

## Projective synchronization for 4D hyperchaotic system based on adaptive nonlinear control strategy

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### Article Info

#### Article history:

Received Dec 20, 2019

Revised Feb 25, 2020

Accepted Mar 12, 2020

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#### Keywords:

4D-Lorenz system

Lyapunov stability theory

Nonlinear control strategy

Projective synchronization

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### ABSTRACT

The main purpose of the paper is to projective synchronous chaotic oscillation in the real four-dimensional hyperchaotic model via designing many adaptive nonlinear controllers. Firstly, in view that there are many strategies in the design process of existing controllers, a nonlinear control strategy is considered as one of the important powerful tools for controlling the dynamical systems. The prominent advantage of the nonlinear controller lies in that it deals with known and unknown parameters. Then, the projective synchronize behavior of a four-dimensional hyperchaotic system is analyzed by using the Lyapunov stability theory and positive definite matrix, and the nonlinear control strategy is adopted to synchronize the hyperchaotic system. Finally, the effectiveness and robustness of the designed adaptive nonlinear controller are verified by simulation.

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## 1. INTRODUCTION

In the field of the nonlinear dynamical system, Lorenz system is the first classical three-dimensional chaotic system which discovers in 1963, it followed that several attempts to create another chaotic system such as Chen system (1999), Lu<sup>ˆ</sup> system (2002), Liu system (2004), Pan system (2009) and so on [1]. In addition, generate higher-dimensional hyperchaotic systems. The hyperchaotic system was firstly introduced in 1979 by Rossler, this system is one of a high-dimensional and strongly coupled nonlinear dynamical system, often exhibits complex nonlinear dynamic behaviors and contains at least two positive Lyapunov exponents [2-3].

In recent years, the control and synchronization of the aforementioned systems have gained enough attention from scholars due to its potential applications in engineering [4-5], electrical circuits, biological systems [6-7], secure communication [8]. It is well known that the idea of synchronization was firstly proposed by [8-11]. The concept of synchronization has been extended to the scope, such as complete synchronization (CS) [1, 3, 8, 12], anti-synchronization (AS) [10, 13], generalized synchronization [14], projective synchronization [15-17] and so on. Various strategies have been introduced until now to realize the stabilization of error dynamic systems, including active control, adaptive control, nonlinear control [18-22], linear feedback control [16]. Among many of these control strategies, nonlinear control has attracted many scholars' attention because of its simplicity, reliability, and effectiveness and widely used as one powerful strategy for synchronization of a different class of nonlinear dynamical systems [23-26]. However, the designed control input must depend on the controlled system functions according to the conventional nonlinear control. In order to simplify the control input, adaptive nonlinear control can be designed to make its control inputs easy to be realized synchronization.

In any control system applications, the main objective is to design a controller for a good performance and robustness. To ensure that the designed adaptive nonlinear controller has a good control

effect, a nonlinear controller is designed for a controlled system based on the Lyapunov stability theory with known and unknown parameters. Then, the designed controllers are used to synchronize the hyperchaotic system. These findings may be of significance in understanding and controlling problems in modern society. Simulation results verify the effectiveness and robustness of the proposed control scheme.

## 2. DESCRIPTION OF 4-D LORENZ SYSTEM

The Lorenz chaotic system is the first famous model of 3-D chaotic behavior which modeled into a four-dimensional by Guangyun et al. In 2017 [27] via state feedback control which consists of two nonlinearity and seven parameters. The new system is depicted as follows.

$$\begin{cases} \dot{x} = a(y - x) - fw \\ \dot{y} = xz - qy \\ \dot{z} = b - xy - cz \\ \dot{w} = ry - dw \end{cases} \quad (1)$$

Where  $x, y, z,$  and  $w$  are state variables and  $a, b, c, d, r, f, q$  are positive parameters the system (1). When the parameters taken the values  $a = 5, b = 20, c = 1, d = 0.1, r = 0.1, f = 20.6, q = 1$ , the system (1) exhibits hyperchaotic behavior due to it has two positive Lyapunov exponents such as  $LE_1 = 0.24, LE_2 = 0.23$ . Figure 1 show attractor the system (1).

