

Estimation Normal Vector of Triangular Mesh Vertex by Angle and Centroid Weights and its Application

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Abstract

To compute vertex normal of triangular meshes more accurately, this paper presents an improved algorithm based on angle and centroid weights. Firstly, four representational algorithms are analyzed by comparing their weighting characteristics such as angles, areas and centroids. The drawbacks of each algorithm are discussed. Following that, an improved algorithm is put forward based on angle and centroid weights. Finally, by taking the deviation angle between the nominal normal vector and the estimated one as the error evaluation standard factor, the triangular mesh models of spheres, ellipsoids, paraboloids and cylinders are used to analyze the performance of all these estimation algorithms. The machining and inspection operations of one mould part are conducted to verify the improved algorithm. Experimental results demonstrate that the algorithm is effective.

Keywords: triangular mesh, vertex normal, centroid, angle, weight

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1. Introduction

With the development of the science and technology and the requirement of the market, high precision, high efficiency, high flexibility, high intelligence and automation become a new method in the manufacturing. The Quality Assurance Technique is a rising synthetical technique which is developed in recent years [1]. In order to ensure the part quality, there are two measurement methods for conventional manufacturing process according to the way whether the machining process and inspection process are done on the same equipment. One is off-line measurement, the other is on-line measurement, also called on-machine measurement [2].

In general manufacturing processes, there are three main steps to obtain acceptable products, i.e. design, machining and inspection. In this case, it is necessary to inspect the quality of the machined parts. In the conventional measuring process the machined surface accuracy is often measured on a CMM (coordinate measuring machine) system [3]. Figure 1(a) illustrates the interrelation between design, manufacturing, and inspection in this case. However this increases the overall manufacturing cost and time to obtain the final product. In addition it is hard to measure the complex, large-sized parts. Furthermore, it inevitably reduces the measurement precision due to the secondary fixture error when the workpiece is transferred from machine center to CMM. The development of the on-line inspection technology provides a suit of effective method for the quality inspecting in the machining process of numerical control. Machining and inspection are integrated in this kind of technology to fulfill the automatic inspection in the machining process [1]. Figure 1(b) illustrates the interrelation between design, manufacturing, and inspection in this case.

Triangular mesh models are widely used in computer animation, CAD/CAM and virtual design. It is of importance for subsequent processing of meshes to calculate the vertex normal of triangular mesh models accurately [4-5]. For both off-line inspection and on-line inspection, trigger probes are widely applied to inspect the precision of freeform surfaces such as moulds, airplane wings, and turbine blades. The most important process for the inspection of a freeform surface is to find accurate coordinates of the measured points. During this process, calculating the normal vector of each measurement point accurately is one of the most important steps for compensating the radius errors and then improving the inspection accuracy, as shown in Figure

2. At present, the authors are developing an on-line inspection system of numerical control machining based on triangular mesh models.

