SVR-based RPD Approach for Complex Processes and its Application in Circuit Optimization

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Abstract

During the lifespan of electronic products, the output voltage and current fluctuate due to the random fluctuations of parameter values of circuit components and environmental noise. Extant methods of circuit designs, such as parameter sweep and sensitivity analysis, are hard to obtain global robust optimization of output characteristics. This paper proposes a SVR-based robust parameter design approach to reach global circuit optimization. First, the approach fits an empirical model of process responses by using SVR. Next, it introduces the fluctuations of controllable factor variations and noise factors into response model by probability density functions, and calculates process means and variances by integration. Finally, it obtains optimal parameter combination by model optimization. An empirical study of the robust design of an inductor-resistor series circuit is conducted. The results show that the proposed approach not only avoids the disadvantage of ignoring interactions between factors when using parameter sweep and sensitivity analysis, but also overcomes the shortcoming of only achieving non-continuous optimization by Taguchi method and the limitation of obtaining local optimization by DRSM, and therefore, enhances the robustness of the circuit outputs.

Keywords: circuit optimization, SVR, RPD, complex processes

1. Introduction

The parameter values of the circuit components randomly fluctuate over time during the lifespan of electronic products. The output characteristics of circuit, such as voltage and current, also fluctuate due to environmental noise. Therefore, the key to product design is to reduce these fluctuations and increase the stability of circuit.

Traditional methods of circuit design include parameter sweep [1] and sensitivity analysis [2]. Parameter sweep examines the influence of the change of a specific component value on outputs by fixing the parameter values of other components. The method uses a one-factor rotation design which needs more runs and cannot examine the interaction effects between components. Sensitivity analysis examines the stability of output voltage or current through the differential transformation of certain input features. However, neither of the methods takes into account the influence of simultaneous fluctuations of multiple component parameter values. Therefore, the output stability of circuit after using these optimization methods needs to be improved.

In fact, how to select appropriate component parameter values which makes circuit output characteristics insensitive to component variations and environmental noise can be considered as a typical robust parameter design (RPD) problem. Moreover, the feature of the relationship between circuit inputs and outputs is complex nonlinearity, not only because the circuit consists of various components (e.g., resistor, capacitor, inductor, and power) that have very different electrical performance, but also because there are random fluctuations of the parameter values during the lifespan of electronic products.

Traditional robust design methods have limited capability to deal with such complex processes. Taguchi method can optimize the process only at certain factor levels, but it fails to gain continuous optimization. Dual Response Surface Methodology (DRSM) is applicable to the optimization of the simple processes which can be fitted by second-order polynomials. Both methods cannot effectively deal with the influence of the random fluctuations of component parameter values on the outputs.

This paper proposes a Support Vector Regression (SVR) based RPD approach for circuit optimization. The approach uses the following steps to achieve the optimization of circuit component parameter values. Firstly, it establishes an empirical model of the complex relationship between circuit component parameter values and the outputs by using SVR. Secondly, it describes the fluctuations of component parameter values by using normal or uniform probability distributions, and describes the influence of component parameter value fluctuations and process noise on circuit outputs by using joint probability distributions. Finally, it finds the parameter values that minimize circuit output fluctuations by using parallel gradient descent method. In the following sections, the paper first reviews extant RPD approaches and gives a brief introduction of SVR theory, and then describes the algorithm steps of the approach. After that, the paper demonstrates the effectiveness of the approach by a case study of the optimization of inductor-resistor series circuit.

2. Brief Introduction to Theories of RPD
2.1. Taguchi Method
It was Taguchi [3] who first introduced the concept of RPD. Taguchi method categorizes process parameters into controllable factors and noise factors. It increases the stability of processes by selecting appropriate controllable factor levels that make quality characteristics (i.e., "responses") of process outputs insensitive to the variations of controllable factors and the influence of noise factors. Signal-to-Noise Ratio (SNR) analysis is one of the main methods to achieve RPD. The basic idea of this method is to (1) plan and run experiments by using inner-outer array; (2) calculate SNR according to the quality characteristics; and (3) find the optimal controllable factor levels that minimize the fluctuations of the responses. SNR is calculated according to the goals of different types of quality characteristics. Taguchi method classifies quality characteristics into three types: the smaller-the-better (STB), the larger-the-better (LTB), and the nominal-the-best (NTB) characteristics. It uses SNR to reflect the robustness of responses, and search for the factor level combination that maximizes the SNR, thereby achieving RPD.

