: 2302-4046

A Novel Discrete Differential Evolution Algorithm

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Abstract

Aiming at the stochastic vehicle routing problems with simultaneous pickups and deliveries, a novel discrete differential evolution algorithm is proposed for routes optimization. The algorithm can directly be used for the discrete domain by special design. Computational simulations and comparisons based on two kinds of problems of different sizes of SVRPSPD are provided. Results demonstrate that the proposed algorithm obtains better results than the basic differential evolution algorithm and the existing genetic algorithm.

Keywords: *SVRPSPD, bitwise operator, discrete differential evolution algorithm*

1. Introduction

The SVRPSPD is a variant of the classical VRP which is often encountered in practice. After VRPSPD is introduced into the literature by Min [1], some scholars contribute on the mathematical formulation and the solution methods. In problems, such as capacity constraint and time windows being taken into account. On the other hand, because VRPSPD has been proved to be NP-hard, the solution methods mostly focus on the heuristics and meta-heuristics. With the development of computer technology, a few heuristics and meta-heuristics have been used to solve VRPSPD [2, 3, 4].

To date, most studies on VRPSPD focus on the level of deterministic model. However, in many real-world applications, one or more parameters of VRPSPD tend to be stochastic. Thus, it is necessary to study the model and the algorithm of stochastic VRPSPD.

The differential evolution (DE) algorithm is one of the latest intelligent optimization algorithms proposed by Storn and Price [5]. As a population-based evolutionary algorithm, DE is originally designed for continuous optimization problems which use simple mutation and crossover operators to generate new candidate solutions, and applies one-to-one competition strategy to select the new individuals. Due to its simplicity, effectiveness and robustness, DE has been successfully applied in solving continuous problems in a variety of fields. However, Owing to continuous nature of DE, the research on DE for combinatorial optimization is very limited. So, it is urgent to propose a discrete differential evolution algorithm for specific problems.

Recently, some scholars have done some researches on DE for combinatorial optimization problems [6, 7, 8]. Few studies have been done on VRP with DE.

In view of above analysis, this paper focuses on designing a novel discrete differential evolution algorithm (DDE). In DDE, individuals are represented as discrete client ordinal, and new mutation operator is defined based on the definition of new algebraic structures. Consequently, DDE can be directly applied to the combinatorial optimization problem where chromosomes are natural numbers.

The remaining paper is organized as follows. In section 2 the stochastic programming model of SVRPSPD is presented. The structure of DE is given in section 3. In section 4 DDE is introduced comprehensively. And in section 5, the computational results over different sizes of SVRPSPD are discussed. Finally, conclusions and some suggestions for future work are summarized.

Received January 7, 2013; Revised February 12, 2013; Accepted February 24, 2013

2. Problem Formulation and Preliminaries

SVRPSPD discussed in this paper can be described as follows: Given a single depot, a set of clients where each client simultaneously has both a delivery demand and a pick-up demand and must be served once by only one vehicle, a homogeneous fleet of vehicles where each vehicle, which delivers the goods from the depot to clients as well as picks up loads back to the depot, has the same capacity and maximum travel time, and must return to the depot if it can not satisfy the demands of clients or if the maximum travel time exceeds.

In detail, we suppose that: *n* denotes the number of the clients; *m* denotes the maximum available number of vehicle can be used of the depot(0); *C* denotes the vehicle capacity; the delivery demand subjects to the normal distribution, that is $d_i \sim N(\mu_i, \sigma_i^2)$; the pick-up demand is determinate, assuming $p_i = r\mu_i$. We also assume the travel time subjects to the normal distribution, that is $t_{ijk} \sim N(d_{ij}/v_{ij}, \sigma_{ij}^2)$, suppose that the service time for every client is proportional to delivery demand of it, that is $T_i = \lambda d_i$. *B* denotes the maximum travel time. The constraints of capacity and maximum travel time could be unsubstantiated, but the probability of constraints satisfaction is more than the given confidence level. Meanwhile, Δ denotes the percentages allowed for overload; $(1-\beta)$ and $(1-\alpha)$ denote the confidence level.

