# Modified Allan Variance Analysis on Random Errors of MINS

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***Abstract***

*Allan variance method is a useful tool for analyzing the random errors, but the confidence on the estimate would be lower when the data length became shorter, therefore the modified Allan variance is deduced to analysis the random errors of MEMS inertial sensors (MINS). The definition and limitation of Allan variance are presented first, and then the modified Allan variance is deduced. Allan variance method is compared with modified Allan variance by identifying the simulated 1/f noises, meanwhile the results are illuminated. In the end, the random errors of MEMS inertial sensors were analyzed by the proposed methods. The characteristics of MEMS accelerometers' and MEMS gyros' stochastic errors are identified and quantified. The derived error model can be applied further to our attitude and heading reference system of the underwater robot.*

*Keywords: Random errors, Modified Allan variance, 1/f noises, MEMS inertial sensors*

## 1. Introduction

Inertial sensors measure the vehicle's acceleration and angular rate, which are then integrated to obtain the vehicle's position, velocity, and attitude[1]. Nowadays, the inertial sensors have been the indispensable sensors for navigation of aero planes, vehicles and robots, etc [2-4]. Especially the MEMS inertial sensors are small, low-cost and integrated. However, the sensors measurements are usually corrupted by different types of error sources, such as sensor noises, scale factor, and bias variations, etc. The errors can be divided into deterministic errors and random errors [5]. The deterministic errors can be reduced by the calibration [6-7], however the random errors have to be reduced by the filters or algorithms depend on the accurate knowledge of the sensors' noise model. Therefore, the analysis and modeling on random errors of MEMS inertial sensors is necessity to improve the accuracy.

The power spectrum density (PSD) is commonly used for the investigation of sensors' stochastic process. PSD is a frequency-domain approach for modeling noise. It is the Fourier transform pair of autocorrelation function, the frequency domain approach of using the PSD to estimate transfer functions is straightforward but difficult for non-system analysis [8]. Some time-domain methods have been devised like the correlation-function approach, which is the dual of the PSD approach and being related as the Fourier transform pair. Nerveless, correlation methods are very model-sensitive and not well suited when dealing with odd power-law processes, higher order processes, or wide dynamic range. The simplest is the Allan variance. The Allan variance method can be used to determine the characteristics of the underlying random processes that give rise to the data noise. The Allan variance is a method of representing the root mean square (RMS) random-drift errors as a function of averaging times. It is simple to compute and relatively simple to interpret and understand. Whereas, the confidence on the estimate would be lower when the data length became shorter [9], and the accuracy would be affected. Accordingly, the modified Allan variance is proposed in this paper to analyze the random errors of the MEMS inertial sensors.

The paper is organized as follows. Section 2 presents the mathematical definition of the Allan variance and the limitation is described, then the modified Allan variance is deduced. Section 3 describes the simulation of the methods, the graphical comparison is shown. Section 4 reports the results of characters of the random errors of MEMS inertial sensors. Conclusions are drawn in section 5.

## 2. Research Method

In order to illustrate the modified Allan variance method, the mathematical definition of common Allan variance is introduced first.

### 2.1. Allan Variance

For the Allan variance, the idea is that one or more white-noise sources of strength $N_i^2$ drive the canonical transfer functions, resulting in the same statistical and spectral properties as the actual device. In this paper, Allan's definition and results are related to five basic noise terms and are expressed in a notation appropriate for inertial sensor data [10-11]. The five basic noise terms are quantization noise, angle random walk, bias instability, rate random walk, and rate ramp.

