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Similar Constructive Method for Solving a nonlinearly Spherical Percolation Model

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Abstract

In the view of nonlinear spherical percolation problem of dual porosity reservoir, a mathematical model considering three types of outer boundary conditions: closed, constant pressure, infinity was established in this paper. The mathematical model was linearized by substitution of variable and became a boundary value problem of ordinary differential equation in Laplace space by Laplace transformation. It was verified that such boundary value problem with one type of outer boundary had a similar structure of solution. And a new method: Similar Constructive Method was obtained for solving such boundary value problem. By this method, solutions with similar structure in other two outer boundary conditions were obtained. The Similar Constructive Method raises efficiency of solving such percolation model.

Keywords: *dual porosity reservoir, quadratic pressure gradient, similar constructive method, similar kernel function, spherical flow, effective wellbore radius*

1. Introduction

In recent years, there has been increasing reporting about dual porosity reservoir. In literatures [1-4], the linearly radial percolation model of dual porosity reservoir was studied. The percolation model was established and solved on basis of ignoring the quadratic pressure gradient. a two-phase porous structure for cancellous bone with transverse isotropism was studied in literature [5]. In the literature [6], for the dual porosity reservoir, a well test analysis model which considered the effect of wellbore storage and skin effect and had a suction well with single rate production in the center of closed circle stratum. And the model was solved by implicit difference scheme of block centered grid. However, percolation models with the quadratic pressure gradient actually can obtain more accurate reservoir performance parameters as before [7]. In literatures [8-10], the research on nonlinear percolation models with quadratic pressure gradient makes these percolation models become more coincident to reality. A nonlinear systems for optimal zero-state-error control was studied in literature [11]. In the literature [12], the research on the nonlinear spherical percolation model of homogeneous reservoir proffers a much development for the nonlinear percolation theory. For the nonlinear percolation model, considering the skin effect by introducing the skin factor into the interior boundary condition results in much difficulty for calculation. In the literature [13], this difficulty was greatly decreased by leading in the effective wellbore radius.

Based on above researches, a nonlinear spherical percolation of dual porosity reservoir which considers three different outer boundary conditions closed, constant pressure, infinity and skin effect by leading in effective wellbore radius will be established. Secondly, a boundary value problem (BVP) of ordinary differential equation in Laplace space will be obtained by substitution of variable and Laplace transformation. Then it will be verified that the solution of such BVP has a similar structure [14~17]. And a new method: Similar Constructive Method will be put forward for solving such type of percolation model as a result. Finally, the nonlinear spherical percolation model will be solved by this method. And the solutions of dimensionless reservoir pressure and dimensionless bottomhole pressure for the percolation model in Laplace

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space will be obtained. The Similar Constructive Method simplifies the solving procedure of such percolation model greatly. The similar structure of solution of the percolation proffers great benefit for editing well test analysis software [18].

2. The Mathematical Nonlinear Spherical Percolation Model

Making basic assumptions as follows:
(1) There exists dual porosity reservoir which is isopachous and isotropic.
(2) There exists interporosity flow channeling from matrix rock system to fracture system.
(3) Fluid is single-phase, laminar flow, slight compressibility and follows Darcy's law.
(4) Ignore the gravity influence;

Leading in effective wellbore radius: $r_{we} = r_w e^{-S}$, a mathematical nonlinear spherical percolation model of dual porosity reservoir is established as follows:

The fundamental percolation system of equations:

$$\begin{cases} \dfrac{\partial^2 p_{D_1}}{\partial {r_D}^2} + \dfrac{2}{r_D}\dfrac{\partial p_{D_1}}{\partial r_D} - C_{LD}\left(\dfrac{\partial p_{D_1}}{\partial r_D}\right)^2 = \dfrac{1}{e^{2S}}\left[\omega \dfrac{\partial p_{D_1}}{\partial t_D} + (1-\omega)\dfrac{\partial p_{D_2}}{\partial t_D}\right] \\ (1-\omega)\dfrac{\partial p_{D_2}}{\partial t_D} + \lambda\left(p_{D_2} - p_{D_1}\right) = 0 \end{cases} \quad (1)$$

where the subfix1 refers to fracture medium and the subfix2 refers to matrix block medium.

