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An optimal nonlinear control for anti-synchronization of Rabinovich hyperchaotic system

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ABSTRACT

This work derives new results for the anti-synchronization of 4D identical Rabinovich hyperchaotic systems by using two strategies: active and nonlinear control. The stabilization results of error dynamics systems are established based on Lyapunov second method. Control is designed via the relevant variables of drive and response systems. In comparison with previous strategies, the current controller (Nonlinear control) focused on the minimum possible limits for relevant variables. The better performance is realizing the anti-synchronization by designing a control with low terms. After obtaining analytical results of the proposed controller, numircal simulation is carried out using Matlab. The graphical results prove validity and applicability of proposed control without known any parameter. The proposed control has certain significance for reducing the time and complexity for strategy implementation.

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1. INTRODUCTION

The impetus for advances in dynamical systems has come from many sources: mathematics, theoretical science, computer simulation, and experimental science. The key requirement for these systems involves a nonlinearity. In 1990, Pecora and Carroll brought to the world the idea of synchronization of dynamical systems [1, 2]. Chaos synchronization has attracted considerable attention due to its important applications in physical systems [3], Encryption [4, 5], and secure communications [6], etc. This greatest success opened the way to discover another phenomenon.

Until now, scientists realize that chaos synchronization can be observed in experiments and in computer models of behavior from all fields of science and engineering [7]. In addition, enormous synchronization phenomena have been applied in various dynamical systems such as complete synchronization (CS) [8-11], anti-synchronization(AS) [12], Hybrid Synchronization (HS) [13], projective synchronization (PS) [14], modified projective synchronization (MPS) [15] and generalized projective synchronization (GPS) [16]. Full synchronization and anti-synchronization are the most commonly used [17] and play an important role in engineering applications [18, 19].

These phenomena are achieved via different various types of anti-synchronization schemes including active control [17], adaptive control, nonlinear control [18-21] and linear feedback control [22-24]. Among the aforementioned schemes, active control and nonlinear control have been widely used as two powerful strategies for the anti-synchronization of different classes of nonlinear dynamical systems [16-20]. The nonlinear control strategy is considered as one of the powerful tools for controlling the dynamical systems. However, the active control suffers from many terms corresponding to relevant variables of drive and response systems. To overcome this problem, the nonlinear control strategy is used with the minimum of terms anti-synchronization whereas nonlinear control strategy has demonstrated excellent performance in anti-synchronization schemes.

In this paper, we implement anti-synchronization between two 4D identical Rabinovich hyperchaotic systems based on active and nonlinear control strategies via Lyapunov second method and observed that the number of terms less than the first strategy. The proposed control with low terms is more interesting and easily applied and implemented.

Major contributions of this work are as follows: (i) active and nonlinear control strategy based Lyapunov second method is utilized for anti-synchronization. (ii) Proposed controllers are exploited without known any parameter. (iii) A necessary and sufficient condition is proposed to show how many relevant variables of drive and response systems can achieve an anti-synchronization underactive and nonlinear controller. (iv) After deriving analytical results, numerical simulation is carried out using Matlab. (v) The validity and applicability of the proposed controllers are proven with graphical results.

The rest of this paper is organized as follows. Section 2 is the description of the hyperchaotic Rabinovich System. Section 3 presents the problem of anti-synchronization for the hyperchaotic Rabinovich. Section 4 is the conclusions of this paper.

2. DECRIPTION OFHYPERCHAOTIC RABINOVICH SYSTEM

Rabinovich system is a four-dimensional hyperchaotic which include ten terms, three of them are nonlinearity with three parameters and descript by the following form [25, 26]:

$$\begin{cases} \dot{x}_1 = -ax_1 + hx_2 + x_2x_3 \\ \dot{x}_2 = hx_1 - x_2 - x_1x_3 + x_4 \\ \dot{x}_3 = -x_3 + x_1x_2 \\ \dot{x}_4 = -kx_2 \end{cases} \tag{1}$$

where x_1, x_2, x_3, x_4 are the state variables and $a = 4, h = 6.75, k = 2$ are positive constants. Figures 1-2 show the attractor of system (1):