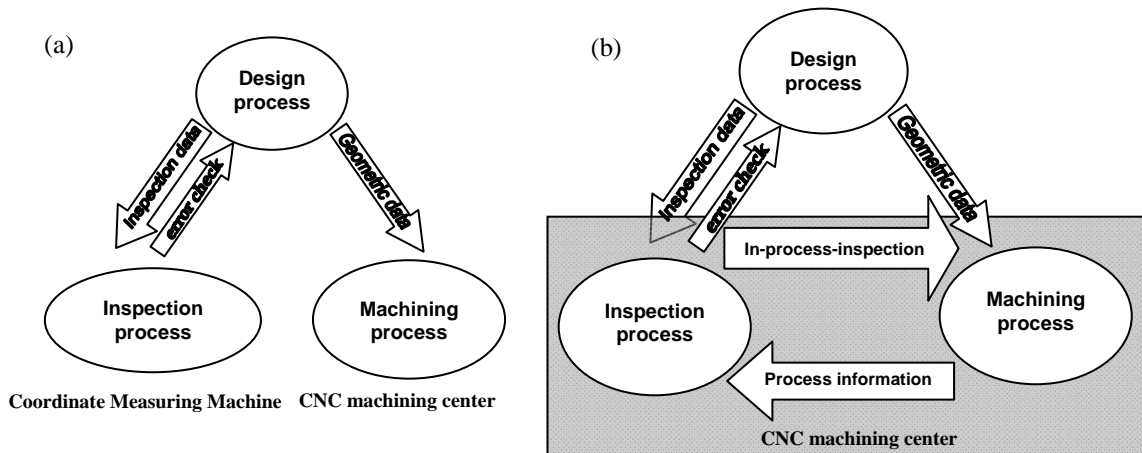


Figure 1. Relationship among Design, Inspection and Machining (a) Off-line Inspection, (b) On-line Inspection.

2. Survey of Previous Algorithms

There has been a great deal of research on the problem calculating the vertex normals of triangular meshes [6-9]. However, the theoretical foundation and weight factors of all these methods vary, so the final calculation accuracy differs considerably. It is necessary to improve the accuracy and stability when calculation the vertex normal to analyze the characteristics of these methods systematically.

The adjacent relation of the vertex v_i among triangle facets is shown in Figure 3. It can be seen that the normal vector of the vertex v_i is dependent on the geometric information such as area, angle α and normal vector of the adjacent triangle facets.

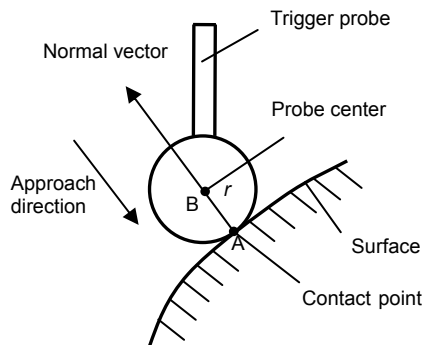


Figure 2. Probe Radius Compensation

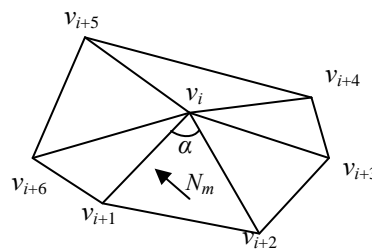


Figure 3. The Adjacent Relation among Triangle Facets

2.1. Algorithm 1

Taubin [6] approximated normal vectors by choosing the weight proportional to the area of triangle facets:

$$n_i = \frac{\sum_{k=1}^k A_k N_k}{\left\| \sum_{k=1}^k A_k N_k \right\|} \tag{1}$$

Where A_k and N_k are the area and the unit normal vector of all adjacent triangle facets.

2.2. Algorithm 2

Shen [7] approximated normal vectors by choosing the weight proportional to the product of the areas and the angles of all triangle facets:

$$n_i = \frac{\sum_{k=1}^k \alpha_i A_k N_k}{\left\| \sum_{k=1}^k \alpha_i A_k N_k \right\|} \quad (2)$$

where α_i is the angle of all adjacent triangle facets.

2.3. Algorithm 3

Chen [8] approximated normal vectors by choosing the weight proportional to the square of the inverse of the distance from vertex v_i to the centroid point g_i of all triangle facets:

$$n_i = \frac{\sum_{k=1}^k w_i N_k}{\left\| \sum_{k=1}^k w_i N_k \right\|} \quad (3)$$

where $w_i = \frac{1}{\|g_i - v_i\|^2}$ is called centroid weight, $g_i = \frac{\sum_{v_j \in f_k} v_j}{3}$ is called the centroid of each adjacent triangle facets.

2.4. Algorithm 4

Peng [9] concluded that the normal vectors are affected greatly by the shapes of all adjacent triangle facets:

$$n_i = \frac{\sum_{k=1}^k \frac{(1-\delta_i) \alpha_i N_k}{A_k}}{\left\| \sum_{k=1}^k \frac{(1-\delta_i) \alpha_i N_k}{A_k} \right\|} \quad (4)$$

where δ_i is the shape parameter of all adjacent triangle facets.

