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A Framework of Fingerprint Scaling

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Abstract

Fingerprint scaling refers to the adjustment of fingerprint images to solve the problem of sensor interoperability. In this paper, we present an innovative framework of fingerprint scaling with minimum modification to existing systems. For the purpose of facilitating system configuration, we have developed a series of scaling methods, including scaling factors, graph- and template-based scaling methods. In graph-based scaling methods, we have explored the application of various technologies in estimation of the average inter-ridge distance. In template-based scaling methods, we have developed an estimation method using Delaunay triangulation algorithm. The experiments show that the performance achieved by using this framework is better than that of original system. With a scaling module, the average EER in our experiments drops from 27.78% to 13.89%.

***Keywords**: fingerprint, scaling, sensor interoperability, ridge distance, Delaunay triangulation*

1. Introduction

Fingerprint-based biometric systems are rapidly gaining acceptance as one of the most popular technologies to verify or recognize users in a wide range of applications: from physical access control to criminal investigation and from inmates managing to corpse identification [1]. A fingerprint system is essentially a pattern recognition system that acquires fingerprint impression from an individual, extracts a notable feature set from the image, compares this feature set against the feature sets stored in the database, and executes an action according to the result of the comparison. A common fingerprint system has four typical modules: (a) sensor module which acquires the fingerprint impression of an individual; (b) feature extraction module which extracts a feature set from the acquired image; (c) matching module in which the extracted feature set is compared with the templates stored in the database through the generation of matching scores; (d) decision-making module in which the matching scores are used to finish verification or recognition [2] .

The framework of fingerprint system is widely used in most applications nowadays. However, the performance of fingerprint system based on this structure was limited by sensor interoperability [3]. Fingerprint sensor interoperability means the ability of a fingerprint system to compensate for the variability introduced in the data of the same person due to employing different sensors [4]. The variations induced from the raw images owing to differences in resolution, scanning area, sensing technology, etc. impact the features extracted from the fingerprint images and propagate into the matching algorithm using these features [5].

Several approaches to sensor interoperability are known from literature. The problem of sensor interoperability is discussed and the impact of changing sensors on the matching performance of a fingerprint system is presented as a case study [3]. Fingerprint sensor interoperability is studied using a multi-sensor database acquired with three different fingerprint sensors [6].A nonlinear calibration scheme based on the Thin-Plate Spline (TPS) model is used to register a pair of fingerprint sensors [4; 7].

Considering the widespread deployment of fingerprint systems in large-scale applications, for example, e-commerce and welfare-disbursement [8], especially applications in WWW environments, it is necessary to carefully investigate sensor interoperability. In particular,

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it is important to understand whether such a fingerprint system can be improved through a simple modification.

This work introduces a novel module to scale fingerprint images, which is embed into a traditional framework of fingerprint system to realize sensor interoperability with minimum modification. The proposed module consists of a sequence of steps including analysis of the information available in the data, estimation of various scaling parameter for sensors used in fingerprint system, and finally a scaling step to make the fingerprint matching more robust on sensor interoperability. This module can be easily integrated with existing fingerprint systems. The efficacy of the proposed approach has been assessed by comparing the performances of embedded scaling module system with original system.

The rest of the paper is organized as follows: Section 2 briefly summarizes the reason why a fingerprint system need this module and highlights the novelty of the proposed technique. Section 3 explains the various methods of fingerprint scaling. Section 4 reports some experimental results. Finally, Section 5 draws some conclusions and future works.

2. Fingerprint Scaling

The large number of existing approaches to fingerprint matching can be coarsely classified in three catalogues: (a) correlation-based matching, (b) minutiae-based matching, and (c) ridge feature-based matching [9].