The initial condition

$$p_{D_i}\Big|_{t_D=0} = 0 \quad (2)$$

The inner boundary condition

$$p_{wD_1} = \left[p_{D_1}\right]\Big|_{r_D=1}, \left[{r_D}^2 \dfrac{\partial p_{D_1}}{\partial r_D}\right]\Bigg|_{r_D=1} = -1 \quad (3)$$

Outer boundary conditions:
When the outer boundary condition is closed:

$$\dfrac{\partial p_{D_i}}{\partial r_D}\Bigg|_{r_D=R_D} = 0 \ . \quad (4)$$

When the outer boundary condition is constant pressure:

$$p_{D_i}\Big|_{r_D=R_D} = 0 \ ; \quad (5)$$

When the outer boundary condition is infinity:

$$p_{D_i}\Big|_{r_D\to\infty} = 0 \ ; \quad (6)$$

Introduced dimensionless variables are defined as follows

$$p_{D_i} = \frac{4\pi k_1 r_{we}\left(p_0 - p_i\right)}{q\mu} \quad , \quad r_D = \frac{r}{r_{we}} = \frac{r}{r_w e^{-S}} \quad , \quad \omega = \frac{C_{t_1}\phi_1}{C_{t_1}\phi_1 + C_{t_2}\phi_2} \quad , \quad C_{LD} = \frac{q\mu}{4\pi k_1 r_w} C_L ,$$

$$t_D = \frac{k_1}{\mu r_w^2\left(\varphi_1 C_{t_1} + \varphi_2 C_{t_2}\right)} t \ , \ \lambda = \alpha \frac{k_2}{k_1} r_w^2 \ , \ R_D = \frac{R}{r_w e^{-S}} .$$

Symbol description:

p — Reservoir pressure (MPa); p_0 — Initial pressure(MPa); r — The distance from any point in the reservoir to the center of well (m); R — The outer boundary radius(m); S —Skin factor, dimensionless; k — Reservoir permeability($\mu\mathrm{m}^2$); C_t — Total compressibility of reservoir, ($1/\mathrm{MPa}$); C_L — Fluid compressibility, ($1/\mathrm{MPa}$); ϕ — Effective porosity, dimensionless; μ — Viscosity factor ($\mathrm{mPa\cdot s}$); ω —Elastic storage ratio, dimensionless; α — Shape dependent constant, ($1/\mathrm{m}^2$); λ — Interporosity flow coefficient, dimensionless; r_w — Shaft radius(m); r_{we} — Effective wellbore radius,(m); q — Production of oil well (m^3/d); t —Time, (h). Subscript: $i=1$ —Fracture system; $i=2$ —Matrix rock system; w—Well; D—Dimensionless.

3. Linearization of the Fixed Solution Problem

In order to linearize the fixed solution problem of Eq.(1)~Eq.(6), namely eliminating the term of $C_{LD}\left(\frac{\partial p_{D_1}}{\partial r_D}\right)^2$, an variable substitution is taken as follows

$$p_{iD}\left(r_D, T_D\right) = -\frac{1}{C_{LD}}\ln\left[1 + Y_i\left(r_D, t_D\right)\right], \left(i = 1,2\right); \ p_w\left(T_D\right) = -\frac{1}{C_{LD}}\ln\left[1 + Y_w\left(t_D\right)\right] . \quad (7)$$

The fixed solution problem of Eq. (1)~Eq. (6) is transformed into

$$\begin{cases} \frac{\partial^2 Y_1}{\partial {r_D}^2} + \frac{2}{r_D}\frac{\partial Y_1}{\partial r_D} = \frac{1}{e^{2S}}\left[\omega\frac{\partial Y_1}{\partial t_D} + \left(1-\omega\right)\frac{\partial Y_2}{\partial t_D}\right] \\ \left(1-\omega\right)\frac{\partial Y_2}{\partial t_D} + \lambda\left(Y_2 - Y_1\right) = 0 \\ Y_i\big|_{t_D=0} = 0 \quad \left(i=1,2\right) \\ Y_w = \left[Y_1\left(r_D, T_D\right)\right]\big|_{r_D=1} \\ \left.\frac{\partial Y_1}{\partial r_D}\right|_{r_D=1} = C_{LD}\left(1+Y_w\right) + \frac{dY_w}{dt_D} \\ \left.\frac{\partial Y_1}{\partial r_D}\right|_{r_D=R_D} = 0 \ \ or \ \ Y_1\big|_{r_D=R_D} = 0 \ \ or \ \ Y_1\big|_{r_D\to\infty} = 0 \end{cases} \quad (8)$$