2.5. The Drawbacks of the above Mentioned Algorithms.

Algorithm 1 reflects only the effects of the areas of triangle facets on the normal vector, but it does not reflect that of the shapes of triangle facets. When long and narrow triangle facets are included in the adjacent triangle facets, the errors of the normal vector will be significant. As is shown in Figure 4, when the areas and the normal vectors of the triangle A and B are same, they will have the same effects on the normal vectors of the vertexes according algorithm 1. However, since the two triangles have the different shapes, the normal vectors of the vertexes are definitely different. So algorithm 1 has its drawback. Algorithm 2 and 4 consider that the larger the angle is, the more contribution it will have for the resulting normal vector. However, Peng proved that the larger the area is, the less contribution it will have if the angle remains constant [9]. Therefore, algorithm 2 will cause greater errors if the areas of all the triangle facets vary considerably. Algorithm 3 argues that the greater the distance from the centroid to the vertex, the less contribution it will have to the resulting normal vector, but this algorithm does not consider the effect of the angle. It can be seen from Figure 5 that if not only the distances from two centroids, g_1 and g_2 , to the vertex v_i in triangle A and B are same, but also the areas of these two triangle facets are same, only the angles are not same, they will have different effects for the resulting normal vector. So algorithm 3 has some drawbacks too. Algorithm 4 is more suitable for irregular triangular mesh models as it considers the comprehensive effects of shapes, areas and angles on the normal vector of the triangular mesh vertex. However, the selection of the shape parameter is still the subject of further research work.

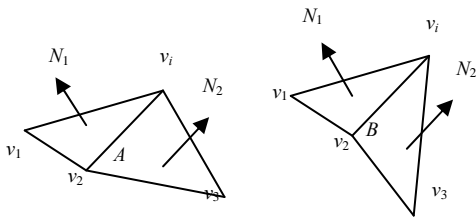


Figure 4. When the Areas and Normal Vectors are Same, but Angles are Different.

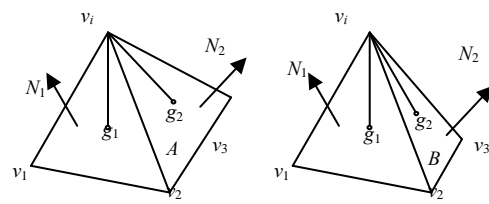


Figure 5. When the Distances and Normal Vectors are Same, but Angles are Different.

3. The Presented Algorithm

This paper presents an improved algorithm that considers the two most important factors that relate the normal of the triangle facet to the normal of the vertex, namely the angle and the distance to the centroid from the vertex. The angle is indicative of the shape of the triangle facet and a large angle indicates that the normal will have a significant contribution to the normal of the vertex, whilst a narrow angle indicates less contribution. The centroid distance also affects the contribution that the normal of the triangle facet will have on the normal of the vertex. If the distance is short then the contribution of the normal will be significant decreasing as the centroid distance increases.

$$n_i = \frac{\sum_{i=1}^k w_i N_k}{\|\sum_{i=1}^k w_i N_k\|} \tag{5}$$

where $w_i = \frac{\alpha_i}{\|g_i - v_i\|}$ is the angle and centroid weight, $g_i = \frac{\sum_{v_j \in f_k} v_j}{3}$ is the centroid

of all adjacent triangle facets. The weight w_i is proportional to the angle, but inversely proportional to the distance from the centroid to the vertex. So this algorithm can reflect the comprehensive effects of the angle and shape on the normal vector of the vertex, however complicated shape factors are not included in the algorithm making it an improvement over algorithm 4.