Assume that there are $N$ consecutive data points, each having a sample time of $t_0$. Forming a group of $n$ consecutive data points (with $n < N/2$), each member of the group is a cluster. Associated with each cluster is a time $T$, which is equal to $nt_0$. If the instantaneous output rate of the inertial sensor is $y(t)$, the cluster average is defined as

$$\bar{y}_k(T) = \frac{1}{T} \int_{t_k}^{t_k+T} y(t)dt \tag{1}$$

Where $\bar{y}_k(T)$ represents the cluster average of the output rate for a cluster which starts from the $k$ th data point and contains the $n$ data points. The definition of the subsequent cluster average is

$$\bar{y}_{k+1}(T) = \frac{1}{T} \int_{t_{k+1}}^{t_{k+1}+T} y(t)dt \tag{2}$$

Where $t_{k+1} = t_k + T$.

Performing the average operation for each of the two adjacent clusters can form the difference

$$\xi_{k+1,k} = \bar{y}_{k+1}(T) - \bar{y}_k(T) \tag{3}$$

For each cluster time $T$, the ensemble of $\xi$ s defined by (3) forms a set of random variables. The quantity of interest is the variance of $\xi$ s over all the clusters of the same size that can be formed from the entire data.

Thus, the Allan variance of length T is defined as

$$\sigma^2(T) = \frac{1}{2(N-2n)} \sum_{k=1}^{N-2n} \left[ \bar{y}_{k+1}(T) - \bar{y}_k(T) \right]^2 \tag{4}$$

Meanwhile, there is a unique relationship that exists between $\sigma^2(T)$ and the PSD of the intrinsic random process. The relationship can be described as

$$\sigma^2(T) = 4 \int_0^\infty df \cdot S_\Omega(f) \cdot \frac{\sin^4(\pi f T)}{(\pi f T)^2} \tag{5}$$

Where $S_\Omega(f)$ is the PSD of the random process $\Omega(T)$.

Equation (5) states that the Allan variance is proportional to the total power output of the random process when passed through a filter with the transfer function of the form $\sin^4(x)/(x)^2$. This particular transfer function is the result of the method used to create and operate on the clusters. Equation (5) will be used to calculate the Allan variance from the rate-noise PSD. The PSD of any physically meaningful random process can be substituted in the integral, and an

expression for the Allan variance $\sigma^2(T)$ as a function of cluster length can be obtained. Therefore，different types of random processes can be confirmed. For indentifying and quantifying various random processes, the log-log plot is used for the square root of $\sigma(T)$ versus time cluster. The error types of inertial sensors mostly include Quantization error, Random walk, Bias instability, Rate random walk and Rate ramp. The relationship is listed in the table 1.

Table 1. the relationship of the coefficient of errors and Allan variance

| Coefficient of the error type | Allan variance | Slope |
|---|---|---|
| Quantization error $Q/(°)$ | $3Q^2/T^2$ | -1 |
| Random walk $N/(°)\mathrm{h}^{-1/2}$ | $N^2/T^2$ | -1/2 |
| Bias instability $B/(°)\,\mathrm{h}^{-1}$ | $(0.6643B)^2$ | 0 |
| Rate random walk $K/(°)\mathrm{h}^{-3/2}$ | $K^2T/3$ | +1/2 |
| Rate ramp $R/(°)\,\mathrm{h}^{-2}$ | $R^2T^2/2$ | +1 |

In practice, the estimation of the Allan variance is based on a finite number of independent clusters that can be formed from the length of data. Defining the parameter $\delta$ as the percentage error in estimating the Allan standard deviation of the cluster due to the finiteness of the number of clusters

$$\delta = \frac{\sigma(T,M) - \sigma(T)}{\sigma(T)} \tag{6}$$

Where $\sigma(T,M)$ denotes the estimate of the Allan standard deviation obtained from $M$ independent clusters; $\sigma(T,M)$ approaches its theoretical value $\sigma(T)$ in the limit of $M$ approaching infinity. The percentage error is equal to

$$\sigma(\delta) = \frac{1}{\sqrt{2\left(\dfrac{N}{n} - 1\right)}} \tag{7}$$

Where, $N$ is the total number of data points in the entire run, and $n$ is the number of data points contained in the cluster.

Equation (7) shows that the estimation errors in the cluster are large as the number of independent clusters in these regions is small. In order to improve the estimation accuracy, the modified Allan variance is presented in the following section.