The problem formulation is as follows:

Let: $x_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ uses arc } (i, j) \\ 0 & \text{otherwise} \end{cases}$ and $z_{ik} = \begin{cases} 1 & \text{if vehicle } k \text{ visits client } i \\ 0 & \text{otherwise} \end{cases}$

$$Minimize \quad z = \sum_{k=1}^{m}\sum_{i=0}^{n}\sum_{j=0}^{n} d_{ij} x_{ijk} \tag{1}$$

$$Subject\ to \ \sum_{k=1}^{m} z_{ik} = 1 \qquad i = 1,2,\cdots,n \tag{2}$$

$$\sum_{j=1}^{n} x_{0jk} \le 1 \qquad k = 1,2,\cdots,m \tag{3}$$

$$\sum_{i=1}^{n} x_{i0k} \le 1 \qquad k = 1,2,\cdots,m \tag{4}$$

$$\sum_{i=0}^{n} x_{ilk} - \sum_{j=0}^{n} x_{ljk} = 0 \quad l = 1,2,\cdots,n;\ k = 1,2,\cdots,m \tag{5}$$

$$P(\sum_{i=0}^{\theta} p_i z_{ik} + \sum_{i=\theta+1}^{n} d_i z_{ik} \ge C(1+\Delta)) = 0 \quad \theta = 0,1,\cdots,n; k = 1,2,\cdots,m \tag{6}$$

$$P(C \le \sum_{i=0}^{\theta} p_i z_{ik} + \sum_{i=\theta+1}^{n} d_i z_{ik} \le C(1+\Delta)) \le \alpha \quad \theta = 0,1,\cdots,n; k = 1,2,\cdots,m \tag{7}$$

$$P(\sum_{i=0}^{n}\sum_{j=0}^{n} t_{ijk} x_{ijk} + \sum_{i=1}^{n} T_i z_{ik} \le B) \ge 1-\beta \quad k = 1,2,\cdots,m \tag{8}$$

$$\sum_{i\in V_k}\sum_{j\in V_k} x_{ijk} \le |V_k| - 1 \ for\ V_k \subseteq V/\{0\}, |V_k| \ge 2, k = 1,2,\cdots,m \tag{9}$$

The objective function (1) minimizes the total distance. Constraints (2) ensure that every client is served by exactly one vehicle. Constraints (3-4) ensure that maximum available number of vehicle is not exceeded. Constraints (5) guarantee that a vehicle exits the client it enters. Constraints (6-7) represent that a small number of overload is allowable, but the probability of overload is less than α . Constraints (8) handle maximum allowable travel time, which allow a certain amount of overtime, but the probability of overtime is less than β . Constraints (9) are sub-tour elimination constraints.

Note that when $\theta = 0$ fixed in constraints (6-7), this model is transformed into the model of SVRP subject to the same constraints. When $\theta = n$ fixed in constraints (6-7), this model is transformed into the model of SVRP with only pickups. When $\theta = 0$ fixed and $m = 1$, that is the depot have only one vehicle, the model is transformed into the model of TSP.

3. Differential Evolution Algorithm

DE is an improved version of GA which belongs to the evolutionary optimization method, where chromosomes are floating-point numbers. The principle for DE is described briefly as follows.

(1) Population Initialization

DE is considered *NP* *d*-dimensional vectors as the initial population to search the best solution. We can denote the group as follows: $\{X_{i,G} \mid X_{i,G} = (x_{1i,G}, x_{2i,G}, \cdots, x_{di,G}), i = 1,2,\cdots,NP, G = 0,1,\cdots,G_{\max}\}$, where G_{max} denotes the maximum evolution generation.

Clearly, $X_{i,0}, i = 1,2,\cdots,NP$ denote the initial population. Generally, the initial population can be chosen randomly from the range of variables.

(2) Mutation Operator

The purpose of mutation is to generate the mutant vector in order to enhance perturbation to the target vector to avoid premature convergence to a non-global local optimum. For each target vector $X_{i,G}, i = 1,2,\cdots,NP$, the mutant vector can be created as: $V_{i,G+1} = X_{r1,G} + F(X_{r2,G} - X_{r3,G})$, where the indexes r_1, r_2, r_3 represent the random and mutually different integers generated within [1,*NP*] and also different from *i*. *F* is a scaling factor, which is a real constant within [0,2].

(3) Crossover Operator

The purpose of crossover is to increase the potential diversity of the evolution group. Based on the mutant vector, the trial vector $U_{i,G+1} = (u_{1i,G+1}, u_{2i,G+1}, \cdots, u_{di,G+1})$ can be constructed as follows:

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1} & rand(0,1) \le CR \; or \; rand(i) = j \\ x_{ji,G} & otherwise \end{cases} \tag{10}$$

In formula (10), *rand*(0,1) is a random value within [0,1]; *rand*(*i*) is a randomly chosen index from $\{1,2,\cdots,d\}$. *G* is the number of current generation, and *CR* is the crossover probability within [0,1].