It is relatively plain and convenience to select the optimal factor level combination by using SNR. But the method has many limitations as well. First, it requires information about the approximate range of factor levels in advance and needs many runs to obtain a satisfied solution [4]. Second, SNR loses a lot of information that is related to process features [5]. The third, it can optimize only at certain factor levels, which leads to a satisfied solution rather than an optimal solution. Therefore, many scholars and experts propose various approaches to improve RPD method among which the representative is the DRSM proposed by Vining and Myers [6].

2.2. DRSM
DRSM [7, 8] investigates process optimums step by step by sequentially adopting first-order polynomial modeling and steepest ascent optimization, and then fits process mean and process variance models by using second-order polynomial in a relatively small range of the factors. After that, it obtains an optimal solution by minimizing the variance under the constraint of mean target. Unlike Taguchi method, DRSM successfully combines parameter design and regression analysis, and obtains regression model between responses and factors through experimental design. It can attain the continuous optimization of factor levels, and is the main method for RPD.

DRSM [9], however, has its limitations when it is applied to RPD with complex processes. Firstly, second-order polynomials fail to fit the complex nonlinear relationship between factors and responses. Secondly, the result of optimization is sensitive to the initial values. It may obtain local rather than global optimization when inappropriate initial values are selected. The third, DRSM doesn't consider the influence of factor variations on responses, therefore, is not a real robust parameter design.

2.3. SVR
SVR is a small-sample based approximate statistical learning approach proposed by Vapnik [10]. It can establish nonparametric models that meet the characteristics of process

under the constraint of small sample. It has been widely used to model complex processes [11-13]. The basic principles of SVR are as follows:

let $x \in R^n$ and $y \in R$ denote the input variables

Vector and output variable of a process; suppose that the function $y = f(x)$ is unknown, the task of model fitting is to use the data from the independent and identical distribution empirical sample S

$$S = \{(x_1, y_1), (x_2, y_2), \ldots, (x_l, y_l)\}, \quad x_i \in x, \, y_i \in y \tag{1}$$

to find the optimum function $f(x, \alpha_0)$ in the function set $\{f(x, \alpha)\}$ which minimizes the excepted risk of prediction:

$$R(\alpha) = \int L(y, f(x, \alpha)) dF(x, y) \tag{2}$$

where α is the generalized parameter, and $L(y, f(x, \alpha))$ is the loss-function which defined as the ε – insensitive function:

$$L(y, f(x, \alpha)) = \left| y - f(x, \alpha) \right|_{\varepsilon > 0}$$
$$= \begin{cases} 0, & \left| y - f(x, \alpha) \right| \le \varepsilon \\ \left| y - f(x, \alpha) \right| - \varepsilon, & \left| y - f(x, \alpha) \right| > \varepsilon \end{cases} \tag{3}$$

If the relationship between input *x* and output *y* is onlinearity, *x* is first mapped onto a high linear dimensional feature space using nonlinear mapping function *T(x)*, and then a linear model is obtained in the feature space:

$$f(x, w) = w \bullet T(x) + b. \tag{4}$$

Then SVR model fitting is formulated as the following optimization problem:

$$\min_{w,b} \frac{1}{2} \|w\|^2 + C \sum_{i=1}^{n} (\xi_i + \xi_i^*)$$
$$st. \; (w \bullet T(x_i) + b) - y_i \le \varepsilon + \xi_i \tag{5}$$
$$y_i - (w \bullet T(x_i) + b) \le \varepsilon + \xi_i^*$$
$$\xi_i, \; \xi_i^* \ge 0, \quad i = 1, 2, \cdots, l$$

Where ξ_i, ξ_i^* is the non-negative slack variables and C is the penalty parameter.
The dual problem of equation (6) is:

$$\min_{\alpha^*} \frac{1}{2} \sum_{i,j=1}^{n} (a_i^* - a_i)(a_j^* - a_j) k(x_i, x_j)$$
$$+ \varepsilon \sum_{i=1}^{l} (a_i^* + a_i) - \sum_{i=1}^{l} y_i (a_i^* - a_i) \quad , \tag{6}$$
$$st. \; \sum_{i=1}^{l} (a_i - a_i^*) = 0, 0 \le a_i^*, a_i \le C, i = 1, 2, \cdots, l$$

with the solution of

$$(\bar{a}, \bar{a}^*) = (\bar{a}_1, \bar{a}_1^*, \cdots, \bar{a}_n, \bar{a}_n^*)^{\mathrm{T}}. \tag{7}$$