Taking another variable substitution as the following

$$u_i\left(r_D,t_D\right)=r_D Y_i\left(r_D,t_D\right),\quad (i=1,2) \tag{9}$$

a linearly fixed solution problem is obtained as the following:

$$\begin{cases} \dfrac{\partial^2 u_1}{\partial r_D{}^2}=\dfrac{1}{e^{2S}}\left[\omega\dfrac{\partial u_1}{\partial t_D}+(1-\omega)\dfrac{\partial u_2}{\partial t_D}\right] \\ (1-\omega)\dfrac{\partial u_2}{\partial t_D}+\lambda\left(u_2-u_1\right)=0 \\ u_i\big|_{t_D=0}=0 \quad (i=1,2) \\ u_w=\left[u_1\left(r_D,t_D\right)\right]\big|_{r_D=1} \\ \left.\dfrac{\partial u_1}{\partial r_D}\right|_{r_D=1}=C_{LD}\left(1+u_w\right)+\dfrac{du_w}{dt_D} \\ \left.\left(u_1-R_D\dfrac{\partial u_1}{\partial r_D}\right)\right|_{r_D=R_D}=0 \;\; or \;\; u_1\big|_{r_D=R_D}=0 \;\; or \;\; u_1\big|_{r_D\to\infty}=0 \end{cases} \tag{10}$$

4. The BVP in Laplace Space

Taking a Laplace transformation of time t_D for the fixed solution problem (10):

$$\overline{u_i}\left(r_D,z\right)=\int_0^\infty e^{-zt_D}u_i\left(r_D,t_D\right)dt_D\,,\quad \overline{u_{w_1}}\left(r_D,z\right)=\int_0^\infty e^{-zt_D}u_{w_1}\left(t_D\right)dt_D \tag{11}$$

yields a BVP of zero order modified Bessel equation:

$$\begin{cases} \dfrac{d^2\overline{u}_1}{dr_D{}^2}=\dfrac{f(z)}{C_D e^{2S}}\overline{u}_1 \\ \overline{u}_2=\dfrac{\lambda}{\dfrac{1-\omega}{C_D}z+\lambda}\overline{u}_1 \\ \left.\left[\left(C_{LD}+z\right)\overline{u}_1-\dfrac{d\overline{u}_1}{dr_D}\right]\right|_{r_D=1}=-\dfrac{C_{LD}}{z} \\ \left.\left(\overline{u}_1-R_D\dfrac{d\overline{u}_1}{dr_D}\right)\right|_{r_D=R_D}=0 \; or \; \overline{u}_1\big|_{r_D=R_D}=0 \; or \; \overline{u}_1\big|_{r_D\to\infty}=0 \end{cases} \tag{12}$$

where $f(z)=\dfrac{1+\omega\alpha}{1+\alpha}z$, $\alpha=\dfrac{1-\omega}{\lambda}z$ and z is Laplace variable.

5. The Similar Constructive Principle

In the followings, we firstly solve the boundary value problem with closed outer boundary condition in Laplace space. Then we obtain the similar constructive method of solution for solving such type of boundary value problem and put forward the general steps for this new method.

As the theory about the uniqueness of the solution of the B.V.P. of ordinary differential equation [19], the BVP (12) has a unique solution. When the outer boundary condition is closed, the solution has the following unified form:

$$\bar{u}_1\left(r_D,z\right)=-\frac{C_{LD}}{z}\cdot\frac{1}{C_{LD}+z-\dfrac{1}{\Phi_1\left(1,z\right)}}\cdot\frac{\Phi_1\left(r_D,z\right)}{\Phi_1\left(1,z\right)} \tag{13}$$

where $\Phi_1\left(r_D,z\right)$ is defined as the Similar Kernel Function:

$$\begin{aligned}\Phi_1(r_D,z)&=\frac{\varphi_{0,0}(r_D,R_D)-R_D\varphi_{0,1}(r_D,R_D)}{\varphi_{1,0}(1,R_D)-R_D\varphi_{1,1}(1,R_D)}\\&=\frac{\sinh\left(\sqrt{\dfrac{f(z)}{e^{2S}}}\left(r_D-R_D\right)\right)+R_D\sqrt{\dfrac{f(z)}{e^{2S}}}\cosh\left(\sqrt{\dfrac{f(z)}{e^{2S}}}\left(r_D-R_D\right)\right)}{\sqrt{\dfrac{f(z)}{e^{2S}}}\left[\cosh\left(\sqrt{\dfrac{f(z)}{e^{2S}}}\left(1-R_D\right)\right)+R_D\sqrt{\dfrac{f(z)}{e^{2S}}}\sinh\left(\sqrt{\dfrac{f(z)}{e^{2S}}}\left(1-R_D\right)\right)\right]}\end{aligned} \tag{14}$$

