

# The Non-equidistant Multivariable New Information Optimization NMGRM(1,n) Based on New Information Background Value Constructing

Youxin Luo\*, Qiyuan Liu, Xiaoyi Che

College of Mechanical Engineering, Hunan University of Arts and Science, Changde, 415000, P.R.China

\*Corresponding author, e-mail: LLYX123@126.com

## Abstract

Applying the principle in which new information should be used fully and modeling method of Grey system for the problem of lower precision as well as lower adaptability in non-equidistant multivariable MGM(1,n) model, taking the mean relative error as objective function, and taking the modified values of response function initial value as design variables, based on accumulated generating operation of reciprocal number, a non-equidistant multivariable new information optimization MGRM(1,n) model was put forward which was taken the  $m$ th component as the initialization. Based on index characteristic of grey model, the characteristic of integral and new information principle, the new information background value in non-equidistant multivariable new information optimization MGRM(1,n) was researched and the discrete function with non-homogeneous exponential law was used to fit the accumulated sequence and the formula of new information background value was given. The new information optimization MGRM(1,n) model can be used in non-equal interval & equal interval time series and has the characteristic of high precision as well as high adaptability. Example validates the practicability and reliability of the proposed model.

**Keywords:** Multivariable non-equidistance sequence, new information background value constructing (NIBVC), non-equidistance NMGRM(1,n) model, accumulated generating operation of reciprocal number (RAGO), optimization

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## 1. Introduction

Grey model is the important part in grey theory. And there are many kinds of model such as GM(1,1), GM(1,N), MGM(1,N) [1-3], GOM(1,1) [4] and GRM(1,1) [5]. In the fields of society, economic and engineering multi-variety is the main characteristic and they have some relation with each other. MGM(1,N) model is the extending for the GM(1,1) model in the  $n$  dimensional variables which is not simply composed by GM(1,1) model and is not different from the one order differential equations in the GM(1,n) model. They are differential equations of  $n$  dimensional variable and the solutions are got by united solution, in which the parameter can reflect relations of the affection and constraint between every variables [6]. Literature [2] takes the first vector of sequence  $\mathbf{x}^{(1)}$  as the initial condition and establishes optimized MGM(1,N) model. Based on the principles of the prior new information literature [7] takes the  $n$ th vector as the initial conditions and established the multi-variables new information MGM(1,N) model. Literature [8] takes the  $n$ th vector of  $\mathbf{x}^{(1)}$  as the initial conditions and establish the multi-variables new information MGM(1,N) model, in which the initial values and backgrounds values are optimized. But these models are the equal interval model. Literature [9] adopts homogeneous index function to fit background values and establish the unequal interval MGM(1,N) model but the non-homogeneous index functions are more common and the above establishment mechanism of the model has some defects. Literature [10] establish the multi-variables unequal MGM(1,N) model and the background values are got by middle values which make the accuracy need to be improved further. Literature [11] adopts the non-homogeneous index functions to fit background values and establish the unequal multi-variables grey MGM(1,N) model and the accuracy is improved greatly. Literature [12] analyze the multi-variables MGM(1,m) model and using vector valued continued fraction theory provides trapezoid formula and reconstruction background values by interpolation and integral values.

Therefore, the accuracy can be improved greatly but the grey model is based on the equal sequence and in fact there are many original data belong to unequal sequence. So the establishing the unequal multi-variables MGM(1,m) model is meaningful. For the grey model accumulated operation is the key and reciprocal is the supplement. For the non-negative discrete data  $x^{(0)}$  the data after AGO process is monotonic increasing. It is reasonable that the curve is monotonic increasing which is used to fit  $x^{(1)}$ . But if  $x^{(0)}$  is monotonic decreasing that the AGO operation determines  $x^{(1)}$  is monotonic increasing. So the fitting  $\hat{x}^{(1)}$  model is monotonic increasing. By the IAGO process to restore the original data it will produce some unreasonable error. So for the monotonic decreasing original  $x^{(0)}$  literature [4] provides inverse accumulated operation and creates the GOM(1,1) model based on the inverse accumulated operation. Literature [5] provides reciprocal accumulated operation and create the GRM(1,1) model based on the reciprocal operation. Based on the reciprocal operation and inverse accumulated operation the sequence  $x^{(1)}$  is monotonic decreasing. So use the monotonic decreasing curve to fit  $x^{(1)}$  and the  $\hat{x}^{(1)}$  will not produced the unreasonable error after AGO or IAGO process. It can improve the accuracy. But the model in the literature [5] is the equal interval and single variable model. The paper absorbed the thoughts in literature [13] and uses the method in the new information unequal GM(1,1) to research the background values in unequal new information MGRM(1,n). it provides the reconstructed method of the background values in MGRM(1,n) model and give the formula of the background values. It takes the mth vector in the original sequence as the initial values and the fixed initial values as the design variables and the relative error as the objective functions. It establish the multi-variables unequal new information grey model MGRM(1,n). It is fit for equal and unequal interval and expands the application. The model has high accuracy and has good value in the theory and practice. The examples show it is practical and reliable.