In equation (7), $k(x, x') = T(x) \bullet T(x')$ is the kernel

function which can reduce the complex of operation in the high dimension feature space after mapping. Finally, the SVR fitting model becomes

$$\begin{cases} f(\boldsymbol{x}, \alpha) = \overline{w} \bullet \boldsymbol{x} + b = \sum_{i=1}^{n} (\overline{a}_i^* - \overline{a}_i) k(\boldsymbol{x}_i \bullet \boldsymbol{x}) + \overline{b} \\ \overline{w} = \sum_{i=1}^{n} (\overline{a}_i^* - \overline{a}_i) k(\boldsymbol{x}_i, \boldsymbol{x}) \\ \overline{b} = y_j - \sum_{i=1}^{n} (\overline{a}_i^* - \overline{a}_i) k(\boldsymbol{x}_i, \boldsymbol{x}) - \varepsilon \end{cases} \tag{8}$$

3. SVR Based RPD
3.1. Basic Ideas
For RPD of complex processes, establishing the empirical models of process means and variances is one of the key steps. Specifically, the empirical models need to meet two requirements. First, the models should reflect the complex relationship between factors and responses in the whole range of factors. Second, the sample needed for modeling should be as small as possible to reduce the cost of optimization. Therefore, SVR models that are suitable for small-sample global modeling can be selected as the basic form of empirical models. Moreover, SVR models have analytic form which helps the consequent computing and optimizing. Therefore, using SVR for modeling has its natural advantage for robust design of circuit with random parameter variations and process noise. The characteristics of SVR enable us to adopt space filling designs such as uniform design [14] with large interval and LHS design. Consequently, not only the sample points selected can cover the whole feasible zone, but also the sample is relatively small.

After modeling and sample point selecting approach are determined, the next step is to consider how to deal with the fluctuations of controllable factor variations and noise factors, both of which influence the response outputs. However, most of the existing studies lay their emphasis on the influence of noise factors. Little effort has been put to examine the influence of the variations of controllable factors [15, 16]. In fact, the fluctuations of controllable factor variations have certain statistical patterns (e.g., in a circuit, the resistance of a resistor normally or uniformly and randomly fluctuates around its nominal value), so we can use probability density functions (e.g., multi-dimensional uniform distribution and multi-dimensional normal distribution, etc.) to describe the fluctuations. In addition, the change of noise factors is typically normal distributed. Therefore, in modeling, we can first establish the single response models among process responses, controllable factors, and noise factors, and then introduce the fluctuations of controllable factor variations and noise factors into response models by probability density function, and calculate process means and variances by integration. The last step is to find multiple minimum points of process variances by using methods such as parallel gradient descent [17], and determine the optimal parameter combination according to the value of process means.

3.2. Algorithm Steps of SVR Based RPD Approach
Based on the theoretical analyses above, we propose the SVR-based RPD for circuit optimization as follows:

Step 1: select the controllable factors $\mathbf{x} = \left[x_1, x_2, \cdots x_m \right]'$ and noise factors $\mathbf{z} = \left[z_1, z_2, \cdots z_n \right]'$, then decide the range of each factor by prior knowledge of the process;

Step 2: run uniform design or uniform grid design experiments with single arrays of controllable factors and noise factors to get raw data, and then standardize the sample data as follows:

$$S = \left\{ \left(\mathbf{x}_1, \mathbf{z}_1, y_1 \right), \cdots, \left(\mathbf{x}_l, \mathbf{z}_l, y_l \right) \right\}, \mathbf{x}_l \in X, \mathbf{z}_l \in Z, y_l \in Y \tag{9}$$

Step 3: choose Gauss function as the kernel function and set appropriate parameters of C and ε, and then fit the SVR model of process output

$$y = f\left([\mathbf{x};\mathbf{z}]\right) + \varepsilon = \sum_{i=1}^{l}\left(\alpha_i^* - \alpha_i\right)k\left([\mathbf{x};\mathbf{z}]_i,[\mathbf{x};\mathbf{z}]\right) + \overline{b} + \varepsilon, \varepsilon \sim \left(0,\sigma_\varepsilon^2\right) \qquad (10)$$

Step 4: set the probability density of controllable factor variations and noise factors according to prior knowledge.