and $\varphi_{i,j}(r_D,l)$ in the expression is defined as guide functions:

$$\varphi_{0,0}(r_D,R_D)=\sinh\left[\sqrt{\frac{f(z)}{e^{2S}}}\left(r_D-R_D\right)\right]$$

$$\varphi_{0,1}(r_D,R_D)=-\sqrt{\frac{f(z)}{e^{2S}}}\cosh\left[\sqrt{\frac{f(z)}{e^{2S}}}\left(r_D-R_D\right)\right]$$

$$\varphi_{1,0}(1,R_D)=\sqrt{\frac{f(z)}{e^{2S}}}\cosh\left[\sqrt{\frac{f(z)}{e^{2S}}}\left(1-R_D\right)\right]$$

$$\varphi_{1,1}(1,R_D)=-\frac{f(z)}{e^{2S}}\sinh\left[\sqrt{\frac{f(z)}{e^{2S}}}\left(1-R_D\right)\right]$$

In fact, the solution for the first fixed equation of the BVP (12) is

$$\bar{u}_1(r_D,z)=d_1e^{\sqrt{\frac{f(z)}{e^{2S}}}r_D}+d_2e^{-\sqrt{\frac{f(z)}{e^{2S}}}r_D} \quad (d_1, d_2 \text{ are arbitrary constants}) \tag{15}$$

where $\bar{u}_{11}\left(r_D,z\right)=e^{\sqrt{\frac{f(z)}{e^{e^{2S}}}}r_D}$, $\bar{u}_{12}\left(r_D,z\right)=e^{-\sqrt{\frac{f(z)}{e^{e^{2S}}}}r_D}$ are two linearly independent solutions. By these two solutions, we make guide functions as follows:

$$\varphi_{0,0}(r_D,l)=\begin{vmatrix} e^{\sqrt{\frac{f(z)}{e^{2S}}}r_D} & e^{-\sqrt{\frac{f(z)}{C_D}}r_D} \\ e^{\sqrt{\frac{f(z)}{e^{2S}}}l} & e^{-\sqrt{\frac{f(z)}{e^{2S}}}l} \end{vmatrix}=2\sinh\left[\sqrt{\frac{f(z)}{e^{2S}}}(r_D-l)\right] \tag{16}$$

$$\varphi_{0,1}(r_D,l)=\frac{\partial}{\partial l}\varphi_{0,0}(r_D,l)=-2\sqrt{\frac{f(z)}{e^{2S}}}\cosh\left[\sqrt{\frac{f(z)}{e^{2S}}}(r_D-l)\right] \tag{17}$$

$$\varphi_{1,0}(r_D,l)=\frac{\partial}{\partial r_D}\varphi_{0,0}(r_D,l)=2\sqrt{\frac{f(z)}{e^{2S}}}\cosh\left[\sqrt{\frac{f(z)}{e^{2S}}}(r_D-l)\right] \tag{18}$$

$$\varphi_{1,1}(r_D,l)=\frac{\partial}{\partial r_D}\varphi_{0,1}(r_D,l)=\frac{\partial}{\partial l}\varphi_{1,0}(r_D,l)=-2\frac{f(z)}{e^{2S}}\sinh\left[\sqrt{\frac{f(z)}{e^{2S}}}(r_D-l)\right] \tag{19}$$

Substituting Eq.(15) into the interior boundary condition of the BVP (12), yields:

$$\left[(C_{LD}+z)-\sqrt{\frac{f(z)}{e^{2S}}}\right]e^{\sqrt{\frac{f(z)}{e^{2S}}}}d_1+\left[(C_{LD}+z)+\sqrt{\frac{f(z)}{e^{2S}}}\right]e^{-\sqrt{\frac{f(z)}{e^{2S}}}}d_2=-\frac{C_{LD}}{z} \tag{20}$$

Then substituting Eq.(15) into the outer boundary condition of the BVP (12), yields:

$$\left(1-R_D\sqrt{\frac{f(z)}{e^{2S}}}\right)e^{\sqrt{\frac{f(z)}{e^{2S}}}R_D}d_1+\left(1+R_D\sqrt{\frac{f(z)}{e^{2S}}}\right)e^{-\sqrt{\frac{f(z)}{e^{2S}}}R_D}d_2=0 \tag{21}$$