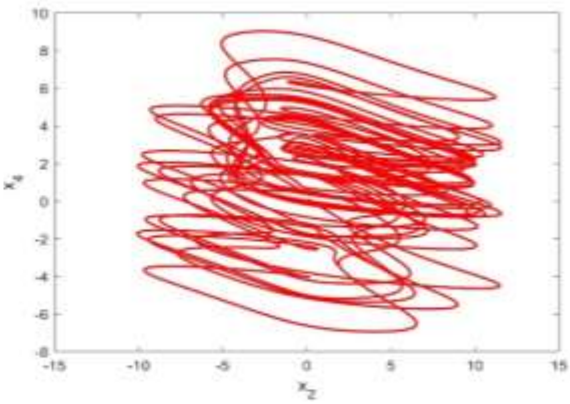

Figure 1. The attractor of system (1) in $x_2 - x_4$ plane

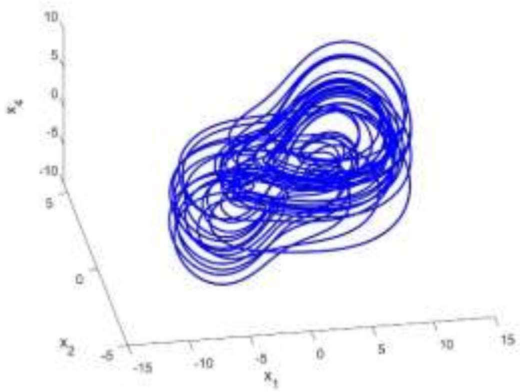

Figure 2. The attractor of system (1) in x_1, x_2, x_4 space

3. ANTI-SYNCHRONIZATION BETWEEN TWO IDENTICAL HYPERCHAOTIC RABINOVICH SYSTEM

In order to achieve anti-synchronization for the Rabinovich system, two systems are needed, the first system (1) is called the drive system, and the second system is called the response system. The response system for the Rabinovich system depicts in (2).

$$\begin{cases} \dot{y}_1 = -ay_1 + hy_2 + y_2y_3 + u_1 \\ \dot{y}_2 = hy_1 - y_2 - y_1y_3 + y_4 + u_2 \\ \dot{y}_3 = -y_3 + y_1y_2 + u_3 \\ \dot{y}_4 = -ky_2 + u_4 \end{cases} \tag{2}$$

where $u = [u_1, u_2, u_3, u_4]^T$ is the controller to be designed, the anti-synchronization error $e \in R^4$ is defined as :
$e_1 = y_1 - \alpha_i x_1$, $e_2 = y_2 - \alpha_i x_2$, $e_3 = y_3 - \alpha_i x_3$, $e_4 = y_4 - \alpha_i x_4$, $\forall\alpha_i = -1$, $\forall\alpha_i = -1$, $i = 1,2,3,4$
So, the error dynamical system is given by:

$$\begin{cases} \dot{e}_1 = -ae_1 + he_2 + e_2e_3 - x_3e_2 - x_2e_3 + 2x_2x_3 + u_1 \\ \dot{e}_2 = he_1 - e_2 + e_4 - y_1e_3 - x_3e_1 + 2y_1x_3 + u_2 \\ \dot{e}_3 = -e_3 + e_1e_2 - x_2e_1 - x_1e_2 + 2x_1x_2 + u_3 \\ \dot{e}_4 = -ke_2 + u_4 \end{cases} \quad (3)$$

3.1 Anti-synchronization based on active control

To realize the anti-synchronization, we need to design suitable nonlinear control. Therefore, the control functions are chosen as the following:

$$\begin{cases} u_1 = -e_2e_3 + x_3e_2 + x_2e_3 - 2x_2x_3 + v_1 \\ u_2 = y_1e_3 + x_3e_1 - 2y_1x_3 + v_2 \\ u_3 = -e_1e_2 + x_2e_1 + x_1e_2 - 2x_1x_2 + v_3 \\ u_4 = v_4 \end{cases} \quad (4)$$

inserting the control (4) in (3) we get:

$$\begin{cases} \dot{e}_1 = -ae_1 + he_2 + v_1 \\ \dot{e}_2 = he_1 - e_2 + e_4 + v_2 \\ \dot{e}_3 = -e_3 + v_3 \\ \dot{e}_4 = -ke_2 + v_4 \end{cases}, \; v = [v_1 \quad v_2 \quad v_3 \quad v_4]^T = A[e_1 \quad e_2 \quad e_3 \quad e_4]^T \quad (5)$$

where v is linear control, A is a constant matrix. To make the system (5) stable, the matrix A should be selected by the following:

$$A = \begin{bmatrix} (a-1) & -h & 0 & 0 \\ -h & (1-2a) & 0 & -1 \\ 0 & 0 & -2 & 0 \\ 0 & k & 0 & -k \end{bmatrix} \quad (6)$$

hence, the error dynamical system (3) with above matrix becomes:

$$\begin{cases} \dot{e}_1 = -e_1 \\ \dot{e}_2 = -2ae_2 \\ \dot{e}_3 = -3e_3 \\ \dot{e}_4 = -ke_4 \end{cases} \quad (7)$$

therefore, the above system has all eigenvalues with negative real parts. These eigenvalues guarantee the stability of the system (7). So, the response system (2) is anti-synchronization with the drive system. Hence, we reach the following results.
Theorem 1: If the matrix (6) is combined with the system (5), then, the response system (2) unfollows the drive system (1) via the following nonlinear active control which consists of (21) terms.

$$\begin{cases} u_1 = (\mathrm{a}-1)e_1 - he_2 - e_2e_3 + x_3e_2 + x_2e_3 - 2x_2x_3 \\ u_2 = -he_1 + (1-2\mathrm{a})e_2 - e_4 + y_1e_3 + x_3e_1 - 2y_1x_3 \\ u_3 = -2e_3 - e_1e_2 + x_2e_1 + x_1e_2 - 2x_1x_2 \\ u_4 = ke_2 - ke_4 \end{cases} \quad (8)$$

Proof: Based on the Lyapunov second method, we construct a positive definite Lyapunov candidate function as:

$$\mathrm{V(e)} = e^T pe = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 + \frac{1}{2}e_4^2 \quad (9)$$

where $P = diag\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right]$, the derivative of the Lyapunov function V(e) with respect to time is:

$$\dot{V}(e) = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4$$

$$\dot{V}(e) = e_1(-e_1) + e_2(-2ae_2) + e_3(-3e_3) + e_4(-ke_4)$$

$$\dot{V}(e) = -e_1^2 - 2ae_2^2 - 3e_3^2 - ke_4^2 = -e^TQe\ ,\ \ Q = diag[\,1, 2a, 3, k] \quad (10)$$

Every diagonal matrix with positive diagonal elements is positive definite. So $Q > 0$. Therefore, $\dot{V}(e)$ is negative definite. And according to the Lyapunov asymptotical stability theory, the nonlinear active controller is implemented and the anti-synchronization of the hyperchaotic system is achieved. The proof is now complete. The theorem 1 shows that proposed control which consists of (21) terms achieved anti-synchronization in Figure 3.