## 2. The Non-Equidistant Multivariable New Information Optimization MGRM(1,n) Based on New Information Background Value Constructing and Accumulated Generating Operation of Reciprocal Number

**Definition 1** if  $\Delta t_j = t_j - t_{j-1} \neq \text{const}$ ,  $i = 1, 2, \dots, n$ ,  $j = 2, \dots, m$ ,  $n$  is variables,  $m$  is

the number of variables, then  $\mathbf{X}_i^{(0)}$  is the unequal sequence. Given  $x_i^{(0)}(t_j) = \frac{1}{x_i^{(00)}(t_j)}$  ( $j=1, 2, \dots, m$ ),

then  $\mathbf{x}_i^{(0)} = (x_i^{(0)}(t_1), \dots, x_i^{(0)}(t_m))$  is the reciprocal sequence of  $\mathbf{x}_i^{(00)}$ .

**Definition 2** given sequence  $\mathbf{X}_i^{(1)} = \{x_i^{(1)}(t_1), x_i^{(1)}(t_2), \dots, x_i^{(1)}(t_j), \dots, x_i^{(1)}(t_m)\}$ , if  $x_i^{(1)}(t_1) = x_i^{(0)}(t_1)$ ,  $x_i^{(1)}(t_j) = x_i^{(1)}(t_{j-1}) + x_i^{(0)}(t_j) \cdot \Delta t_j$ ,  $j = 2, \dots, m$ ,  $i = 1, 2, \dots, n$ ,  $\Delta t_j = t_j - t_{j-1}$ , then  $\mathbf{X}_i^{(1)}$  is the one order accumulated operation (1-AGO) of the unequal sequence  $\mathbf{X}_i^{(0)}$ .

If the original data matrix of the multi-variables is:

$$\mathbf{X}^{(0)} = \{\mathbf{X}_1^{(0)}, \mathbf{X}_2^{(0)}, \dots, \mathbf{X}_n^{(0)}\}^T = \begin{bmatrix} x_1^{(0)}(t_1) & x_1^{(0)}(t_2) & \cdots & x_1^{(0)}(t_m) \\ x_2^{(0)}(t_1) & x_2^{(0)}(t_2) & \cdots & x_2^{(0)}(t_m) \\ \cdots & \cdots & \cdots & \cdots \\ x_n^{(0)}(t_1) & x_n^{(0)}(t_2) & \cdots & x_n^{(0)}(t_m) \end{bmatrix} \quad (1)$$

Where  $\mathbf{X}^{(0)}(t_j) = [x_1^{(0)}(t_j), x_2^{(0)}(t_j), \dots, x_n^{(0)}(t_j)]$  is the objective values at  $t_j$  for  $\mathbf{X}^{(0)}(t_j)$  ( $j=1, 2, \dots, m$ ). Sequence  $[x_i^{(0)}(t_1), x_i^{(0)}(t_2), \dots, x_i^{(0)}(t_j), \dots, x_i^{(0)}(t_m)]$  ( $i = 1, 2, \dots, n$ ,  $j = 1, 2, \dots, m$ ) is unequal which means  $t_j - t_{j-1}$  is not const.