For correlation-based techniques [10], let $I^{(\Delta x,\Delta y,\theta)}$ represent a rotation of the input image *I* by an angle *θ* around the origin and shifted by *Δx* and *Δy* pixels in directions *x* and *y*, respectively. Then the similarity between the two fingerprint images *template (T)* and *input (I)* can be measured as

$$S\left(T,I\right)=\max_{\Delta x,\Delta y,\theta} CC\left(T,I^{(\Delta x,\Delta y,\theta)}\right) \tag{1}$$

where $CC(T, I) = T^T I$ is the cross-correlation between *T* and *I*. The cross-correlation is a well known measure of image similarity and the maximization in (1) allows us to find the optimal registration. For minutiae-based techniques, the most common minutiae matching algorithms consider each minutia as a triplet $m=\{x,y,\theta\}$ that indicates the (x,y) minutia location coordinates and the minutia angle *θ*. In the pattern recognition literature [11], the minutiae matching problem has been generally addressed as a point pattern matching problem. For ridge feature-based techniques, such as FingerCode method [12], a feature vector is composed of an ordered enumeration of the features extracted from the local information contained in each sector specified by the tessellation. Matching two fingerprints is then translated into matching their respective FingerCodes, which is simply performed by computing the Euclidean distance between two FingerCodes.

These techniques, however, suffer from the following shortcoming: most techniques aim at single-sensor based matching only, the translation and rotation of images are considered in all algorithms, but multi-sensor based matching, especially image scaling, is in all probability incompatible to existing techniques.

For example, a minutiae-based matching method aligns the two fingerprints in order to maximize the number of matching minutiae [13]. Correctly aligning two fingerprints certainly requires displacement (in *x* and *y*) and rotation (*θ*) to be recovered.

Let map(.) be the function which maps a minutiae *m'* (from *I*) into *m”* according to a given geometrical transformation; for example by considering a displacement of [*Δx,Δy*] and a counterclockwise rotationθ around the origin:

$$map_{\Delta x,\Delta y,\theta}\left(m'=\{x',y',\theta'\}\right)=m''=\{x'',y'',\theta'+\theta\} \tag{2}$$

where $\begin{bmatrix} x'' \\ y'' \end{bmatrix}=\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix}+\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$

Then the matching problem can be solved using the “tolerance boxes” technique [14]. Figure 1 shows an example of a comparison of a fingerprint pair.

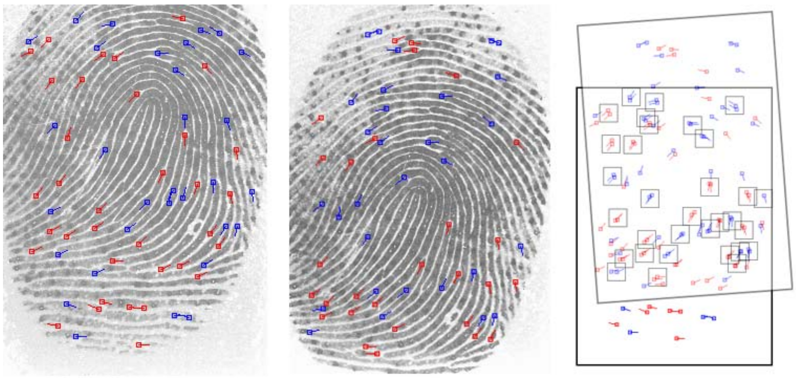

Figure 1. An example of matching the minutiae set in (a) with minutiae set in (b) is shown in (c)

However, in multi-sensor situation, the sensing area of these scanners may vary greatly from a few square millimeters to a few square inches. The resolution of the acquired image can vary anywhere between 250 dpi and 512 dpi; scanners that acquire 1000 dpi images are also available in the market [4; 7]. It is very hard to map the two fingerprints using (2), which will result in a significant drop of performance. Figure 2 shows an example of fingerprints comparison in multi-sensor situation.

Figure 2. In multi-sensor situation, an example of matching the minutiae set in (a) with minutiae set in (b) is shown in (c).

The same problem also can be found in correlation- or ridge feature-based system [6]. Because the size of fingerprint in the images originated from the same sensor is similar, in fact, the problem of fingerprint scaling does not exist in traditional fingerprint systems. Fingerprint images that are acquired from the same sensor will have the same resolution and scanning area. Thus we can ignore the procedure of scaling. The rotation and shifting are two main facets considered in traditional fingerprint systems. Nevertheless, fingerprint scaling has to be dealt with when we consider sensor interoperability.

The scaling of fingerprint has been mentioned [8; 15; 16], where the scaling is considered as a geometrical transformation for single-sensor situation.