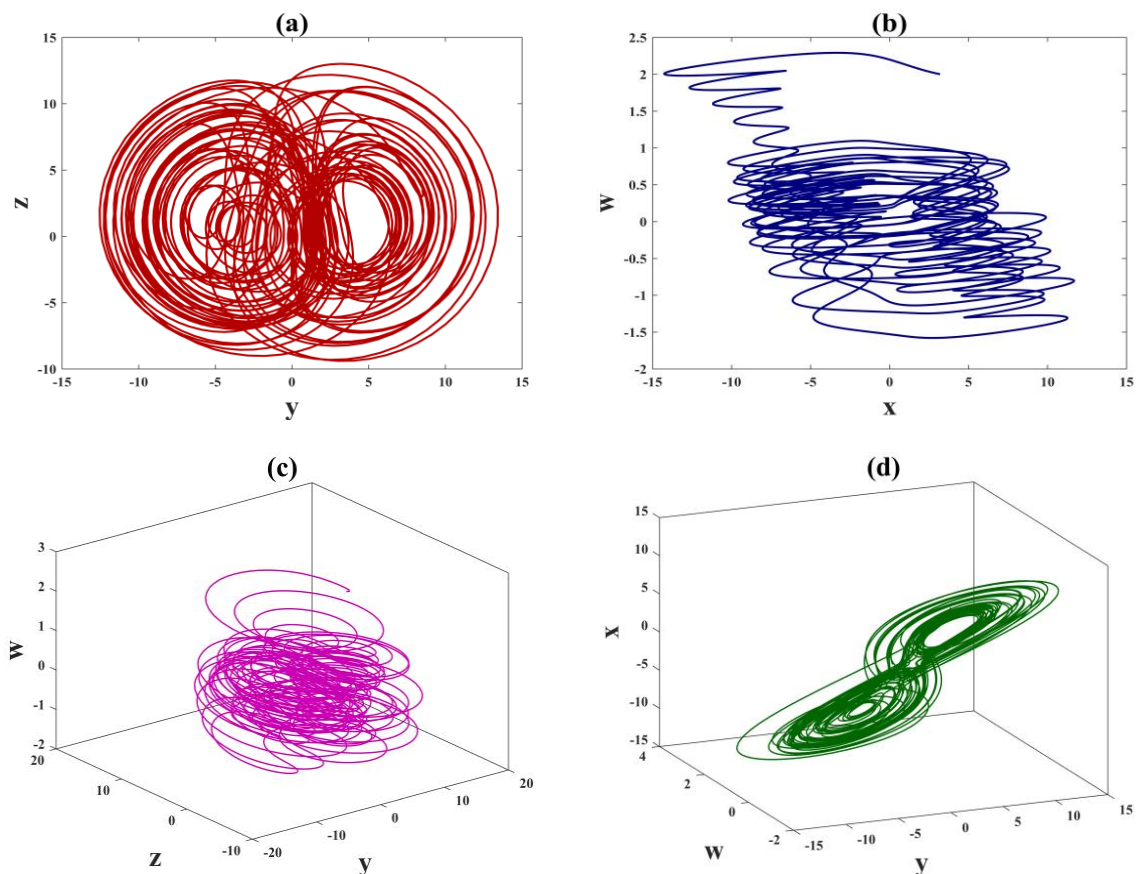


Figure 1. Show attractor the system (1) a:  $y, z$ -plane, b:  $x, w$ -plane, c:  $y, z, w$ -space, d:  $y, w, x$ -space.