4. Experimental Results

4.1. Calculation of Normal Vector

To compare the proposed algorithm with the four previous algorithms, a series of experiments have been conducted using four kinds of quadric surfaces that can be expressed by analytic formula. The surfaces are: sphere ($X^2 + Y^2 + Z^2 = r^2$), ellipsoid ($X^2/a^2 + Y^2/b^2 + Z^2/c^2 = 1$), paraboloid ($X^2 + Y^2 = Z$) and cylinder ($X^2 + Y^2 = r^2$). The triangular mesh models and corresponding files of the STL format are generated by a 3D CAD software system, as shown in Figure 6. The deviation angle between the nominal normal vector and the estimated one is taken as the error evaluation standard factor.

According to the theory of differential geometry, the nominal normal vectors of above mentioned four kinds of surfaces are:

$$(X, Y, Z), (X/a^2, Y/b^2, Z/c^2), (2X, 2Y, -1), (X, Y, 0)$$

The normal vectors of the surfaces are calculated using five algorithms, respectively. The arithmetic mean error \bar{e} , the maximum error e_{max} , the minimum e_{min} and the standard deviation σ are also obtained for evaluating the accuracy and stability. The results are shown in Table 1 and the lowest values of the deviations are all underlined. The values of all e_{min} are 0 and they are omitted in Table 1. It must be pointed out that the vertexes on the borders are not calculated because of the incomplete geometric information on the border of the paraboloids and the cylinders.

From Table 1, some conclusions can be drawn by comparing the deviations of the presented algorithm with that of other four previous algorithms:

Table 1. Comparison of Angular Discrepancy for Five Algorithms

Surface	Vertex number	Error	Algorithm 1	Algorithm 2	Algorithm 3	Algorithm 4	The presented algorithm
Sphere ($r=1$)	1955	$\bar{e} / (^\circ)$	0.2039	0.1089	0.0505	0.0472	0.0651
		$e_{\max} / (^\circ)$	0.9455	0.1715	0.1589	0.11551	0.0985
		$\sigma / (^\circ)$	0.2294	0.0537	0.0348	0.0352	0.0276
Sphere ($r=5$)	2487	$\bar{e} / (^\circ)$	0.1661	0.0863	0.0398	0.0377	0.0513
		$e_{\max} / (^\circ)$	0.8389	0.1362	0.1423	0.0936	0.0777
		$\sigma / (^\circ)$	0.1956	0.0425	0.0298	0.0284	0.0216
Sphere ($r=10$)	4902	$\bar{e} / (^\circ)$	0.0932	0.0445	0.0202	0.0198	0.0260
		$e_{\max} / (^\circ)$	0.6020	0.0706	0.1041	0.0507	0.0394
		$\sigma / (^\circ)$	0.1249	0.0219	0.0191	0.0153	0.0109
Sphere ($r=15$)	7322	$\bar{e} / (^\circ)$	0.0657	0.0300	0.0136	0.0135	0.0174
		$e_{\max} / (^\circ)$	0.4929	0.0476	0.0858	0.0349	0.0264
		$\sigma / (^\circ)$	0.0949	0.0147	0.0144	0.0105	0.0073
Sphere ($r=20$)	3083	$\bar{e} / (^\circ)$	0.0502	0.0224	0.0102	0.0101	0.0130
		$e_{\max} / (^\circ)$	0.4232	0.0355	0.0739	0.0263	0.0198
		$\sigma / (^\circ)$	0.0768	0.0109	0.0116	0.0078	0.0054
Sphere ($r=50$)	9872	$\bar{e} / (^\circ)$	0.1382	0.0700	0.0322	0.0308	0.0414
		$e_{\max} / (^\circ)$	0.7541	0.1107	0.1289	0.0774	0.0628
		$\sigma / (^\circ)$	0.1695	0.0345	0.0259	0.0233	0.0175
Ellipsoid ($a=3, b=4, c=3$)	1224	$\bar{e} / (^\circ)$	0.4557	0.2172	0.1877	0.2088	0.0937
		$e_{\max} / (^\circ)$	3.1092	1.4870	0.3174	0.3210	0.1677
		$\sigma / (^\circ)$	0.6726	0.2854	0.0883	0.0999	0.0384
Paraboloid	737	$\bar{e} / (^\circ)$	0.9060	0.5401	0.3998	0.3822	0.2902
		$e_{\max} / (^\circ)$	3.0275	2.5409	0.7786	0.7172	0.5391
		$\sigma / (^\circ)$	1.010	0.7760	0.2062	0.1848	0.1130
Cylinder ($r=1$)	504	$\bar{e} / (^\circ)$	0.0327	0.0234	0.0069	0.0127	0.0129
		$e_{\max} / (^\circ)$	0.2410	0.2093	0.0390	0.0913	0.0603
		$\sigma / (^\circ)$	0.0651	0.0559	0.0096	0.0237	0.0157