The same problem, with the aim of improving matching performance, has been discussed in a previous work [17]. The Hough transform for line detection can be generalized for point matching [17]. The space of transformations consists of quadruples ($s,\theta,\Delta x,\Delta y$), where each parameter is discretized in a finite set of values:

$$s \in \{s_1, \ldots, s_K\}, \theta \in \{\theta_1, \ldots, \theta_L\}, \Delta x \in \{\Delta x_1, \ldots, \Delta x_M\}, \quad (3)$$

And

$$\Delta y \in \{\Delta y_1, \ldots, \Delta y_M\},$$

This technique is limited by the following drawbacks: (a) the scaling is embedded in matching module, which means we have to consider the scaling in every matching method; (b) adding the scaling parameter to an existing system is expensive, meaning it is difficult to adapt these techniques to correlation- or ridge feature-based system; (c) it is very hard to estimate accurate scaling parameter, and the performance of scaling depends on feature exaction result.

This work addresses the problem of fingerprint scaling that was described above. Our approach substantially differentiates from the previous works in the following aspects: 1) The fingerprint scaling should be viewed as a necessary module in fingerprint systems, instead of just a technique corresponding to certain matching method, 2) The fingerprint scaling is purposed to deal with sensor interoperability in a multi-sensor system, rather than solve the aligning problem in a single-sensor system.

3. The Scaling Approach

3.1. A System Model

Considering the facility of improving sensor interoperability in existing fingerprint systems, it is essential to modify the system with minimum adjustment in system model. A typical fingerprint verification system involves two stages: during enrollment, the user's fingerprint is acquired and its distinctive features are extracted and stored as a template; and during verification, a new fingerprint is acquired and compared to the stored template to verify the user's claimed identity [18]. In our system, nothing is changed in the enrollment stage, as shown in Figure 3.

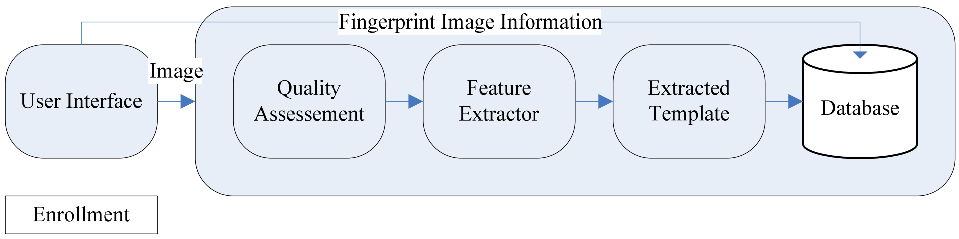

Figure 3. Enrollment stage of fingerprint system with scaling.

Depending on the application context, a fingerprint system may operate either in the verification or identification mode. The only modification in the verification or identification mode is that a scaling module is inserted before feature extraction. Figure 4 shows the verification and identification model of a fingerprint system with scaling.

Let scale(.) be the function which scales a image *f'* into *f"* according to a given linear transformation; for example, by considering a parameter *s*:

$$scale_s\left(f' = \{h', v'\}\right) = f'' = \{h'', v''\} \quad (4)$$

where $\begin{bmatrix} h'' \\ v'' \end{bmatrix} = s \begin{bmatrix} h' \\ v' \end{bmatrix}$, and *{h, v}* refers to the horizontal and vertical size of the image. So the scaling problem can be solved by estimating the parameter *s*.