### 2.1. Modified Allan variance

According to the equation (7), more samples adopted would improve the confidence of the resulting stability estimate. The term overlapping samples is defined. The calculation is performed by utilizing all possible combinations of the data set, as shown in the figure 1. The figure shows the different strides. For non-overlapped Allan variance the stride T is the averaging period and equals $m \cdot T_0$. In case of overlapped Allan variance the stride $T_0$ equals the sample period. It makes maximum use of a data set by forming all possible overlapping samples

at each averaging time $T$. It can be estimated from a set of $M$ frequency measurements for averaging time $T = m \cdot T_0$, where $m$ is the averaging factor and $T_0$ is the basic measurement interval. And the expression of the modified Allan variance can be deduced as follows:

$$\sigma^2(T) = \frac{1}{2m^4(K-3m+2)} \sum_{j=1}^{M-3m+2} \left\{ \sum_{i=j}^{j+m-1} \left( \sum_{k=i}^{i+m-1} [\bar{y}_{k+m}(T) - \bar{y}_k(T)] \right) \right\}^2 \qquad (8)$$

The modified Allan variance is the same as the normal Allan variance for m = 1. It includes an additional phase averaging operation due to the inner loop. In other words, one first averages the phase data before performing the Allan deviation calculation.
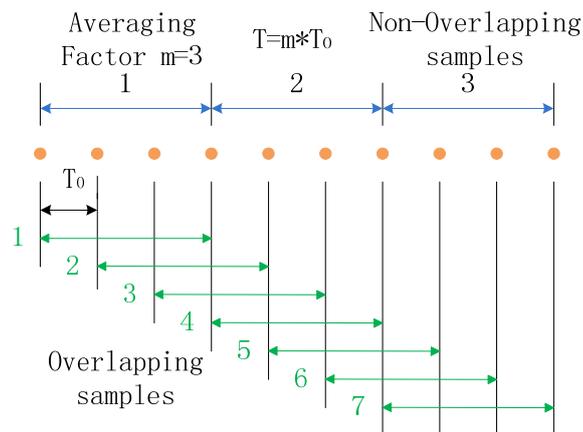


Figure 1. Comparison of non-overlapping and overlapping sampling

## 3. Results and Analysis
In this section, the simulations for comparing are discussed and then the tests are implemented. The results are illuminated in the end.

### 3.1. Simulation
In order to compare the Allan variance with the modified Allan variance, the simulation is implemented. The stand random signals are produced by the Matlab to compare. The FFT method is adopted to simulate the random noises. The random errors as quantization noise, angle random walk, bias instability, rate random walk, and rate ramp, are corresponded to five kinds of 1/f noises [12]. The table 2 lists the parameters of the simulated noises. The period is 1s and the total sample numbers is 10000. Then the graphs about the simulated noises and the comparisons of the two methods are shown in Figure 2- Figure 6.