(4) Selection Operator

Based on the estimation of the group, the selection operator executes according to the fitness value of the target vector and its corresponding trial vector. The population of the next generation is obtained by adopting the following greedy selection criterion:

$$X_{i,G+1} = \begin{cases} U_{i,G+1} & f(U_{i,G+1}) > f(X_{i,G}) \\ X_{i,G} & otherwise \end{cases} \tag{11}$$

It is obvious from the above description that the traditional DE algorithm is designed for the continuous optimization problems, not suitable for application to combinatorial optimization problems.

4. Discrete Differential Evolution Algorithm

This paper proposes a novel DDE for SVRP. The essential difference is mutation operator with DE.

4.1. Representation and Fitness Function

In this paper, we adopt the client-ordinal-based representation which has been widely used in the literature for VRP. The feasible routing can be decoded to the chromosome (i_{11}, i_{12},..., i_{1s}; 0, i_{21}, i_{22},..., i_{2t};...; 0, i_{m1}, i_{m2},..., i_{mw}) with the length n+m-1.

In DE, the fitness function is used to evaluate the adaptability to environment of chromosome.First of all, we will deal with the constraints (6-8), let them transform into their equivalent representations. We can have the results that constraints (6-8) are equivalent to the following formula, respectively [9]:

$$\Phi^{-1}(1-\alpha)\sqrt{\sum_{i=\theta+1}^{n}\sigma_i^2 z_{ik}} + \sum_{i=0}^{\theta} p_i z_{ik} + \sum_{i=\theta+1}^{n}\mu_i z_{ik} \le C, \theta = 0,1,\cdots,n; k = 1,2,\cdots,m \tag{12}$$

$$\Phi^{-1}(1)\sqrt{\sum_{i=\theta+1}^{n}\sigma_i^2 z_{ik}} + \sum_{i=0}^{\theta} p_i z_{ik} + \sum_{i=\theta+1}^{n}\mu_i z_{ik} \le C(1+\Delta), \theta = 0,1,\cdots,n; k = 1,2,\cdots,m \tag{13}$$

$$\Phi^{-1}(1-\beta)\sqrt{\sum_{i=0}^{n}\sum_{j=0}^{n} x_{ijk}\sigma_{ij}^2 + \sum_{i=1}^{n} z_{ik}\lambda^2\sigma_i^2} + \sum_{i=1}^{n}\lambda\mu_i z_{ik} + \sum_{i=0}^{n}\sum_{j=0}^{n} x_{ijk} d_{ij} \Big/ v_{ij} \le \mathrm{B} \tag{14}$$

In this paper, we use formula $f_l = bz' / z_l$ as the fitness function, where f_l is the fitness value of the chromosome l, b is a given constant, z' is the best distribution costs corresponding to the chromosome in initial population, z_l is the distribution costs of the chromosome l.

4.2. Mutation Operator

In DDE, a simple mutation operator is designed in order to generate discrete values. Before designing the new mutation operator, first of all, we introduce two bitwise operators of the computer language to define the new algebraic structure on the set of vectors whose elements are natural numbers. We use N_d for the d-dimension vector set and define a binary operator from $N_d \times N_d$ to N_d .

Definition 1: Assuming $A = (a_1, a_2, \cdots, a_d) \in N_d$, $B = (b_1, b_2, \cdots, b_d) \in N_d$, and giving numerical constant $F \in (0,1)$, we define operators as follows:

$$\&: \&(A,B) \underline{\underline{\Delta}} (a_1 \& b_1, a_2 \& b_2, \cdots, a_d \& b_d) , \quad |:|(A,B) \underline{\underline{\Delta}} (a_1 | b_1, a_2 | b_2, \cdots, a_d | b_d)$$

$$F \otimes A \underline{\underline{\Delta}} \begin{cases} (a_j, a_2, \cdots, a_{j-1}, a_1, a_{j+1}, \cdots a_{d-1}, a_d) & rand(0,1) > F \\ (a_1, a_2, \cdots, a_{j-1}, a_d, a_{j+1}, \cdots, a_{d-1}, a_j) & otherwise \end{cases}$$

where $rand$(0,1) is a random number within [0,1], j is a random natural number within (1,d).