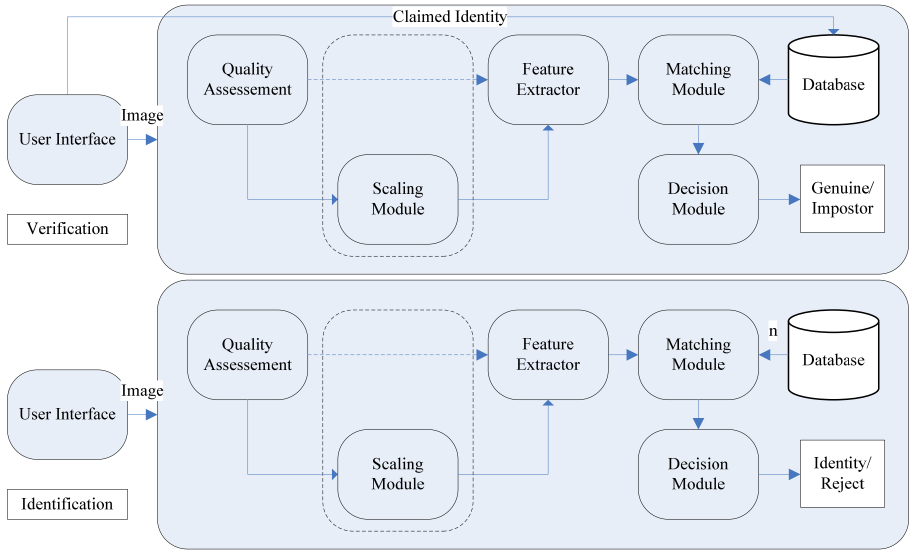

Figure 4. Verification and identification stages of fingerprint system with scaling.

3.2. Estimating the Scaling Parameter

It is obvious that the scaling parameter has very close relationship with the resolution and scanning area of sensors used in acquiring fingerprint images. There are two sensors in a recognition procedure. One sensor is used in enrollment stage, and other one is used in verification or identification stage. Let us examine the scaling parameter in the following two cases.

3.2.1. Sensors are Both Known

Suppose there is a Scaling Factor (SF) related to each sensor. The single SF means nothing, but the ratio of two SF reflects the relationship between sensors. So we can list a table of all sensors and SFs, as shown in Table 1.

Table 1. Scaling Factors for Sensors

ID	Sensor	Scaling Factor
1	Sensor 1	SF_1
2	Sensor 2	SF_2
3	Sensor 3	SF_3

The simple case occurs when the sensors are both known. For example, sensor *i* is used in enrollment stage, and sensor *j* is used in verification or identification stage. In this case the scaling parameter will be:

$$s_{ij} = \frac{SF_i}{SF_j} \tag{5}$$

3.2.2. At Least One Sensor is Unknown

In this case, the scaling parameter has to be estimated using data in the database and input image. There are two possible situations. As we can see clearly in Fig.3, features extracted are stored as a template in enrollment stage. Sometimes, the fingerprint image is also saved in database.

If there is image in the database, we can estimate the scaling parameter using biographic information. Since a template is a very compact representation of the fingerprint, the estimation using template and input image will be more complicated.

3.2.2.1. Graph-Based Information

The estimation approach consists of a sequence of steps including computation of various aspects of the fingerprint images based on information available in the images. A final rendering step is executed to estimate scaling parameter. An estimation method will be introduced as follows [19].

Average inter-ridge distance is an important characteristic of fingerprint and is often used in fingerprint enhancement and classification procedures [20]. For the same finger, average inter-ridge distance is stable correspondingly, so we could calculate the scaling parameter through making the average inter-ridge distances of two fingerprint images unified. Figure 5 illustrates the process of this method.

Let IRD be the average inter-ridge distance of a fingerprint, the scaling parameter *s* can be estimated by

$$s = \frac{IRD_e}{IRD_i} \tag{6}$$

where IRD_e means average inter-ridge distance of enrollment fingerprint, and IRD_i means that of input fingerprint.

In order to estimate the average inter-ridge distance of a fingerprint, a spectral-based algorithm has been adopted to find reasonable values. The algorithm transforms the fingerprint image using discrete Fourier transform (DFT) at first [20], and then estimates region through maximizing local discrete information entropy [21], finally calculates average inter-ridge distance by weighted Euclidean distance [22]. More detail can be found in [23].

The experimental results showed that the estimation method for scaling parameter is appropriate, and its robustness allows the typical fingerprint image to be covered.