Table 2. The parameters of the simulated noises

|  | Quantization error | Random walk | Rate random walk | Bias instability | Rate ramp |
|---|---|---|---|---|---|
| Std deviation | 0.3 | 1 | 0.2 | 0.1 | 0.1 |
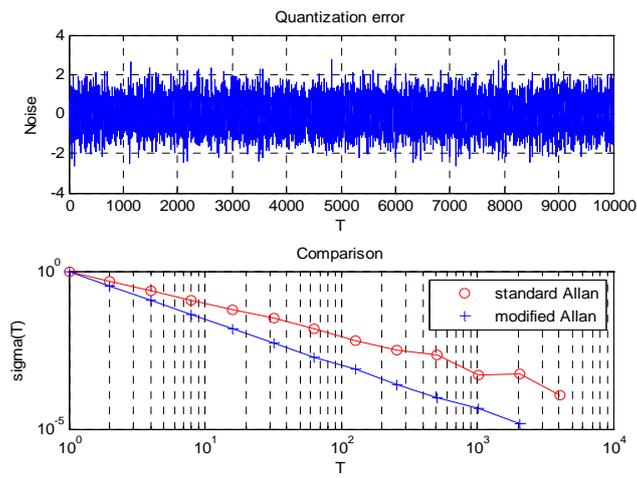
Figure 2. The simulation of quantization error
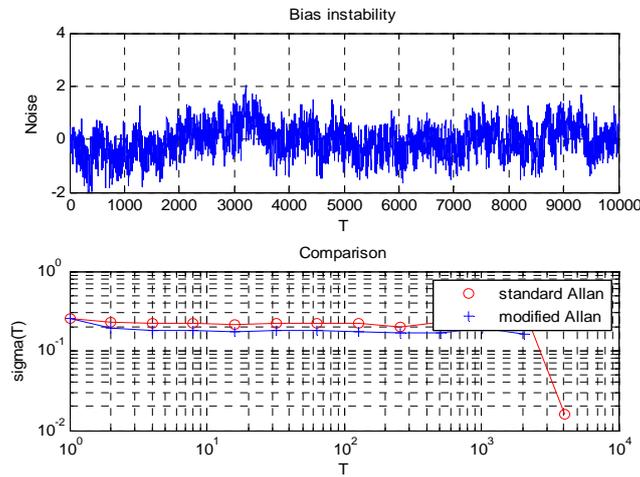


Figure 3. The simulation of bias instability
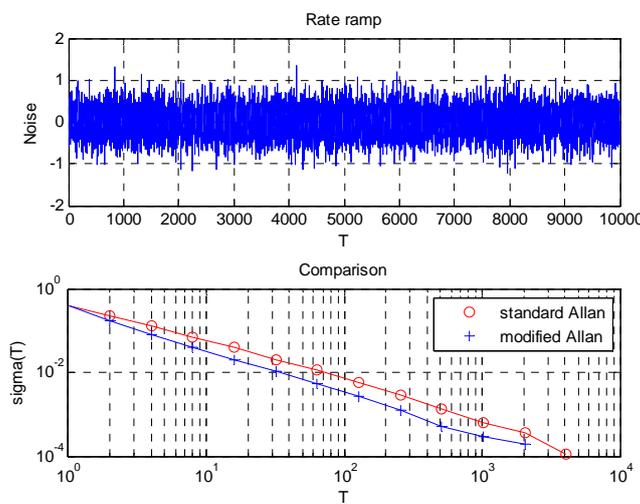


Figure 4. The simulation of rate ramp
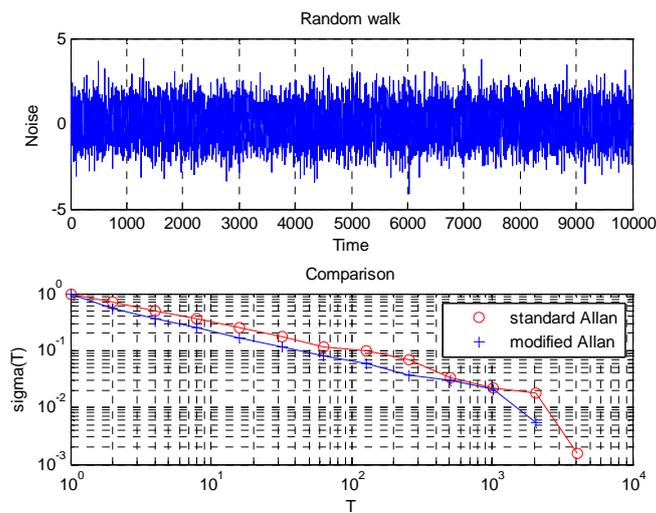
Figure 5. The simulation of random walk
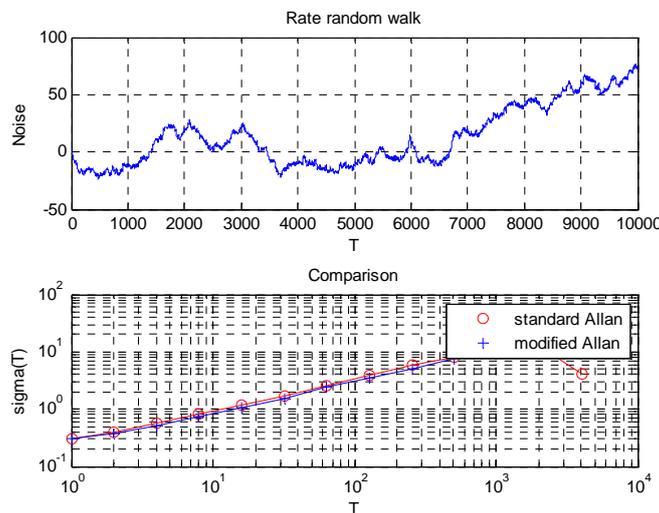