## 3. THE PROJECTIVE SYNCHRONIZATION OF 4-D LORENZ SYSTEM

In this section, the projective synchronization between two identical 4-D hyperchaotic Lorenz system is implemented via nonlinear control strategy, the drive and response systems are given by (2) and (3) respectively as:

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1) - fw_1 \\ \dot{y}_1 = x_1z_1 - qy_1 \\ \dot{z}_1 = b - x_1y_1 - cz_1 \\ \dot{w}_1 = ry_1 - dw_1 \end{cases} \tag{2}$$

$$\begin{cases} \dot{x}_2 = a(y_2 - x_2) - fw_2 + u_1 \\ \dot{y}_2 = x_2z_2 - qy_2 + u_2 \\ \dot{z}_2 = b - x_2y_2 - cz_2 + u_3 \\ \dot{w}_2 = ry_2 - dw_2 + u_4 \end{cases} \tag{3}$$

Where  $(x_i, y_i, z_i, w_i) \in R^4$ ,  $U = [u_1, u_2, u_3, u_4]^T$  Represents control input to be designed, by using the following law.

$$\begin{cases} \dot{e}_1 = x_2 - \alpha_1x_1 \\ \dot{e}_2 = y_2 - \alpha_2y_1 \\ \dot{e}_3 = z_2 - \alpha_3z_1 \\ \dot{e}_4 = w_2 - \alpha_4w_1 \end{cases} \tag{4}$$

For  $\forall \alpha_i = 1, i = 1,2,3,4$ , the corresponding error dynamical system is given as:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - fe_4 + u_1 \\ \dot{e}_2 = -qe_2 + e_1z_1 + e_3x_1 + e_1e_3 + u_2 \\ \dot{e}_3 = -ce_3 - e_1y_1 - e_2x_1 - e_1e_2 + u_3 \\ \dot{e}_4 = -de_4 + re_2 + u_4 \end{cases} \tag{5}$$

**3.1. Projective synchronization with unknown parameters**

In this subsection, we design adaptive nonlinear control to make the system (3) synchronizes with the system (2) with unknown parameters.

**Theorem 1.** If adaptive nonlinear control is selected as follows:

$$\begin{cases} u_1 = 0 \\ u_2 = -ae_1 - e_1z_1 - re_4 \\ u_3 = e_1y_1 \\ u_4 = fe_1 \end{cases} \tag{6}$$

Then the system (3) follows the trace of the system (2), which means that the projective synchronization can be achieved by the designed controller (6).

**Proof:** Insert the controller (6) in the error dynamical system (5), we obtain

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - fe_4 \\ \dot{e}_2 = -qe_2 + e_3x_1 + e_1e_3 - ae_1 - re_4 \\ \dot{e}_3 = -ce_3 - e_2x_1 - e_1e_2 \\ \dot{e}_4 = -de_4 + re_2 + fe_1 \end{cases} \tag{7}$$

To achieve projective synchronization for the system (7) theoretically, select the Lyapunov function as the following:

$$V(e) = e^T P e, P = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \tag{8}$$

The time derivative of the Lyapunov function is:

$$\dot{V}(e) = -ae_1^2 - qe_2^2 - ce_3^2 - de_4^2 = -e^T Q e$$

$$\text{Herein } Q = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}.$$

So, every diagonal matrix with positive diagonal elements is a positive definite. Therefore,  $\dot{V}(e)$  is also a positive definite function. The projective synchronization is realized between system (3) and system (2) based on the proposed control (6). This theoretical result is illustrated by numerical simulation as shown in Figure 2.