In order to establish the model we accumulated the original data and formed the new matrix

$$\mathbf{X}^{(1)} = \{\mathbf{X}_1^{(1)}, \mathbf{X}_2^{(1)}, \dots, \mathbf{X}_n^{(1)}\}^T = \begin{bmatrix} x_1^{(1)}(t_1) & x_1^{(1)}(t_2) & \cdots & x_1^{(1)}(t_m) \\ x_2^{(1)}(t_1) & x_2^{(1)}(t_2) & \cdots & x_2^{(1)}(t_m) \\ \cdots & \cdots & \cdots & \cdots \\ x_n^{(1)}(t_1) & x_n^{(1)}(t_2) & \cdots & x_n^{(1)}(t_m) \end{bmatrix} \quad (2)$$

Where  $x_i^{(1)}(t_j) (i=1,2,\dots,n)$  satisfied the definition 2, namely,

$$x_i^{(1)}(t_j) = \begin{cases} \sum_{j=1}^k x_i^{(0)}(t_j)(t_j - t_{j-1}) & (k=2,\dots,m) \\ x_i^{(0)}(t_1) & (k=1) \end{cases} \quad (3)$$

The  $n$  variables one order differential equations of unequal MGRM(1,n) model based on reciprocal accumulated operation is:

$$\begin{cases} \frac{dx_1^{(1)}}{dt} = a_{11}x_1^{(1)} + a_{12}x_2^{(1)} + \cdots + a_{1n}x_n^{(1)} + b_1 \\ \frac{dx_2^{(1)}}{dt} = a_{21}x_1^{(1)} + a_{22}x_2^{(1)} + \cdots + a_{2n}x_n^{(1)} + b_2 \\ \cdots \\ \frac{dx_n^{(1)}}{dt} = a_{n1}x_1^{(1)} + a_{n2}x_2^{(1)} + \cdots + a_{nm}x_n^{(1)} + b_n \end{cases} \quad (4)$$

Given  $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$ ,  $\mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix}$ , and the formula (4) can be described as:

$$\frac{d\mathbf{X}^{(1)}(t)}{dt} = \mathbf{A}\mathbf{X}^{(1)}(t) + \mathbf{B} \quad (5)$$

Based on the prior new information principles of grey theory if the first vector of  $\mathbf{x}_i^{(1)}(t_1)$  is taken as the initial conditions we do not make full use of the new information. So taken the  $m$ th vector of  $\mathbf{x}_i^{(1)}(t_m)$  as the initial conditions we can make full use of the newest information. The continuous time response equation in formula (5) is:

$$\mathbf{X}^{(1)}(t) = e^{\mathbf{A}t} \mathbf{X}^{(1)}(t_m) + \mathbf{A}^{-1}(e^{\mathbf{A}t} - \mathbf{I})\mathbf{B} \quad (6)$$

Where  $e^{\mathbf{A}t} = \mathbf{I} + \sum_{k=1}^{\infty} \frac{\mathbf{A}^k}{k!} t^k$ ,  $\mathbf{I}$  is the unit matrix.

In order to identify A and B, we integrate the formula (5) between  $[t_{j-1}, t_j]$  and get:

$$x_i^{(0)}(t_j)\Delta t_j = \sum_{l=1}^n a_{il} \int_{t_{j-1}}^{t_j} x_i^{(1)}(t)dt + b_i \Delta t_j \quad (i=1,2,\dots,n; j=2,3,\dots,m) \quad (7)$$

Given  $z_i^{(1)}(t_j) = \int_{t_{j-1}}^{t_j} x_i^{(1)}(t)dt$ . The traditional background values are equal to  $z_i^{(1)}(t_j)\Delta t_j$  of trapezoid the area. When the interval is little and the change is not large the construction of the background values is appropriate. When the sequence changes large the error of the background values is large. So the traditional construction of the background value in new information model is un-appropriate. So if we take  $z_i^{(1)}(t_j) = \int_{t_{j-1}}^{t_j} x_i^{(1)}(t)dt$  as the background values to get the estimated parameters it will be much better for the whitening equations (4). According to the index law of grey predictive model and the principle [13] of the unequal interval new information GM(1,1) model, we supposed  $x_i^{(1)}(t) = G_i e^{a_i(t-t_m)} + C_i$  where  $a_i, G_i, C_i$  is undefined coefficient and they satisfy  $x_i^{(1)}(t_j) = G_i e^{a_i(t_j-t_m)} + C_i$ . The known data can be got by grey model.