Figure 6. The simulation of rate random walk

The identified results are listed in the Table 3. According to the graphs and Table 3, it can be concluded that the modified Allan variance is super than the Allan variance in accuracy. Meanwhile, we also calculate the computation time of the two methods. The Allan variance takes the 0.09s but the modified Allan variance takes the 1.1s. The use of the modified Allan variance improves the accuracy, but at the expense of greater computational time.

Table 3. The results of two methods

| Comparisons | Quantization error | Random walk | Rate random walk | Bias instability | Rate ramp |
|---|---|---|---|---|---|
| Allan variance | 0.35 | 0.79 | 0.24 | 0.06 | 0.16 |
| Modified Allan variance | 0.27 | 0.95 | 0.19 | 0.11 | 0.09 |

### 3.2. Test Results

The testing platform is a prototype attitude and heading reference system, which will be used in the underwater robot of the nuclear power plants. The system is integrated by the MEMS inertial sensors and magnetometer. MEMS inertial sensors are composed of MEMS accelerometers and MEMS gyros. The performance is related to the random errors characters of the MEMS inertial sensors, therefore the experiment is designed to acquire the model. We put the system on stable plate of the Thermal unit to keep the sensors in the same temperature, and the data is collected. The experiment procedure is shown in the Figure 6.

Figure 6. The experiment for analyzing on random errors of inertial sensors

Then the collected date of the MEMS accelerometers and MEMS gyros are analyzed by the modified Allan variance. The log-log plots are shown in Figure 7 and Figure 8.
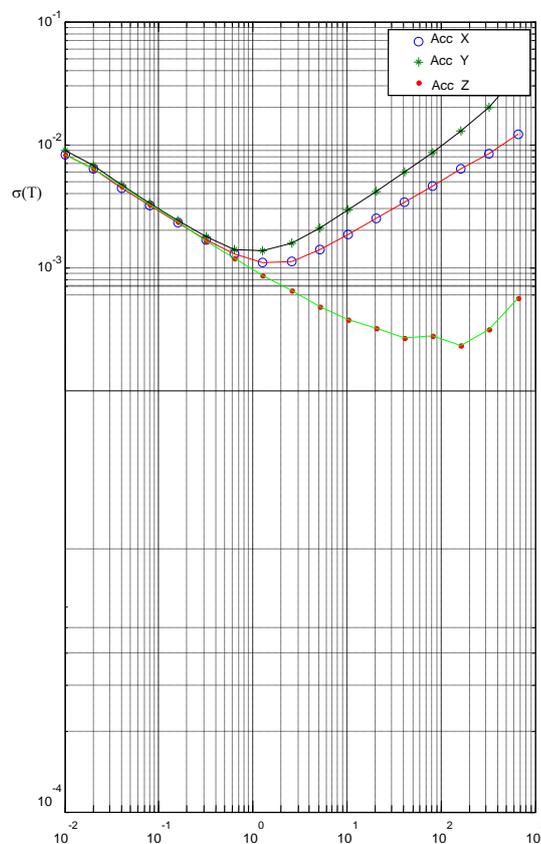
Figure 7. Allan deviation plot for 3 accelerometer axes
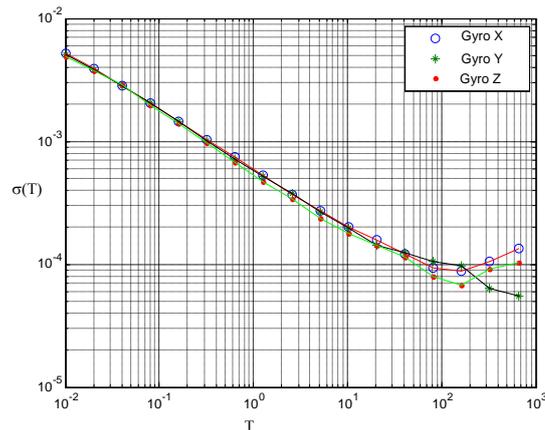
Figure 8. Allan deviation plot for 3 gyro axes