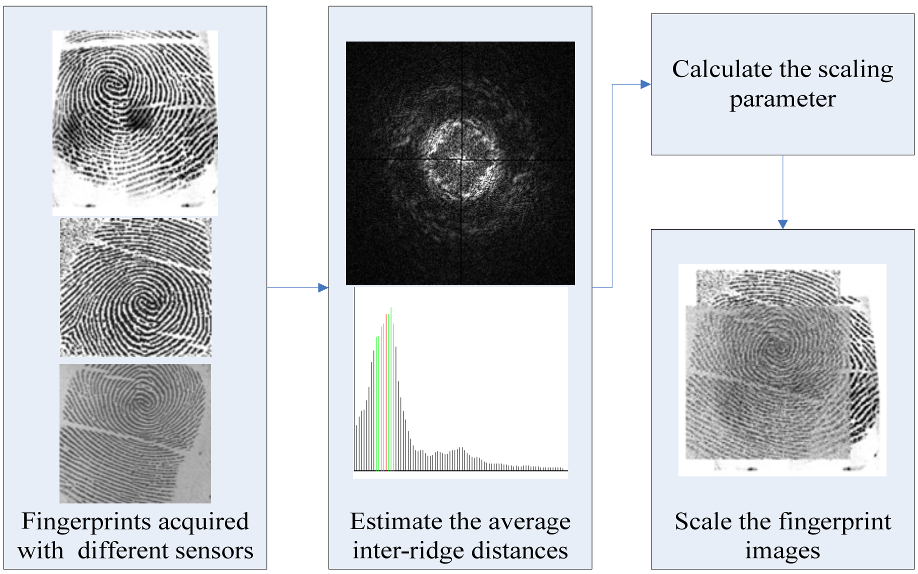

Figure 5. An estimation method using average inter-ridge distance.

3.2.2.2. Template-Based Information

Because there is some information lost in feature exaction process, many researchers and practitioners in the biometric community believed that a template does not include enough information to reconstruct the original fingerprint image. However, this belief was recently questioned by a few works [24]. Using fingerprint image reconstruction technique, we can convert the template scaling problem to graph-based one.

The other way to solve the template scaling problem is to estimate the scaling parameter using template directly. Because the template is based on various algorithms, the estimation method has to be designed according to on the template.

Minutiae-based method is the most popular and widely used technique, being the basis of fingerprint comparison made by fingerprint examiners [25]. Minutiae are extracted from the fingerprint and stored as sets of points in the two-dimensional plane. In this paper, an estimation method based on minutiae template will be proposed as follows.

The problem of triangulation is a fundamental one in computational geometry with applications in surface or function interpolation [26; 27]. Here, the Delaunay triangulation is used to associate a unique topological structure with the fingerprint minutiae [28].

Given a set S of points p_1, p_2,…, p_N, we can compute the Delaunay triangulation of *S* by computing its Voronoi tessellation. The Voronoi tessellation partitions the space into cells with all the points in the cell around pi being closer to pi than to any other point in *S* [29]. Given the Voronoi tessellation, the Delaunay triangulation can be formed by connecting the centers of every pair of neighboring Voronoi regions. Once the minutiae have been extracted, their Delaunay triangulation is computed. Figure 6 demonstrates the Delaunay triangulation of the minutiae extracted from one of the fingerprints.

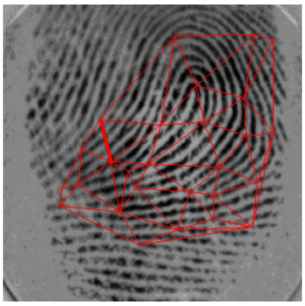

Figure 6. The Delaunay triangulation of the minutiae.

Delaunay triangulation has certain properties described as follows: (a) the Delaunay triangulation of a non-degenerate set of points is unique, which guarantees the same triangulation net deducted from a minutia set; (b) a circle through the three points of a Delaunay triangle contains no other points; (c) the minimum angle across all the angles in all the triangles in a Delaunay triangulation is greater than the minimum angle in any other triangulation of the same points [28].

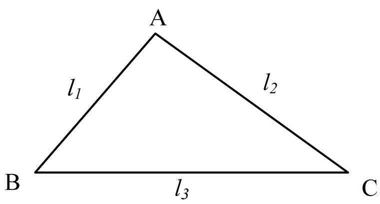

Figure 7. Invariants using the minutiae triangles.