Opposite accumulated operation is:

$$x_i^{(0)}(t_j) = \frac{x_i^{(1)}(t_j) - x_i^{(1)}(t_{j-1})}{\Delta t_j} = \frac{G_i(1 - e^{a_i \Delta t_j})}{\Delta t_j} e^{-a_i(t_j-t_m)} = g_i e^{-a_i(t_j-t_m)} \quad (8)$$

Where

$$g_i = \frac{G_i(1 - e^{a_i \Delta t_j})}{\Delta t_j} = \frac{G_i(1 - (1 + (a_i \Delta t_j) + \frac{(a_i \Delta t_j)^2}{2!} + \dots))}{\Delta t_j}$$

When  $a_i$  and  $\Delta t_j$  is little we expand the first two form of  $e^{a_i \Delta t_j}$  is:

$$g_i = \frac{G_i(1 - e^{a_i \Delta t_j})}{\Delta t_j} = \frac{G_i(-a_i \Delta t_j)}{\Delta t_j} = -G_i a_i, \quad \frac{x_i^{(0)}(t_j)}{x_i^{(0)}(t_{j-1})} = \frac{e^{-a_i(t_j-t_m)}}{e^{-a_i(t_{j-1}-t_m)}} = e^{-a_i \Delta t_j}$$

Then,

$$a_i = \frac{\ln x_i^{(0)}(t_j) - \ln x_i^{(0)}(t_{j-1})}{\Delta t_j} \quad (j=2,3,\dots,m) \quad (9)$$

Put formula (9) into formula (8):

$$\left\{ \begin{aligned} g_i &= \frac{x_i^{(0)}(t_j)}{e^{-a_i(t_j-t_m)}} = \frac{x_i^{(0)}(t_j)}{[x_i^{(0)}(t_j)/x_i^{(0)}(t_{j-1})]^{\frac{t_m-t_j}{\Delta t_j}}}, G_i = \frac{x_i^{(0)}(t_j)\Delta t_j [x_i^{(0)}(t_j)/x_i^{(0)}(t_{j-1})]^{\frac{t_m-t_j}{\Delta t_j}}}{1 - \frac{x_i^{(0)}(t_{j-1})}{x_i^{(0)}(t_j)}} \end{aligned} \right. \quad (10)$$

From the initial condition  $x_i^{(1)}(t_m) = G_i e^{a_i(t_m-t_m)} + C_i = G_i + C_i$  and get:

$$C_i = x_i^{(0)}(t_m) - G_i = x_i^{(0)}(t_m) - \frac{x_i^{(0)}(t_j)\Delta t_j [x_i^{(0)}(t_j)/x_i^{(0)}(t_{j-1})]^{\frac{t_m-t_j}{\Delta t_j}}}{1 - \frac{x_i^{(0)}(t_{j-1})}{x_i^{(0)}(t_j)}} \quad (11)$$

Put formula (9) and (11) into the background values  $\int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j) dt$  and get:

$$\begin{aligned} z_i^{(1)}(t_j) &= \int_{t_{j-1}}^{t_j} x_i^{(1)} dt = -\frac{\Delta t_j x_i^{(0)}(t_j)}{a_i} + C_i \Delta t_j \\ &= \frac{(\Delta t_j)^2 x_i^{(0)}(t_j)}{\ln x_i^{(0)}(t_j) - \ln x_i^{(0)}(t_{j-1})} + x_i^{(0)}(t_m) \Delta t_j - \frac{x_i^{(0)}(t_m) (\Delta t_j)^2 [x_i^{(0)}(t_j)/x_i^{(0)}(t_{j-1})]^{\frac{t_m-t_j}{\Delta t_j}}}{1 - \frac{x_i^{(0)}(t_{j-1})}{x_i^{(0)}(t_j)}} \end{aligned} \quad (12)$$

Let  $\mathbf{a}_i = (a_{i1}, a_{i2}, \dots, a_{in}, b_i)^T$  ( $i = 1, 2, \dots, n$ ), the  $a_i$  can be got by the least square method.

$$\hat{\mathbf{a}}_i = [\hat{a}_{i1}, \hat{a}_{i2}, \dots, \hat{a}_{in}, \hat{b}_i]^T = (\mathbf{L}^T \mathbf{L})^{-1} \mathbf{L}^T \mathbf{Y}_i, i = 1, 2, \dots, n \quad (13)$$