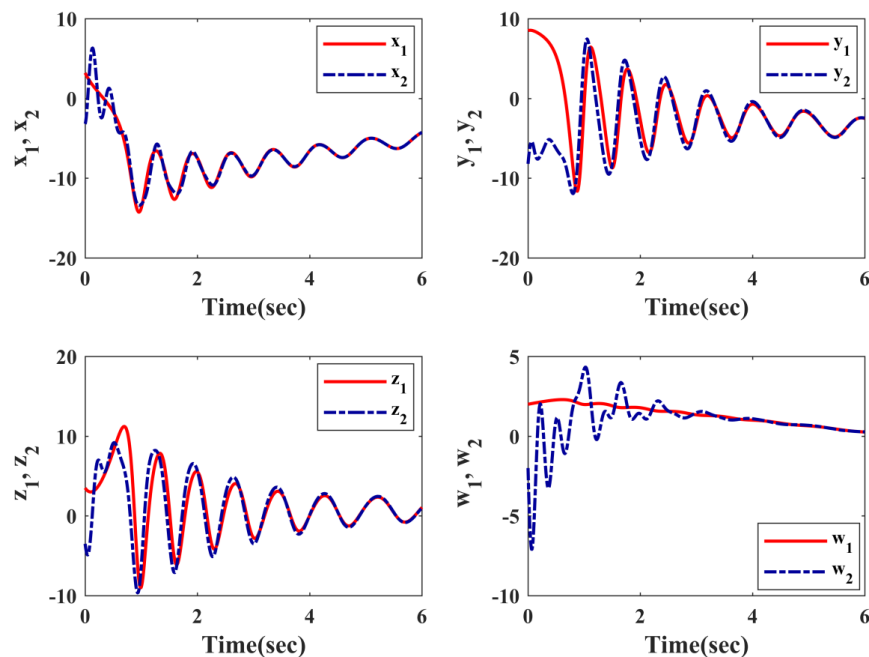


Figure 2. Projective synchronizes for the system (5) with controller (6)

### 3.2. Projective synchronization with known parameters

In the previous subsection (subsection 3.1), the projective synchronization with unknown parameters was performed theoretically and numerically without any problems. This subsection will be considered the projective synchronization with known parameters and note that there some problems in this case. And how we deal with these problems and what are treating? The following theory answer this question.

**Theorem 2.** If the adaptive nonlinear controller is designed as follows.

$$\begin{cases} u_1 = -e_2 z_1 + e_3 y_1 \\ u_2 = -b e_1 - r e_4 \\ u_3 = 0 \\ u_4 = f e_1 \end{cases} \quad (9)$$

Then system (5) will be projective synchronized.

**Proof:** When substituting the controllers (9) in the system (5), we get

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - f e_4 - e_2 z_1 + e_3 y_1 \\ \dot{e}_2 = -q e_2 + e_1 z_1 + e_3 x_1 + e_1 e_3 - b e_1 - r e_4 \\ \dot{e}_3 = -c e_3 - e_1 y_1 - e_2 x_1 - e_1 e_2 \\ \dot{e}_4 = -d e_4 + r e_2 + f e_1 \end{cases} \quad (10)$$

Based on theoretically analysis, the Lyapunov function and its derivative yields the (11) and (12), respectively.

$$V(e) = \frac{1}{2} \sum_{i=1}^4 e_i^2 \tag{11}$$

$$\dot{V}(e) = -ae_1^2 - qe_2^2 - ce_3^2 - de_4^2 + (a - b)e_1e_2 = -e^T Q_1 e \tag{12}$$

Where

$$Q_1 = \begin{bmatrix} a & -\frac{(a-b)}{2} & 0 & 0 \\ -\frac{(a-b)}{2} & q & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$$

To ensure that the system (10) is asymptotically stable, the symmetric matrix  $Q$  should be positive definite if the following conditions must hold:

$$\begin{cases} a, c, d > 0 \\ 4aq > (a - b)^2 \end{cases} \tag{13}$$

But, the second inequality of (13) is incorrect when the parameters are known. So, this control is failing. For this reason, we attempt to overcome this problem via two treatments:

- a) Modifying the control only.
- b) Modifying the control with Lyapunov function.