Given a minutiae triangle, we compute two invariants which are then used to compute similar triangles. The invariants are based on the sides and angles of the minutiae triangle, as illustrated in Figure 7 and Algorithm.1.

Algorithm. 1 Represent a minutiae triangle
Input: l_1, l_2, l_3, $\angle A, \angle B, \angle C$
Output: A minutiae triangle representation
- Sort the angles of the triangle: $\angle A \geq \angle B \geq \angle C$

- Compute invariants: $-1 \le \cos(A) \le 1$ and $0 \le \frac{l_1}{l_2} \le 1$
- Represent a minutiae triangle by the following format: (m_ID, iv_1, iv_2, c_1)
  where *m_ID* is an identification code for the minutiae triangle of fingerprint, and $iv_1=\cos(A)$, $iv_2=l_1/l_2$ and $c_1=l_3$.

Given the templates of enrolled and query images, two corresponding Delaunay triangulation nets can be produced. Suppose the sets of minutiae triangles are represented as

$$Enroll = \{Em\_i \mid i = 1, 2, ..., p\} \tag{7}$$

$$Query = \{Qm\_j \mid j = 1, 2, ..., q\} \tag{8}$$

where p and q are the number of minutiae triangle. We can find similar triangles between enrolled and query images by

$$\left|Em\_i.iv_1 - Qm\_j.iv_1\right| \le t_1 \tag{9}$$

$$\left|Em\_i.iv_2 - Qm\_j.iv_2\right| \le t_2 \tag{10}$$

where t_1 and t_2 are thresholds. Suppose there are n pares of similar triangles, the scaling parameter s can be estimated by

$$s = \frac{1}{n}\sum_{i=1}^{n}\frac{Em\_i.c_1}{Qm\_i.c_1} \tag{11}$$

4. Experimental Results

The proposed scaling parameter estimation approach based on biographic information has been evaluated on fingerprint images acquired through two 500 dpi optical/capacitive scanners during the collection of First International Competition for Fingerprint Verification Algorithms (FVC 2000) database DB1 and DB2 [23]. The database consists of fingerprint impressions obtained from 100 non-habituated, cooperative subjects. Every subject was asked to provide 8 impressions of the same finger [30].

In this paper, some systematic experiments are reported as follows. First, some scaling factors are estimated, and then the performance of system using scaling module is compared to the original one and the results are discussed.

4.1. Examples of Scaling Factors

We estimated the scaling factors of sensors in two ways. One is to manually transform the impressions of the same finger originated from different sensors. The other is to estimate the scaling factors according to average inter-ridge distance.

It is an ideal situation that there are several impressions of the same finger originated from different sensors. In this situation, the finger will become a reference to unify all impressions. The scaling factors can be calculated accurately. In this paper, we illustrate the computation of scaling parameter through an experiment.

Although there are several databases containing fingerprint images acquired from two different sensor technologies, such as MSU dataset [31] and MCYT database [32], we need the images originated by more than two sensors. CASIA Fingerprint Databases for Cross-matching are constructed with three representative sensors, but the fingerprint images in DB1 are rescaled into 300x330 with 420dpi from the originally captured 500x550 images at the optimal resolution of 700dpi. In order to evaluate the scaling parameter, we made a database for this project. Currently, we are acquiring data from 3 different sensors (Ewaytek EWD79006-A, FingerPrints FPC1011C, Fytech ZY202B) in order to study the interoperability issues associated with them. Table 2 summarizes the global features of the three sensors.

Table 2. The Global Features of Three Sensors

	Sensor	Sensor Type	Image Size	Resolution
DB1	EWD79006-A	Semi-conductive	256*256	250dpi
DB2	FPC1011C	Capacitive	152*200	363dpi
DB3	ZY202B	Optical	320*320	500dpi

Each database includes 74 impressions acquired from the same 74 fingers respectively. Figure 8 shows some sample images from each one of them.

Figure 8. Sample images taken from DB1, DB2 and DB3. In order to show the different image sizes of each database, the three images are displayed at the same scale factor.

In this experiment, we scaled the images manually. First, we marked the minutiae on fingerprint images. Then, we scaled the images to unify the range of minutiae. Finally, the scaling factor was estimated by computing ratio of image sizes. Table 3 shows the scaling factors of these three sensors.