Where

$$\mathbf{L} = \begin{bmatrix} z_1^{(1)}(t_2) & z_2^{(1)}(t_2) & \dots & z_n^{(1)}(t_2) & \Delta t_2 \\ z_1^{(1)}(t_3) & z_2^{(1)}(t_3) & \dots & z_n^{(1)}(t_3) & \Delta t_3 \\ \dots & \dots & \dots & \dots & \dots \\ z_1^{(1)}(t_m) & z_2^{(1)}(t_m) & \dots & z_n^{(1)}(t_m) & \Delta t_m \end{bmatrix} \quad (14)$$

$$\mathbf{Y}_i = [x_i^{(0)}(t_2)\Delta t_2, x_i^{(0)}(t_3)\Delta t_3, \dots, x_i^{(0)}(t_m)\Delta t_m]^T \quad (15)$$

So the  $\mathbf{A}$  and  $\mathbf{B}$  can be got

$$\hat{\mathbf{A}} = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} & \dots & \hat{a}_{1n} \\ \hat{a}_{21} & \hat{a}_{22} & \dots & \hat{a}_{2n} \\ \dots & \dots & \dots & \dots \\ \hat{a}_{n1} & \hat{a}_{n2} & \dots & \hat{a}_{nn} \end{bmatrix}, \quad \hat{\mathbf{B}} = \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \dots \\ \hat{b}_n \end{bmatrix} \quad (16)$$

The calculated values of the new information MGRM(1,n) model is:

$$\hat{\mathbf{X}}_j^{(1)}(t_j) = e^{\hat{\mathbf{A}}(t_j-t_m)} \mathbf{X}^{(1)}(t_m) + \hat{\mathbf{A}}^{-1} (e^{\hat{\mathbf{A}}(t_j-t_m)} - \mathbf{I}) \hat{\mathbf{B}}, j = 1, 2, \dots, m \quad (17)$$

Taken the  $m$ th vector as the initial values and use  $X_i^{(0)}(t_m) + \beta_i$  to substitute  $X_i^{(0)}(t_m)$  where  $\beta$  is the vector. The fitting values of the restored data are:

$$\hat{\mathbf{X}}_i^{(0)}(t_j) = \begin{cases} \lim_{\Delta t \rightarrow 0} \frac{\mathbf{X}_i^{(1)}(t_1) - \mathbf{X}_i^{(1)}(t - \Delta t)}{\Delta t}, & j = 1 \\ (\hat{\mathbf{X}}_i^{(1)}(t_j) - \hat{\mathbf{X}}_i^{(1)}(t_{j-1})) / (t_j - t_{j-1}), & j = 2, 3, \dots, m \end{cases} \quad (18)$$

The model values of the original data by definition 1 is  $\hat{x}_i^{(00)}(t_j) (j = 1, 2, \dots, m)$

The absolute error of the  $i$ th variables is  $\hat{x}_i^{(00)}(t_j) - x_i^{(00)}(t_j)$

The relative error of the  $i$ th variables is  $e_i(t_j) = \frac{\hat{x}_i^{(00)}(t_j) - x_i^{(00)}(t_j)}{x_i^{(00)}(t_j)} * 100$

The average relative error of the  $i$ th variables is  $\frac{1}{m} \sum_{j=1}^m |e_i(t_j)|$

The average error of the whole data is

$$f = \frac{1}{nm} \sum_{i=1}^n \left( \sum_{j=1}^m |e_i(t_j)| \right) \quad (19)$$

Taken the average error  $f$  as the objective function and  $\beta$  is the design variables, we use the Matlab7.5 to optimize the Fmincon or other method eg. Genetic Algorithm [15] Hybrid Particle Swarm Algorithm [16]. to solve it.

### 3. Examples

**Example 1** From literature [10] the data of affection of water absorption rate for the mechanical properties of PA66 is as table 1. According to the different water absorption rate of PA66 we get the mechanical properties such as bending strength, flexural modulus, tensile strength which is changed with the water absorption rate.  $X_1^{(0)}$  is bending strength (Mpa),  $X_2^{(0)}$  is flexural modulus (Gpa),  $X_3^{(0)}$  is tensile strength (Mpa). The data is as table 1.