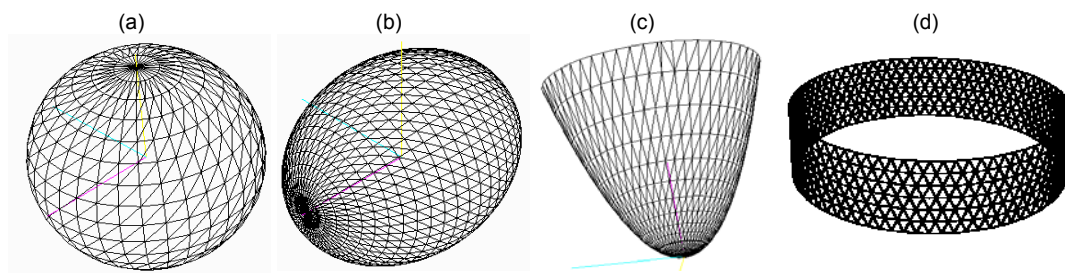


Figure 6. Triangular Meshes of Quadric Surfaces (a) Sphere, (b) Ellipsoid, (c) Paraboloid, (d) Cylinder.

a) For spherical surfaces, though the arithmetic mean error \bar{e} of the presented algorithm are larger than that of algorithm 3 and 4, the maximum error e_{\max} and the standard deviation σ are smaller than others, so the presented algorithm is more consistent. At the same time, it can be seen that with the increasing radius of the sphere, the deviations of all algorithms will decrease when the precision of all meshes is the same. The consistency in estimating the normals of vertices (measured by σ) is an important factor when determining the curvature and the rate of change of the curvature for a complex surface.

b) For ellipsoids and paraboloids, the arithmetic mean error \bar{e} , the maximum error e_{\max} , the minimum e_{\min} and the standard deviation σ of the presented algorithm are all the smallest among the five algorithms. So the calculation using the presented algorithm is the most accurate for ellipsoidal and paraboloidal shapes.

c) For cylindrical surfaces, algorithm 3 has the best performance. The arrangement of vertices in the cylindrical mesh creates, uniquely in this study, very regular facets, with every angle being approximately 60° . However algorithm 3 does not consider the angle in the calculation of the weighting showing that for a regular mesh such as this, it is the best algorithm when the affect of the angles need not be considered, but as the mesh becomes more irregular

and the angles vary it is less effective. In particular the maximum error e_{max} is significantly larger with irregular meshes. The presented method is better than the other 3 algorithms demonstrating its effectiveness. Further work is being carried out on the shapes of the facets to further investigate this issue.

In summary, for spherical, ellipsoid and paraboloid shapes, the calculation using the presented algorithm is the best among five algorithms. It has the best accuracy and consistency, and has the lowest dispersion degree. Therefore, the presented algorithm is effective for it takes into account the angle together with the centroid.

4.2. Machining and Inspection Experiment

To further validate the presented algorithm, machining operation and inspection operations which include on-line and off-line methods are conducted using a mould part with a free-form surface. Firstly, according to the CAD model with STL format, 40 measurement points on the part surface are selected using the on-line inspection system, as shown in Figure 7. Then the normal vectors of these points are calculated using the presented algorithm. According to these, the numerical control codes are generated for the on-line inspection. Using the CAD model, the numerical control codes are also generated for machining and the machining operation is conducted on the machining center, as shown in Figure 8(a). Then the cutting tool is replaced by a trigger probe and the on-line inspection operation is conducted on the same machine, as shown in Figure 8(b).