Table 3. Scaling Factors for Sensors

ID	Sensor	Scaling Factor
1	EWD79006-A	0.96
2	FPC1011C	1.37
3	ZY202B	1.00

The other more common situation is we can not get the images of the same finger originated from different sensors, meaning we can not manually unify the fingerprints. In this situation, the average inter-ridge distance of database will be a good standard to estimate the scaling factors.

For example, FVC2000 database is very popular in technology evaluation of fingerprint verification algorithms [30]. In FVC2000, the images are a collection of three sensors. Table 4 summarizes the global features of the three sensors.

Table 4. The Global Features of Three Sensors

	Sensor	Sensor Type	Image Size	Resolution
DB1	Secure Desktop	Optical	300*300	500dpi
DB2	TouchChip	Capacitive	256*364	500dpi
DB3	DFR-90	Optical	448*478	500dpi

We do not know whether there are impressions in different DB originated from the same finger. But the scaling factors can be estimated by calculating the average inter-ridge distance of each DB. As a result, the average inter-ridge distances and the scaling factors are shown as Table 5.

Table 5. Average Inter-ridge Distances and Scaling Factors for Sensors

ID	DB	Sensor	Average inter-ridge distance	Scaling Factor
1	DB1	Secure Desktop	9.55	1.00
2	DB2	TouchChip	8.92	1.07
3	DB3	DFR-90	10.58	0.90

We have performed extensive experiments on the stability of the average inter-ridge distance. Figure 9 summarizes the results. The graph shows the changes in the average inter-ridge distance as we increase the number of fingers. We can see that the average inter-ridge distance is fairly stable.

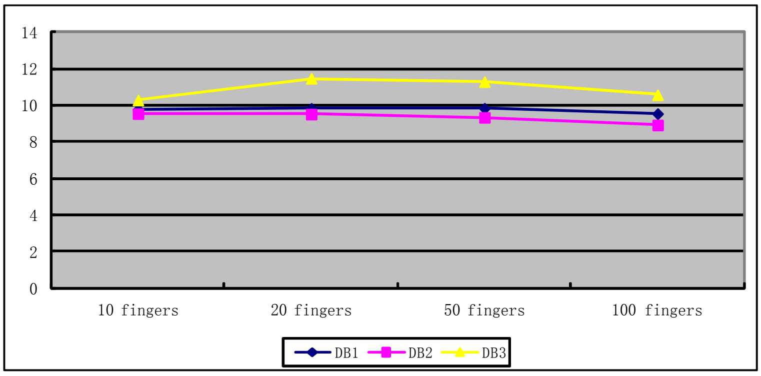

Figure 9. The stability of the average inter-ridge distance.

4.2. Systematic Experiments

This section reports the results of experiments aimed at exploring the feasibility of embedding a scaling module into a traditional fingerprint recognition system. The system uses minutiae as the main feature. The database used in this experiment is our experimental sets reported in last section.

The performance of a fingerprint verification system is mainly described by two values, i.e., false acceptance rate (FAR) and false rejection rate (FRR). FAR and FRR are defined as

$$FAR = P(D_1 \mid \omega_2) \tag{16}$$

and

$$FRR = P(D_2 \mid \omega_1) \tag{17}$$

where w_1 and w_2 represent the classes of true genuine matches and impostor matches, respectively, D_1 and D_2 denote the decisions of genuine matches and impostor matches, respectively.

The Equal Error Rate (EER) is computed as the point where *FAR(t)=FRR(t)*. It is a very important value to evaluate the performance of system. A ROC (Receiving Operating Curve) is obtained, where FAR is plotted as a function of FRR.

We tested the matching performance with different enrollment and request sensors. It can be proved that a significant performance upgrade has been achieved through using the scaling module. Table 6 shows the EER in various situations. Figure 10 highlights the EER without/with scaling module at various sensor pair. With a scaling module, the average EER in our experiments drops from 27.78% to 13.89%.