Specifically, assume the controllable factor variations follow the normal distribution, and let x_{pi} denotes the variation of x_i, then the probability density functions of the controllable factor variations turn into

$$g_i(x_{pi}) = \frac{1}{\sigma_{pi}\sqrt{2\pi}}\exp[-\frac{(x_{pi}-x_i)^2}{2\sigma_{pi}^2}], \quad i=1,2,\cdots m \qquad (11)$$

Assume noise factors follow the normal distribution with the following probability density functions:

$$f_j(z_j) = \frac{1}{\sigma_j\sqrt{2\pi}}\exp[-\frac{(z_j-\mu_j)^2}{2\sigma_j^2}], \quad j=1,2,\cdots n \qquad (12)$$

and assume the independence both among different controllable factors and between controllable factors and noise factors, the mean, variance and mean square error (*MSE*) of the output *y* are denoted as

$$\begin{cases} E(y) = \int\cdots\iint y g_1\left(x_{p1};x_1\right)\cdots g_m\left(x_{p1};x_m\right)f_1\left(z_1;\mu_1\right)\cdots f_n\left(z_n;\mu_n\right)d\mathbf{x}_p d\mathbf{z} \\ D(y) = \int\cdots\iint\left[y-E(y)\right]^2 g_1\left(x_{p1};x_1\right)\cdots g_m\left(x_{p1};x_m\right)f_1\left(z_1;\mu_1\right)\cdots f_n\left(z_n;\mu_n\right)d\mathbf{x}_p d\mathbf{z} \\ MSE = \left(\left(E(y)-t\right)^2 + D(y)\right) \end{cases} \qquad (13)$$

Step 5: minimize $D(y)$ or $MSE(y)$ by using the concurrent gradient descent algorithm or genetic algorithm to get the optimal levels of controllable factors and therefore reach the RPD of the process.

4. Case Study

In order to demonstrate the effectiveness of the approach we proposed above, a case study of the optimization of inductor-resistor series circuit is conducted in this section. Figure 1 shows the circuit diagram, which includes an inductor *L*, a resistor *R* and AC power with voltage *V* and frequency *f*.
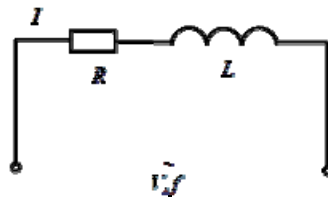

Figure 1. Inductor-resistor series circuit

The relationship between the circuit output *I* and *L, R, V, f* follows

$$I = \frac{V}{\sqrt{R^2 + (2\pi f L)^2}}$$

(14)

According to the prior knowledge, the controllable factors are x_1 (*L*)and x_2 (*R*), and the noise factors are z_1(*V*) and z_2(*f*), with the ranges $x_1 \in [2\text{mH}, 6\text{mH}], x_2 \in [9\Omega, 11\Omega]$, $z_1 \in [95\text{V}, 105\text{V}], z_2 \in [45\text{Hz}, 55\text{Hz}]$, All the factors are standardized into [-1,1] before optimization.

The goal of the circuit optimization is to reach the mean value of *I* by 10A, and meanwhile, minimize the fluctuation of *I* caused by the variations of x_1, x_2, and z_1 and z_2.

Firstly, we run 100 times uniform design experiments to get the sample set of *S*, and then fit the SVR model according to step 3 in section 3.2:

$$y = f(x, z) + \varepsilon = \sum_{i=1}^{100} (\alpha_i^* - \alpha_i) k(x_i, x) + \bar{b}$$

(15)

where *y* is the output characteristic denotes as *I*.