By the method in the paper we establish the unequal interval prior new information optimized MGRM(1,3) based on the reciprocal accumulated operation and new information background values construction.

$$\mathbf{A} = \begin{vmatrix} -0.6678 & 0.0256 & 0.5816 \\ -11.6300 & 0.5301 & 17.6201 \\ 0.0107 & 0.0093 & -0.0806 \end{vmatrix}, \quad \mathbf{B} = \begin{vmatrix} 0.0084 \\ 0.2605 \\ 0.0106 \end{vmatrix}, \quad \beta = \begin{vmatrix} -0.001342 \\ 0.0099866 \\ -0.00079966 \end{vmatrix}$$

Table 1 the affection of water absorbance to the mechanical properties of pure PA66

number	1	2	3	4	5	6	7	8	9
Water Absorbance ratio $t_j / \%$	0	0.0607	0.1071	0.1662	0.2069	0.4344	0.5243	0.8524	0.9756
$X_1^{(0)}$	83.4	84.9	84.5	84.2	84.4	78.4	75.4	59.5	54.1
$X_2^{(0)}$	2.63	2.64	2.61	2.65	2.66	2.52	2.32	1.90	1.72
$X_3^{(0)}$	84.2	84.4	86.3	84.3	81.3	74.9	75.7	73.2	66.9

The fitting data of  $X_3^{(0)}$  is:

$$\hat{X}_3^{(0)} = [84.9022, 84.3999, 83.5059, 82.5976, 81.7182, 79.2236, 76.1869, 71.9091, 67.2779]$$

The absolute error of  $X_3^{(0)}$  is:

$$q = [0.7022, -0.00011079, -2.7941, -1.7024, 0.41823, 4.3236, 0.48687, -1.2909, 0.37791]$$

The relative error of  $X_3^{(0)}$  :

$$e = [0.83396, -0.00013127, -3.2377, -2.0195, 0.51443, 5.7725, 0.64316, -1.7635, 0.56489]$$

The relative average error of  $X_3^{(0)}$  is 1.7055% and the accuracy is high. The largest relative error is 5.7725%. The relative average error of  $X_3^{(0)}$  is 1.9684%.

**Example2** the loading is 600N, the relative sliding speed is 0.314m/s, 0.417m/s, 0.628m/s, 0.942m/s, 1.046m/s. The experimental data is as table 2.

Table 2. The experimental data of **TiN** Thin film coating [14]

sequence $j$	1	2	3	4	5
Sliding speed (m/s)	0.314	0.471	0.628	0.942	1.046
Coefficient of friction $\mu$	0.251	0.258	0.265	0.273	0.288
Wear rate $\omega$ ( $\times 10^5 \text{mgm}$ )	7.5	8	8.5	9.5	11

When  $t_k$  is the sliding speed,  $X_1^{(0)}$  is the frictional coefficient, attrition rate is  $X_2^{(0)}$ , we establish the MGRM(1,2) model based on the reciprocal accumulated operation and new information background values and the parameters of the model is:

$$\mathbf{A} = \begin{vmatrix} -1.2817 & 36.3479 \\ -0.0646 & 1.6210 \end{vmatrix}, \mathbf{B} = \begin{vmatrix} 4.5553 \\ 0.1640 \end{vmatrix}, \beta = \begin{vmatrix} -0.15965 \\ -0.0063673 \end{vmatrix}$$

The fitting values (%) of  $X_1^{(0)}$

$$\hat{X}_1^{(0)} = [0.2563, 0.258, 0.26277, 0.27476, 0.29059]$$

The absolute error (%) of  $X_1^{(0)}$

$$q = [0.0053004, -1.1851e-006, -0.0022276, 0.0017623, 0.0025868]$$

The relative error (%) of  $X_1^{(0)}$

$$e = [2.1117, -0.00045936, -0.84059, 0.64554, 0.89821]$$

The average relative error of  $X_1^{(0)}$  is 0.8993%, and the average values of relative error is 0.96729%. However the average values in the non-optimized model is 1.0586%. It can be concluded that the model in the paper has high accuracy.

#### 4. Conclusion

In the system of multi-variables non-equal sequence which the multi-variables are affected and constrained. Based on index characteristic of grey model, the characteristic of integral and new information principle, the new information background value in non-equidistant multivariable new information optimization MGRM(1,n) was researched and the discrete function with non-homogeneous exponential law was used to fit the accumulated sequence and the formula of new information background value was given. taking the mean relative error as objective function, and taking the modified values of response function initial value as design variables, based on accumulated generating operation of reciprocal number, a non-equidistant multivariable new information optimization MGRM(1,n) model was put forward which was taken the  $m$ th component as the initialization. The new information optimization MGRM(1,n) model can be used in non-equal interval & equal interval time series and has the characteristic of high precision as well as high adaptability. Example validates the practicability and reliability of the proposed model.

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