Based on the above definition, the mutation operator can be presented as follows:

$$V_{i,G+1} = X_{r1,G} \mid F \otimes (X_{r2,G} \,\&\, X_{r3,G}), i = 1,2,\cdots,NP, G = 0,1,\cdots,G_{\max} - 1 \tag{15}$$

In formula(15), $X_{i,G}$ is the target vector, $X_{r1,G}$, $X_{r2,G}$ and $X_{r3,G}$ are vectors distinct and different of the target vector chosen randomly from the group. F is a mutant scale factor, belonging to the interval [0,1]. The above formula consists of two components and an example is given in Figure 1.

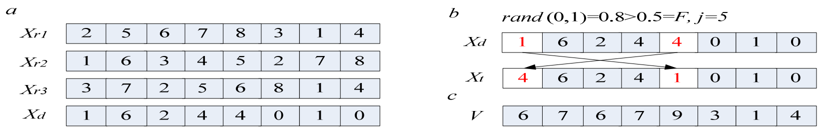

Figure 1. Mutation operator: (*a*): select the vectors X_{r1}, X_{r2} *and* X_{r3} randomly, then calculate $X_d = X_{r2} \& X_{r3}$; (*b*): select uniform number rand (0,1) between (0,1) and random number *j* randomly, calculate $X_t = F \otimes X_d$ for *F*=0.5; (*c*): calculate $V = X_{r1} | X_t$.

4.3. Crossover Operator and Selection Operator

In this paper, according to formula (10), the trial individuals are generated one by one. The selection is executed according to formula (11) that the population of the next generation is produced individually by using the greedy selection criterion.

4.4. Revise Operator.

We see that the feasible chromosome genes of VRP must be different with each other. As described in the previous, illegal individuals may be produced during the evolutionary process. So, an auxiliary operator based on integer order criterion (IOR) is applied to amend the infeasible chromosomes [10].

5. Computational Results and Analysis

To validate the effectiveness of the proposed DDE algorithm, two kinds of problems of different sizes of SVRPSPD are selected, that is small-scale problem (30 clients), and medium-sized problem (50 clients), which are solved by DDE, traditional DE and GA [3], respectively in the same conditions. The relative parameters of models and algorithms in the paper are listed in Table 1.

Table 1. The Relative Parameters by Models And Algorithms

N	*K*	*C*	*NP*	*Gmax*	*CR*	*F*	*Pc*	*Pm*	Δ	λ	*r*	*B*	α (β)
30	8	600	100	200	0.3	0.5	0.855	0.055	0.1	1.8	0.4	30400	0.05
50	15	600	100	200	0.3	0.5	0.855	0.055	0.1	1.8	0.4	30400	0.05

All the algorithms in this paper are implemented with C programming language. For each instance, 10 independent replications are conducted to obtain statistics. The computational results are shown in Table 2, where SD denotes the standard deviation of the distance value, T denotes the computational time.

Table 2. Average Results of the DDE, DE and GA Algorithms

	DDE			*DE*			*GA*		
N	*Distance*	*SD*	*T(s)*	*Distance*	*SD*	*T(s)*	*Distance*	*SD*	*T(s)*
30	388.673	44.795	14	424.118	34.330	13.5	462.746	20.701	6.8
50	964.637	95.072	38.7	1107.13	61.566	38.3	1163.115	48.487	18.5

From Table 2, it follows that DDE can obtain the better results than DE and GA. Therefore, we can conclude that DDE outperforms DE and GA on the considered problem. But the DDE computational time is much longer. This happens because in the process of DDE, the illegal chromosomes are revised during each generation, and as the problem size increases, the number of amendments increases.

6. Conclusion

The aim of this paper is to provide a more realistic modeling approach to VRP and a more applicable algorithm for solving it. Thus, firstly, this paper proposes a stochastic programming model for SVRPSPD with uncertain demand and travel time, and then presents a novel DDE for routing optimization. In DDE, a client-ordinal-based representation is applied, and novel mutation operator is developed for this representation. Furthermore, the performance of DDE is discussed by numerical experiments. Simulation results and comparisons demonstrate the superiority of the proposed DDE algorithm in terms of solution quality and effectiveness. Particularly, the new mutation operator designed for direct application to combinatorial optimization problems is satisfying.

Acknowledgements

This work was financially supported by the Tianjin Philosophy Social Science Research Project (No.TJGL12-0790) and the Doctor Foundation of Tianjin Normal University Grant (No.52WW1102).

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