Secondly, we assume that the distribution of variation of x_1, x_2,which are denoted by x_{p1} and x_{p2}, follow the normal distribution as

$$\begin{cases} g_i(xp_i) = \dfrac{1}{\sigma_{pi} \times \sqrt{2\pi}} \exp[-\dfrac{(xp_i - \mu_{pi})^2}{2 \times \sigma_{pi}^2}] \\ \sigma_{pi} = 0.01 \\ \mu_{pi} = x_i \\ i = 1, 2 \end{cases}$$

(16)

and the distribution of z_1, z_2 follow the normal distribution as

$$\begin{cases} f(z_i) = \dfrac{1}{\sigma_i \sqrt{2\pi}} \exp[-\dfrac{(z_i - \mu_i)^2}{2\sigma_i^2}] \\ \sigma_i = 0.1 \\ u_i = 0 \\ i = 1, 2 \end{cases}$$

(17)

Then we get *E(y)*, *D(y)* and *MSE* as follows

$$\begin{cases} E(y) = \iint \iint y g_1(x_{p1}; x_1) g_2(x_{p2}; x_2) f_1(z_1; 0) f_2(z_2; 0) dx_{p1} dx_{p2} dz_1 dz_2 \\ D(y) = \iint \iint [y - E(y)]^2 g_1(x_{p1}; x_1) g_2(x_{p2}; x_2) f_1(z_1; 0) f_2(z_2; 0) dx_{p1} dx_{p2} dz_1 dz_2 \\ MSE = \left((E(y) - 10)^2 + D(y) \right) \end{cases}$$

(18)

Thirdly, after integration, we get the mesh plot of the relationship of *E(y)*, *D(y)* and *MSE*, shown in Figure 2.
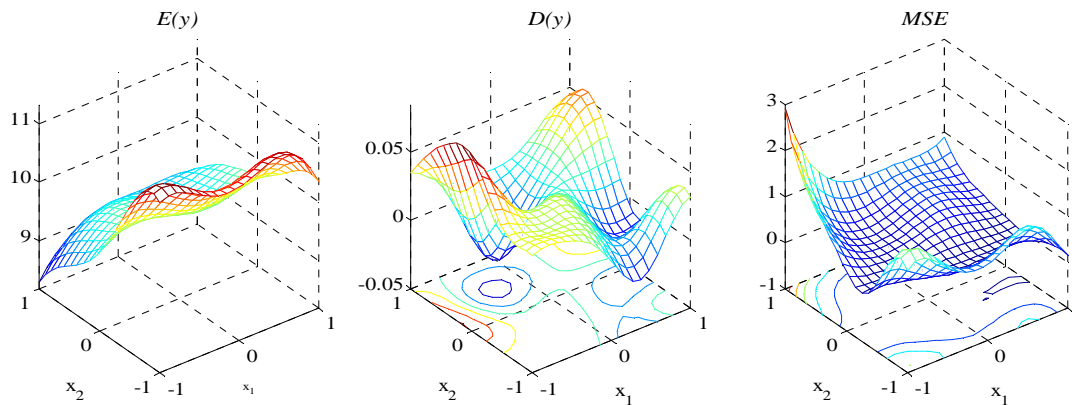
Figure 2. Surface of $E(y)$, $D(y)$ and MSE

As can be seen from Figure 2, within the range of controllable factors, the varieties of $E(y)$, $D(y)$ and MSE are typically complex, especially for MSE, who has several local minimum. If we optimize the circuit by using DRSM, it will fall into a certain local optimum inevitably. While by using the proposed approach, all the local optimum can be reached through parallel gradient search (as shown in Table 1), hence we can get the global optimum consequentially.

Table 1. Minimum of MSE with corresponding $E(y)$, $D(y)$, x_1, and x_2

x_1	x_2	$E(y)$	$D(y)$	MSE
0.7895	-0.2632	9.9429	0.000134	0.028916
0.8947	-0.3684	10.0054	0.000239	0.027638
0.6842	-0.1579	9.8688	0.003216	0.035053
0.1579	-0.2632	10.1868	0.002748	0.037414

Table 1 shows that MSE has several local optimums, the global optimum of which is 0.027638, where the controllable factors x_1=0.8974 and x_2=-0.3684. After mapping the controllable factors into their initial ranges, we get the optimal values of the inductor-resistor series circuit: L=5.7894mH, R=9.6316Ω, and the corresponding I =10.0054A .

5. Conclusion

This paper proposes a SVR based RPD approach for the robust parameter design of circuit. The approach overcomes the shortcoming of ignoring interactions between factors in parameter sweep and sensitivity analysis methods, avoids the limitation of failure to achieve continuous optimization by using Taguchi method, and prevents to obtain local optimization by using DRSM. The paper describes detailed application steps of the approach, and takes into account the influence of input variations on responses, which make the experiments more realistic and the optimization steps more reasonable. The results of the case study also demonstrate the applicability and the effectiveness of the approach in circuit optimization.

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