**First treatment:** The first treatment is by modifying the control itself through the modifying the first equation of control (9) i.e., add the term  $(3ae_2)$ . we obtain the following control:

$$\begin{cases} u_1 = -e_2z_1 + e_3y_1 + 3ae_2 \\ u_2 = -be_1 - re_4 \\ u_3 = 0 \\ u_4 = fe_1 \end{cases} \tag{14}$$

And the error dynamical system (5) with this control is becoming,

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - fe_4 - e_2z_1 + e_3y_1 + 3ae_2 \\ \dot{e}_2 = -qe_2 + e_1z_1 + e_3x_1 + e_1e_3 - be_1 - re_4 \\ \dot{e}_3 = -ce_3 - e_1y_1 - e_2x_1 - e_1e_2 \\ \dot{e}_4 = -de_4 + re_2 + fe_1 \end{cases}$$

Theoretically, based on Lyapunov function (11), the derivative for this function yield:

$$\dot{V}(e) = -ae_1^2 - qe_2^2 - ce_3^2 - de_4^2 + (4a - b)e_1e_2 = -e^T Q_2 e \tag{15}$$

$$Q_2 = \begin{bmatrix} a & \frac{(b-4a)}{2} & 0 & 0 \\ \frac{(b-4a)}{2} & q & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$$

Substitute the value of parameters in above matrix we get

$$Q_2 = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{10} \end{bmatrix}$$

So,

$$\dot{V}(e) = -5e_1^2 - e_2^2 - e_3^2 - 0.1e_4^2,$$

Then  $\dot{V}(e) < 0$ . Therefore, we succeed to achieve control for the system (5) after we are modifying control (9). This theoretical result is illustrated by numerical simulation as shown in Figure 3.

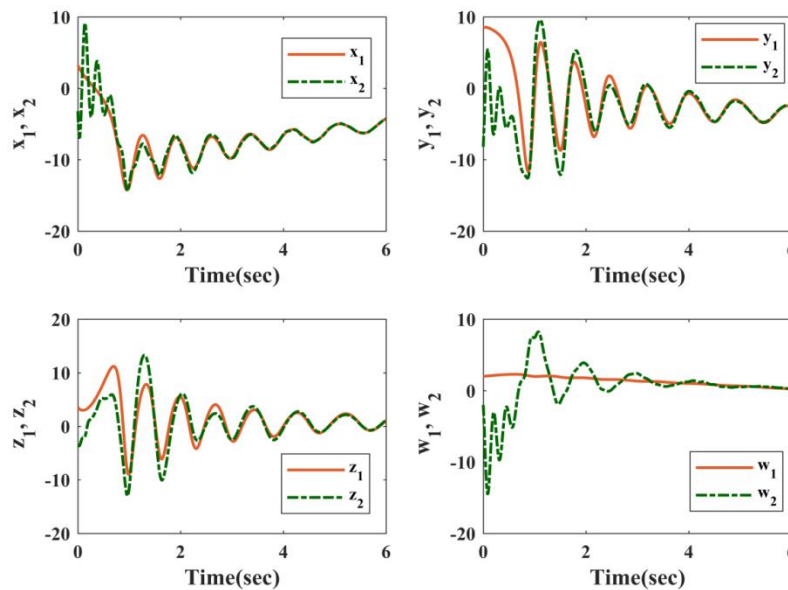


Figure 3. Projective synchronizes for the system (5) with the controller (14)

**Second treatment:** The second treatment is by modifying both control and Lyapunov function via modifying the second equation for control (5) i.e., replace the term  $(-be_1)$  be  $(-\frac{1}{4}be_1)$ . In other words, multiply the term  $(-be_1)$  be the constant  $(\frac{1}{4})$ , and the same time modifying the matrix  $P$  in (8).

$$\begin{cases} u_1 = -e_2z_1 + e_3y_1 \\ u_2 = -\frac{1}{4}be_1 - re_4 \\ u_3 = 0 \\ u_4 = fe_1 \end{cases} \tag{16}$$