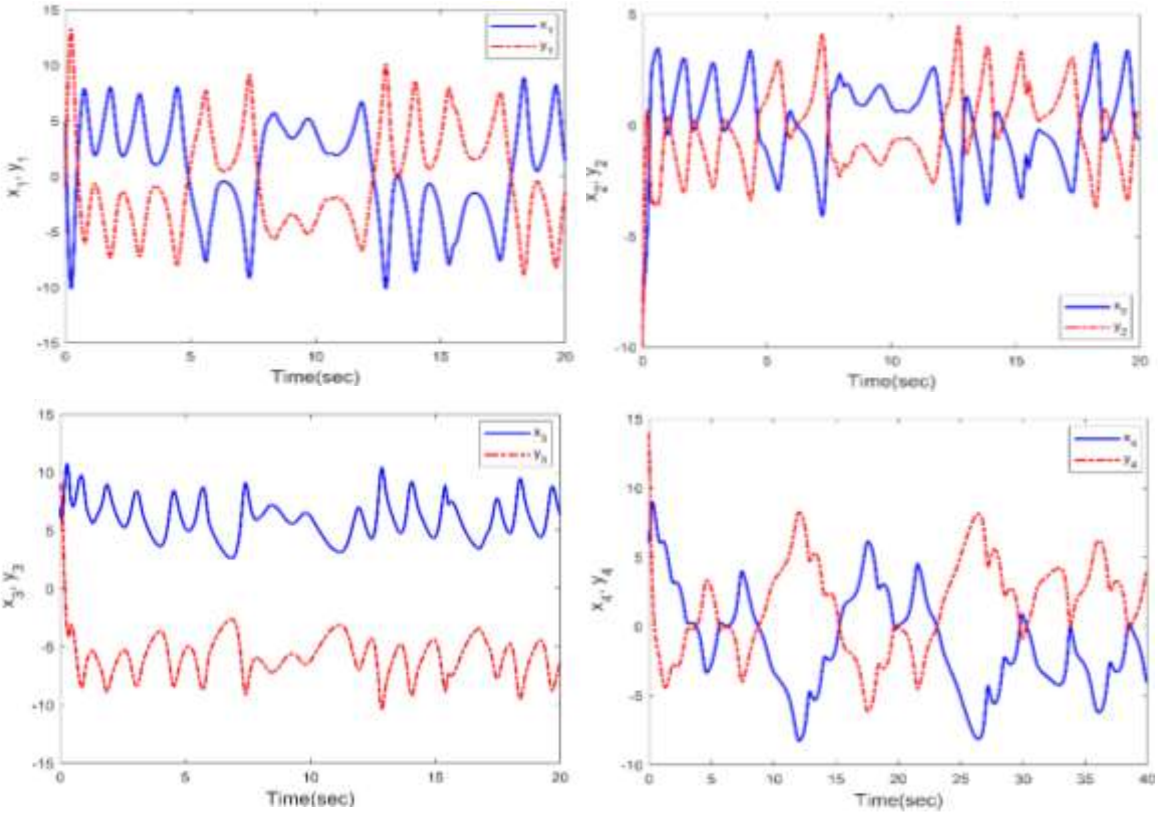

Figure 3. Anti-synchronization between systems (1) and (2) with control (8)

3.2. Anti-Synchronization based on nonlinear control strategy

In this section, anti-synchronization between system (1) and system (2) is considered by using another strategy which is called nonlinear control.

Theorem 2: The system (3) is stable, if design a controller consists of (12) terms as follows:

$$\begin{cases} u_1 = 2(x_2e_3 - x_2x_3) \\ u_2 = -2((h - x_3)e_1 + y_1x_3) + x_1e_3 \\ u_3 = -2(e_1e_2 + x_1x_2\,) + y_1e_2 \\ u_4 = (k-1)e_2 - e_4 \end{cases} \quad (11)$$

Proof: With this choice, the error dynamical system (3) becomes

$$\begin{bmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\\ \dot{e}_4\end{bmatrix}=\begin{bmatrix}-a & h-x_3 & e_2+x_2 & 0\\ -h+x_3 & -1 & -y_1+x_1 & 1\\ -e_2-x_2 & -x_1+y_1 & -1 & 0\\ 0 & -1 & 0 & -1\end{bmatrix}\begin{bmatrix}e_1\\ e_2\\ e_3\\ e_4\end{bmatrix}$$

i.e.

$$\begin{cases}\dot{e}_1=-ae_1+he_2+e_2e_3-x_3e_2+x_2e_3\\ \dot{e}_2=-he_1-e_2+e_4-y_1e_3+x_3e_1+x_1e_3\\ \dot{e}_3=-e_3-e_1e_2-x_2e_1-x_1e_2+y_1e_2\\ \dot{e}_4=-e_2-e_4\end{cases} \tag{12}$$

The Lyapunov function and it is derivative as (13) and (14) respectively:

$$\mathrm{V(e)}=e^Tpe \tag{13}$$

$$\dot{V}(e)=-ae_1^2-e_2^2-e_3^2-e_4^2=-e^TQe \tag{14}$$

So $Q>0$. Therefore, $\dot{V}(e)$ is negative definite. The theorem 2 showed that proposed control which consists of (12) terms achieved anti-synchronization in Figure 4.