Combinating Eq.(16)~Eq.(19), the determinant of coefficients of Eq.(20) and Eq.(21) is obtained as the following:

$$\Delta=\begin{vmatrix} \left[-(C_{LD}+z)+\sqrt{\frac{f(z)}{e^{2S}}}\right]e^{\sqrt{\frac{f(z)}{e^{2S}}}} & \left[-(C_{LD}+z)-\sqrt{\frac{f(z)}{e^{2S}}}\right]e^{-\sqrt{\frac{f(z)}{e^{2S}}}} \\ \left(-1+R_D\sqrt{\frac{f(z)}{e^{2S}}}\right)e^{\sqrt{\frac{f(z)}{e^{2S}}}R_D} & \left(-1-R_D\sqrt{\frac{f(z)}{e^{2S}}}\right)e^{-\sqrt{\frac{f(z)}{e^{2S}}}R_D} \end{vmatrix}$$
$$=(C_{LD}+z)\varphi_{0,0}(\alpha,\beta)-(C_{LD}+z)R_D\varphi_{0,1}(\alpha,\beta)+\varphi_{1,0}(\alpha,\beta)-R_D\varphi_{1,1}(\alpha,\beta) \tag{22}$$

Because the BVP (12) has an unique solution, so $\Delta\neq 0$. As the Crammer rule, values of d_1 and d_2 are obtained as follows:

$$d_1=\frac{-C_{LD}/z}{\Delta}\left(1+R_D\sqrt{\frac{f(z)}{e^{2S}}}\right)e^{-\sqrt{\frac{f(z)}{e^{2S}}}R_D} \tag{23}$$

$$d_2 = \frac{C_{LD}/z}{\Delta}\left(1 - R_D\sqrt{\frac{f(z)}{e^{2S}}}\right)e^{\sqrt{\frac{f(z)}{e^{2S}}}R_D} \tag{24}$$

Substituting Eq. (22) ~ Eq. (24) into Eq. (15) and combinating Eq. (16) ~Eq. (19), we obtain

$$\bar{u}_1(r_D, z) = -\frac{C_{LD}}{z}\cdot\frac{\varphi_{0,0}(r_D, R_D) - R_D\varphi_{0,1}(r_D, R_D)}{\Delta}. \tag{25}$$

Let Eq. (14) be the Similar Kernel Function of the solution for the BVP (12) with the closed outer boundary condition. Then Eq. (25) can be translated into the similar structure of solution that is Eq.(13) for the BVP (12) with the closed outer boundary condition.

As the obtained Similar Kernel Function of Eq.(14) and the solution's similar structure of Eq.(13), we can analyze the fixed connection among the solution, the basic equation and boundary conditions as follows: The solution of such type of BVP has a similar structure which can be expressed as a continued fraction product form. The similar structure is determinated by coefficients of non-homogeneous interior boundary condition. In addition, the Similar Kernel Function in the similar structure is determinated by guide functions and coefficients of homogeneous outer boundary condition. And guide functions are made by two linearly independent solutions of the basic equation of the BVP. We define this method as the Similar Constructive Method.

Then we can generalize the procedure of the Similar Constructive Method as follows:

Step 1: Construct the guide functions by two linearly independent solutions of the basic equation from the BVP (12), expressed as Eq. (16) ~Eq. (19).

Step 2: Construct the Similar Kernel Function by the coefficients of homogeneous outer boundary condition $1, -R_D$ and guide functions $\varphi_{i,i}(r_D, l), (i, j = 0,1)$, expressed as Eq. (14).

Step 3: Construct the solution of the BVP (12) by the Similar Kernel Function $\Phi_1(x)$ and the coefficients of the non-homogeneous interior boundary condition which are $C_{LD} + z$, 0 and $-\frac{C_{LD}}{z}$, expressed as Eq. (13).

6. Solve the BVP by the Similar Constructive Method

Next, we solve the BVP when its outer boundary conditions are constant pressure and infinity by then Similar Constructive Method.

Step 1: By $\bar{u}_{11}(r_D, z)$ and $\bar{u}_{12}(r_D, z)$, guide functions are made as Eq. (16) ~Eq. (19).