Table 6. the EER without/with scaling module at various sensor pair

EER	Sensor 1 + Sensor 2	Sensor 1 + Sensor 3	Sensor 2 + Sensor 3
Without Scaling	40.28%	9.72%	33.33%
With Scaling	18.06%	8.33%	15.28%

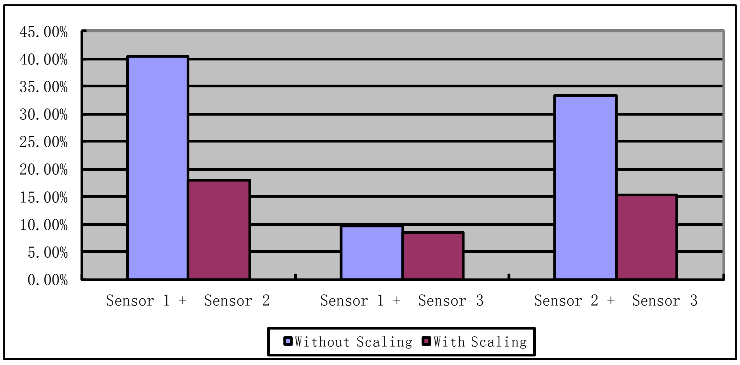

Figure 10. A histogram for the EER of without/with scaling module.

Figure 11 reports the ROC results obtained under the various situations. We can see clearly that the experiments have obtained very positive results.

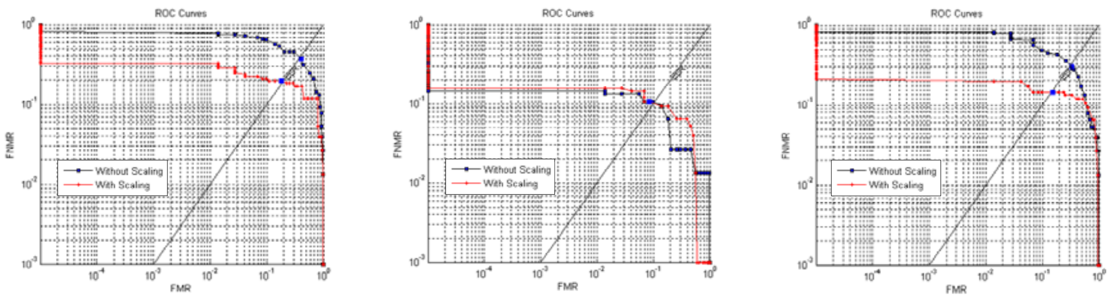

Figure 11. The ROC graphs of without/with scaling module using (a) sensor 1 and sensor 2, (b) sensor 1 and sensor 3, (c) sensor 2 and sensor 3.

5. Conclusions and Future Works

In this work, we experimented with the idea that sensor interoperability of fingerprint system can be improved by introducing a novel scaling module. For the purpose of facilitating system configuration, we have developed a series of scaling methods, including scaling factors and the graph- or template-based scaling parameter estimation.

In graph-based scaling methods, we have explored the application of various technologies in estimation of the average inter-ridge distance. In template-based scaling methods, we have developed an estimation algorithm using Delaunay triangulation algorithm. Finally, we have developed a practical measure for the scaling factors. The advantage of using such a scaling module is the improved robustness against sensor interoperability, an important property overlooked in previous work.

Experimental results indicated that the accuracy and robustness of fingerprint system can be improved effectively by embedding such a scaling module into the traditional framework of fingerprint systems under multi-sensor situation. An important feature of the scaling module is that it can be easily adapted to work with any kind of existing fingerprint system. Namely, the problem of sensor interoperability is solved by this method in a creative way.

Meanwhile, there are still some limitations in embedding a scaling module to solve the problem of sensor interoperability: 1) Like other fingerprint systems, scaling module assumes that the features of impressions originated from the same finger are stable. This assumption may not always hold under multi-sensor situation; 2) When the quality of fingerprint images is very poor, or the valid area of fingerprint image is very small, the scaling module may not work reliably. 3) The scaling module can not improve the sensor interoperability caused by the variations induced from sensing technology or non-linear distortion. The related works can be found in [7].

Our future work will focus on: 1) developing new algorithms for scaling images; 2) proposing more effective techniques for estimating scaling parameter; 3) extending our experiments for handling more sensors with more scaling factors.

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