And the error dynamical system (5) with this control is becomes,

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - fe_4 - e_2z_1 + e_3y_1 \\ \dot{e}_2 = -qe_2 + e_1z_1 + e_3x_1 + e_1e_3 - \frac{1}{4}be_1 - re_4 \\ \dot{e}_3 = -ce_3 - e_1y_1 - e_2x_1 - e_1e_2 \\ \dot{e}_4 = -de_4 + re_2 + fe_1 \end{cases} \tag{17}$$

Now, according to Lyapunov's second method, the Lyapunov function will be modified as follows:

$$V(e) = e^T P_1 e = \frac{1}{2}[4e_1^2 + 4e_2^2 + 4e_3^2 + 4e_4^2] \tag{18}$$

Where the diagonal matrix  $P_1$  modified as:

$$P_1 = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Consequently, the derivative of Lyapunov function becomes:

$$\dot{V}(e) = -4ae_1^2 - 4qe_2^2 - 4ce_3^2 - 4de_4^2 + (4a - b)e_1e_2 = -e^T Q_3 e \tag{19}$$

$$\text{when } Q_3 = \begin{bmatrix} 4a & -\frac{(4a-b)}{2} & 0 & 0 \\ -\frac{(4a-b)}{2} & 4q & 0 & 0 \\ 0 & 0 & 4c & 0 \\ 0 & 0 & 0 & 4d \end{bmatrix} = \begin{bmatrix} 20 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0.4 \end{bmatrix}.$$

So,  $Q_3$  is positive definite. Therefore, we succeed to achieve control for the system (9) after we update modifies the control with Lyapunov function theoretically. Figure 4 shows these results numerically.

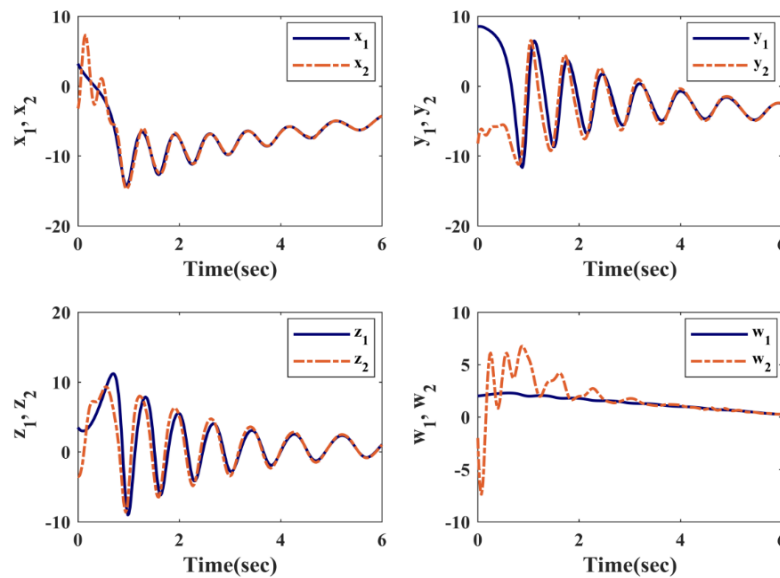


Figure 4. Projective synchronizes for the system (5) with the controller (16)

#### 4. CONCLUSION

In this paper, the projection synchronization between two identical four-dimensional hyperchaotic systems is dealt with. Two control with known and unknown parameters are proposed based on nonlinear control strategies, instead of the sliding mode control which is widely used in literature. In order to obtain an appropriate controller, many modifying on control are achieved through control itself and the Lyapunov function. As a result of two controllers based on theoretical analysis and numerical simulations, it is observed that two proposed control methods in the paper produce better outcomes than the sliding mode control.

#### ACKNOWLEDGEMENTS

The authors are very grateful to the University of Mosul/College of Computer Sciences and Mathematics for their provided facilities, which helped to improve the quality of this work.

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