Step 2: Construct the Similar Kernel Functions as the coefficients of homogeneous outer boundary condition and guide functions:

When the outer boundary condition is constant pressure, the Similar Kernel Function is constructed by the coefficients of homogeneous outer boundary condition as

$$\Phi_2(r_D, z) = \frac{\varphi_{0,0}(r_D, R_D)}{\varphi_{1,0}(1, R_D)} = \frac{1}{\sqrt{\frac{f(z)}{e^{2S}}}}\cdot\frac{\sinh\left[\sqrt{\frac{f(z)}{e^{2S}}}(r_D - R_D)\right]}{\cosh\left[\sqrt{\frac{f(z)}{e^{2S}}}(1 - R_D)\right]} \tag{26}$$

Similarly, when the outer boundary condition is infinite, the Similar Kernel Function is constructed as

$$\Phi_3(r_D,z) = \lim_{R_D\to\infty}\frac{\varphi_{0,0}(r_D,R_D)}{\varphi_{1,0}(1,R_D)} = \frac{-1}{\sqrt{\frac{f(z)}{e^{2S}}}}e^{-\sqrt{\frac{f(z)}{e^{2S}}}(r_D-1)} \tag{27}$$

Step 3: Construct the solution of the BVP (12) by the coefficients of the non-homogeneous interior boundary condition and the Similar Kernel Function.

When the outer boundary condition is constant pressure, the solution of the BVP (12) is

$$\bar{u}_1(r_D,z) = -\frac{C_{LD}}{z}\cdot\frac{1}{C_{LD}+z-\frac{1}{\Phi_2(1,z)}}\cdot\frac{\Phi_2(r_D,z)}{\Phi_2(1,z)} \tag{28}$$

And when the outer boundary condition is infinity, the solution of the BVP (12) is

$$\bar{u}_1(r_D,z) = -\frac{C_{LD}}{z}\cdot\frac{1}{C_{LD}+z-\frac{1}{\Phi_3(1,z)}}\cdot\frac{\Phi_3(r_D,z)}{\Phi_3(1,z)} \tag{29}$$

Therefore, defining a Similar Kernel Function as follows:

$$\Phi(r_D,z) = \begin{cases}\Phi_1(r_D,z), & \text{Closed outer boundary condition;}\\ \Phi_2(r_D,z), & \text{Constant pressure outer boundary condition;}\\ \Phi_3(r_D,z), & \text{Infinite outer boundary condition;}\end{cases} \tag{30}$$

the solution that has a unified form of the BVP (12) is obtained as follows

$$\bar{u}_1(r_D,z) = -\frac{C_{LD}}{z}\cdot\frac{1}{C_{LD}+z-\frac{1}{\Phi(1,z)}}\cdot\frac{\Phi(r_D,z)}{\Phi(1,z)} \tag{31}$$

Substituting Eq. (31) into the second basic equation of the BVP (12), yields:

$$\bar{u}_2(r_D,z) = -\frac{C_{LD}}{z}\cdot\frac{\lambda}{\frac{1-\omega}{e^{2S}}z+\lambda}\cdot\frac{1}{C_{LD}+z-\frac{1}{\Phi(1,z)}}\cdot\frac{\Phi(r_D,z)}{\Phi(1,z)} \tag{32}$$

Making Laplace numerical inversion of Eq.(31) and Eq.(32) while using Eq.(7), Eq.(9) and Eq.(11), the dimensionless reservoir flowing pressure and the bottom bole flowing pressure are obtained as follows:

the dimensionless fracture pressure:

$$p_{D_1}(r_D,t_D) = -\frac{1}{C_{LD}}\ln\left[1+\frac{u_1(r_D,t_D)}{r_D}\right] \tag{33}$$

the dimensionless matrix block pressure:

$$p_{D_2}\left(r_D,t_D\right)=-\frac{1}{C_{LD}}\ln\left[1+\frac{\lambda}{\left(1-\omega\right)z+\lambda}\cdot\frac{u_1\left(r_D,t_D\right)}{r_D}\right] \quad (34)$$

the dimensionless bottom hole pressure:

$$p_{wD}\left(r_D,t_D\right)=-\frac{1}{C_{LD}}\ln\left[1+u_{w1}\left(t_D\right)\right] \quad (35)$$

7. Conclusion

In this paper, the nonlinearly spherical percolation model of dual porosity reservoir that considers the effective wellbore radius and three types of outer boundary conditions: closed, constant pressure, infinity has been studied. The BVP (12) in Laplace space is solved by a new method: Similar Constructive Method. It is not difficult to find out that solutions of the BVP (12) with three types of outer boundary conditions have a similar structure. The Similar Constructive Method provides a new way that is both convenient and efficient to solve the nonlinear percolation problem of dual porosity reservoir.

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