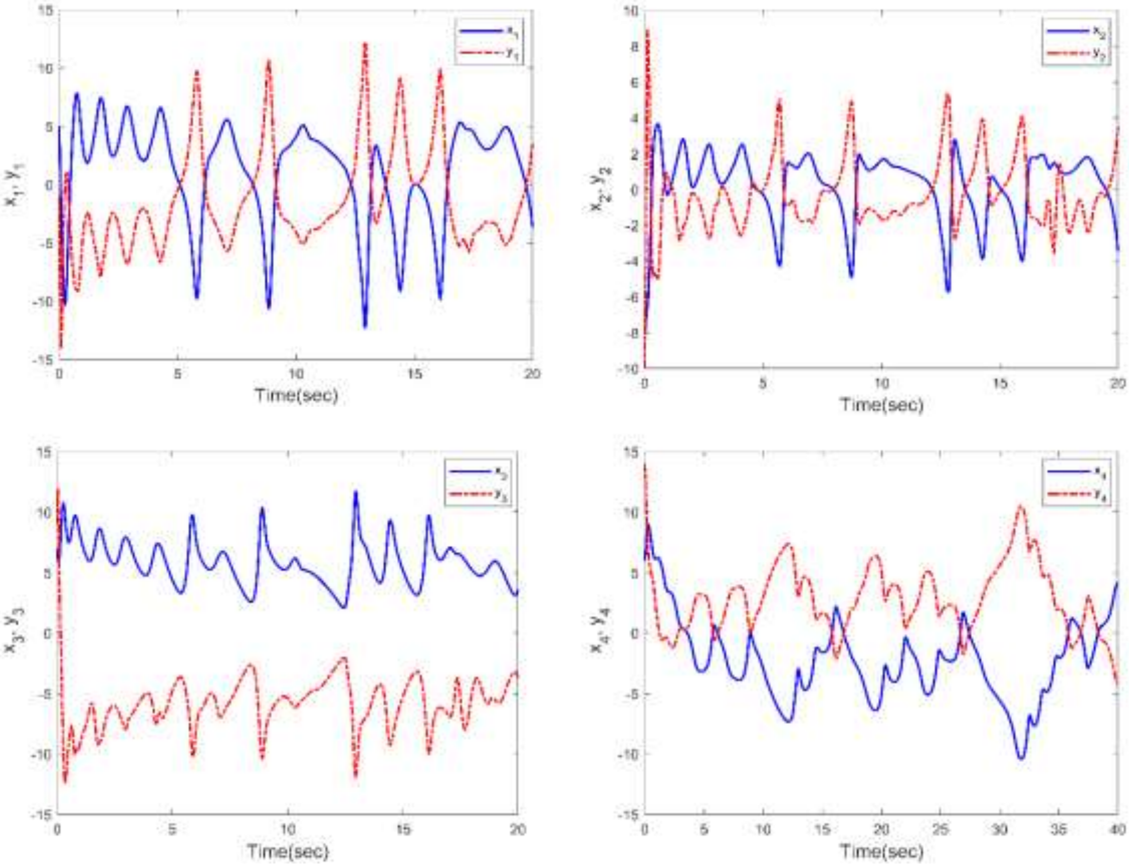

Figure 4. Anti-synchronization between systems (1) and (2) with control (11)

Theorem 3: If the controller is designed with (12) terms as follows:

$$\begin{cases}u_1=-2(he_2+e_2e_3-x_3e_2+x_2x_3)\\ u_2=x_1e_3-e_4-2y_1x_3\\ u_3=2(e_1-x_1)x_2\ +y_1e_2\\ u_4=ke_2-e_4\end{cases} \tag{15}$$

then, the response system (2) unfollow the drive system (1) via the following nonlinear control.
Proof: When substituting the controllers (15) in the system (3), we get:

$$\begin{cases} \dot{e}_1 = -ae_1 + (-h + x_3)e_2 - (e_2 + x_2)e_3 \\ \dot{e}_2 = (h - x_3)e_1 - e_2 + (-y_1 + x_1)e_3 \\ \dot{e}_3 = (e_2 + x_2)e_1 + (-x_1 + y_1)e_2 - e_3 \\ \dot{e}_4 = -e_4 \end{cases} \quad (16)$$

Construct Lyapunov function as:

$$\mathrm{V(e)} = e^T pe = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 + \frac{1}{2}e_4^2$$

Then

$$\dot{V}(e) = -ae_1^2 - e_2^2 - e_3^2 - e_4^2 = -e^T Qe$$

So, $\mathrm{V(e)} > 0$ and $\dot{V}(e) < 0$, the nonlinear controller is implemented. The theorem 3 showed that proposed control which consists of (12) terms achieved anti-synchronization in Figure 5.