According to the Figure 7, the random errors of accelerometers mostly include rate ramp in the short time, and the bias instability comes up among the ten seconds to dozens of seconds in X and Y axis, the Z accelerometer has the bias instability about the one hundred seconds. Then the velocity random walk is the main noise in the larger time. Likewise, the mostly noise is random walk in the average time of 100 seconds, and the bias instability comes up after the 100 seconds. Then the error characters of MEMS accelerometers and MEMS gyros are summarized in the Table 4 and Table 5. In order to obtain a more reliable error coefficient, the multiple sampling is implemented under the same experimental conditions. We can obtain an approximate coefficient of the error term. So the results of the analysis are valid.

Table 4. Identified error coefficients for MEMS Accelerometers

| | X-Acc | Y-Acc | Z-Acc |
|---|---|---|---|
| Rate random walk (m/s) | $2.83 \times 10^{-3}$ | $3.03 \times 10^{-3}$ | $3.04 \times 10^{-3}$ |
| Bias instability ($m/s^2$) | $1.19 \times 10^{-3}$ | $1.48 \times 10^{-3}$ | $2.60 \times 10^{-4}$ |
| Velocity random walk ($m/s^3$) | $3.55 \times 10^{-3}$ | $4.27 \times 10^{-3}$ | $1.08 \times 10^{-4}$ |

Table 5. Identified error coefficients for MEMS gyros

| | X-Gyro | Y-Gyro | Z-Gyro |
|---|---|---|---|
| Random walk (°/s$^{1/2}$) | $1.87 \times 10^{-3}$ | $1.86 \times 10^{-3}$ | $1.88 \times 10^{-3}$ |
| Bias instability (°/s) | $9.07 \times 10^{-5}$ | $1.01 \times 10^{-4}$ | $7.33 \times 10^{-5}$ |

## 4. Conclusion

The Allan Variance is a simple and efficient method for identifying and characterizing different stochastic processes and their coefficients, but the confidence on the estimate would be lower when the data length became shorter. The modified Allan variance is proposed to improve the accuracy. Five basic noise terms, which are quantization noise, angle random walk, bias instability, rate random walk, and rate ramp, are simulated by the FFT method, and 1/f noises are identified by the Allan variance and modified Allan variance. The results of modified Allan variance are super but at the expense of greater computational time. Then the MEMS inertial sensors of attitude and heading reference system are tested and the data are analyzed by the modified Allan variance. From the testing results the conclusion can be drawn that the dominant noise type is rate random walk for MEMS accelerometers for the short time clusters, and for the medium time clusters the bias instability is the dominant error. For the long time clusters, the dominate error is velocity random walk. For MEMS gyros there are only two kinds of random error. The dominant error is random walk for the short time and the

dominant error is bias instability for the long time. When the main stochastic errors are identified and quantified, an error model can be derived and can be applied further to our attitude and heading reference system.

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