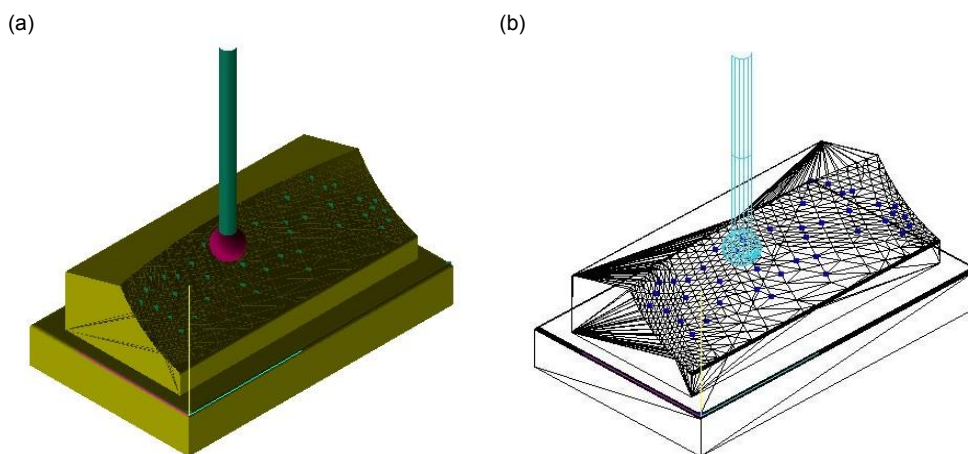


Figure 7. CAD Model and Measurement Points (a)Solid, (b)Triangular Mesh.

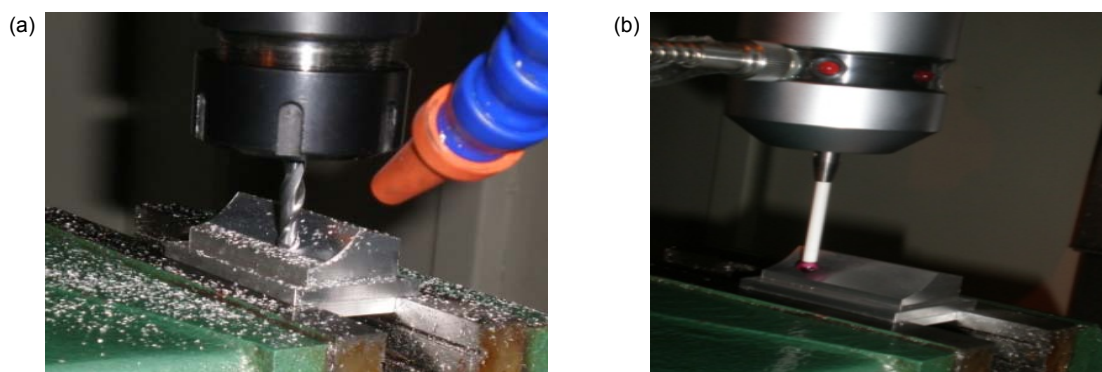


Figure 8. Machining and Inspection Operations on the Same Machining Center (a) Machining, (b) Inspection.

Finally, the off-line inspection operation is conducted with a commercial CMM, as shown in Figure 9. By comparing the inspection results with these two methods, it can be found that the on-line inspection method has almost the same accuracy as the off-line one with a CMM. Therefore, the presented algorithm is effective for on-line inspection of parts during manufacturing.



Figure 9. Inspection Operation on CMM.

5. Conclusion

Four previous representational algorithms of normal vector of triangle mesh vertex are reviewed in this paper. Then an improved algorithm is presented by analyzing the drawback of all the algorithms. This algorithm can reflect the comprehensive effects of angles, areas and shapes of the triangle facets. Experiments results prove that its calculation has a greater accuracy and consistency than other four algorithms. The machining and inspection operations of one mould part are conducted to verify the improved algorithm.

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