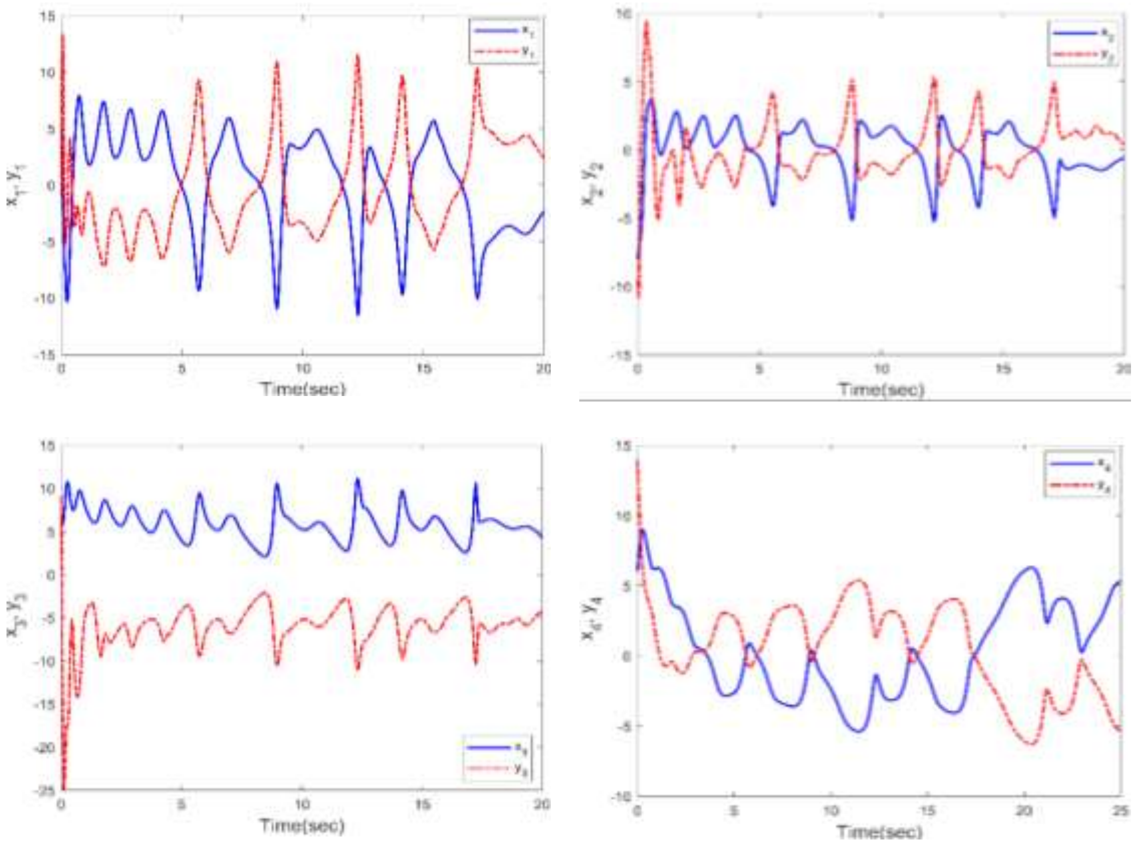

Figure 5. Anti- synchronization between systems (1) and (2) with control (15)

Theorem 4: The system (3) is achieved. If the controller is designed as:

$$\begin{cases} u_1 = -2(x_2 - e_2)x_3 \\ u_2 = -2he_1 + y_1e_3 - 2y_1x_3 \\ u_3 = -2(e_2 - x_2)e_1 + x_1(e_2 - 2x_2) \\ u_4 = (k - 1)e_2 - e_4 \end{cases} \quad (17)$$

then, the response system (2) unfollows the drive system (1) via the following nonlinear control.
Proof: Using system (3) with the controller (17) is given by:

$$\begin{cases} \dot{e}_1 = -ae_1 + (h + x_3)e_2 + (e_2 - x_2)e_3 \\ \dot{e}_2 = -(h + x_3)e_1 - e_2 + e_4 \\ \dot{e}_3 = (x_2 - e_2)e_1 - e_3 \\ \dot{e}_4 = -e_2 - e_4 \end{cases} \tag{18}$$

The same results were found in theorem (3). The theorem 4 showed that proposed control which consists of (12) terms achieved anti-synchronization in Figure 6.

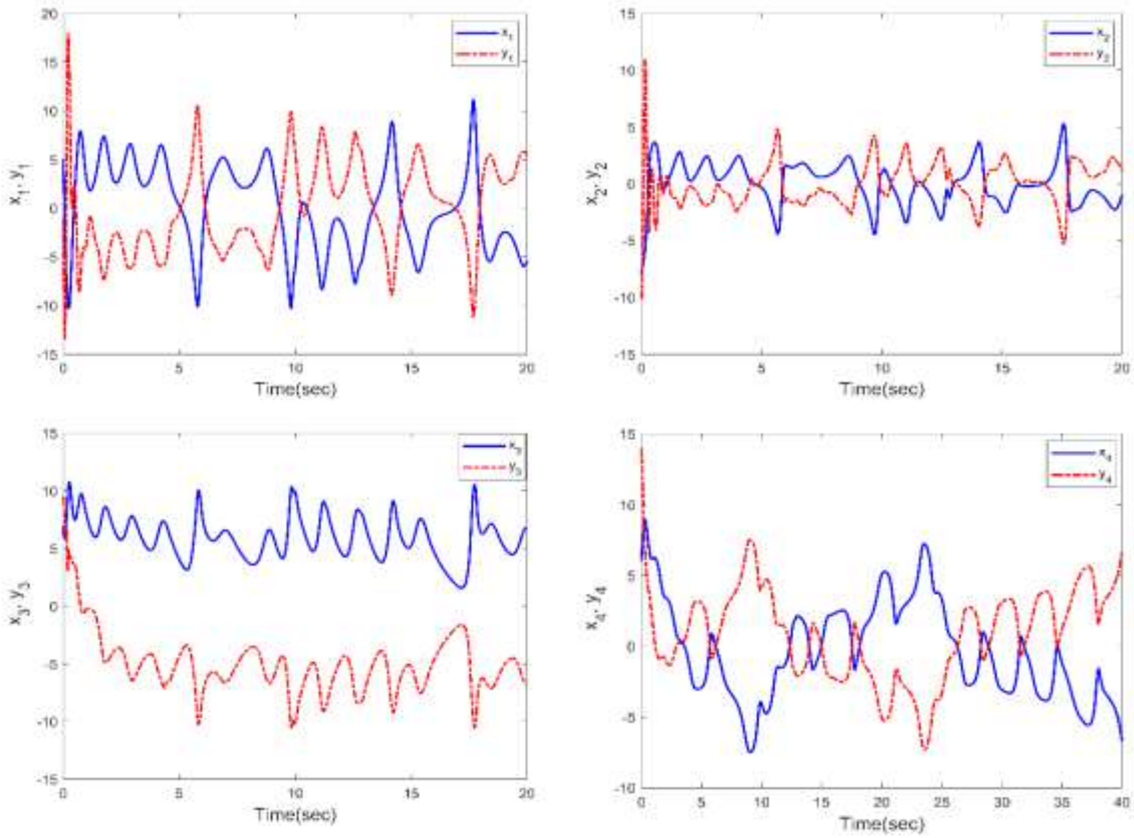

Figure 6. Anti-synchronization between systems (1) and (2) with control (17)

4. CONCLUSION

In the paper, the anti-synchronization problem for 4-D Rabinovich hyperchaotic system is considered, based on two strategies: active and non-linear controller. The stability of error dynamical systems are established based on the Lyapunov theory and compared between these strategies It was found that both of them lead to anti-synchronization, but the performance of the number of terms of the nonlinear controller is less than the active control.

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