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Modified MVR-CORDIC Algorithm and it's Application in Attitude Measurement

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Abstract

Aiming at attitude measurement with 51 MCU, The speed and efficiency of basic MVR-CORDIC (Modified Vector Rotational-Coordinate Rotation Digital Computer) are lower, we lists the steps used to measure the heading in rotate mode. Considered the features of the 51-MCU and the MVR-CORDIC algorithm, some new schemes about iteration number and searching algorithm are presented to improve the speed. this modified algorithm is tested and compared in 51-MCU, the experiment result shows that adaptive iteration makes iteration times to 67 percents of the general numbers and the modified searching algorithm reduces 57 percents of the normal iteration time. The experimental results show that the Angle indicator system has high resolution; accuracy of pitch Angle within the ±0.4°; the accuracy of roll within the ±0.4° when pitch angle is not more than 80°, which meet the measuring system accuracy.

Keywords*: attitude measurement, MVR-CORDIC, adaptive iteration*

1. Introduction

The technology of slanting measure is to determine the tilt and tendency of the object in the space. Tilt-sensor is a device which is used to measure the incline of the objects compared with the horizontal. This technology may be applied in many aspects, such as inertia measuring system of spacecraft, determining extensional of robot manipulator, incline measuring of vehicle and hull, judging of rock tendency, contrail detecting of broach, etc.

Generally the acceleration sensor and signal conditioning circuit integrated with components called accelerometer. Acceleration sensor is a kind of Angle sensor, which measures the value of acceleration of sports or gravity and converts to electric signals of sensors. The purpose of acceleration measurement is to get the attitude Angle of the object.

In the Angle sensor signal processing, functional operation algorithm to the precision of the output value plays a decisive role. If the general look-up table or interpolation calculation is employed, the accuracy and computing speed should give way to each other in the Arctan and open radical sign function operation, with more hardware resources be occupied [1]. There are a variety of Arctan function calculation methods, like look-up table method [2], polynomial approximation method [3], rational approximation [4], CORDIC algorithm, etc. the method of Look-up table is speedy, simple and easy-to-do, but take up too big memory capacity; Polynomial approximation algorithm can achieve higher precision, but the general approximation order of polynomial number is higher, large amounts of calculation, and consume a large amount of hardware resources; Rational approximation, compared with polynomial approximation method, is more likely to achieve higher precision, but a large amount of computing time is consumed in block operation .The Coordinated Rotation Digital Computer (CORDIC) algorithm is well-known iterative technique to perform various basic arithmetic operations [1]-[3]. The algorithm is very attractive for hardware implementation because it uses only elementary shift-and-add steps to perform vector rotation in a two-dimensional (2-D) plane. Hence, the CORDIC algorithm can be applied to many DSP applications where rotation-based arithmetic functions are heavily utilized, such as linear system solver [4], [5], digital lattice filter [6], [7], singular value problems [8].

In this paper, we propose a modified vector rotational CORDIC (MVR-CORDIC) algorithm [9], It is very suitable for applications that use the CORDIC algorithm in only forward rotation mode (also known as vector rotation mode), i.e, the rotation angles are fixed and known

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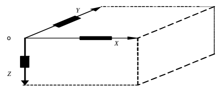
in advance, such as digital lattice filter [6], [7], [10] and discrete linear transformation [11], [12]. The major feature of the aforementioned applications is that the directional sequence, $\mu(i)$, which controls the rotation direction of each elementary angle in the micro rotation phase, can be computed in advance. By reformatting and searching for new sequences, we can reduce the iteration number significantly, while not increase the quantization noise level. This can be achieved by modifying the basic micro rotation procedure of conventional CORDIC algorithm. Then, we can improve the speed performance of the conventional CORDIC algorithm.

2. The Proposed Algorithm

2.1 MVR - CORDIC principle

CORDIC algorithm is a kind of vector rotation for the calculation of iterative algorithm. Its basic idea is to decompose a pre-rotated specific Angle, with linear combination of a set of prescribed basic Angle, namely, the multiple basic Angle rotation .i.e:



(1)

(N =W=basic Angle element quantity ; $\mu(i) \in \{-1,1\}$, the first time rotating direction; $\alpha(i) = \arctan(2^{-i})$, the corresponding basic Angle elements; ξ, the after-iterative residual error of the angle).

In contrast with the basic CORDIC algorithm, MVR - CORDIC characterizes in: (1) skipping some micro- rotation Angle; (2) repeating some micro-rotation Angle; (3) reducing iterations, in order to greatly improve the iterative speed and iteration precision. Thus, decomposition expression of MVR - CORDIC has been changed from (1) into (2):

$$\theta = \sum_{i=0}^{R_m-1} \mu(i) * \alpha(S(i)) + \xi \tag{2}$$

(R_m , iterations, generally $R_m << W$; $\mu(i) \in \{-1,0,1\}$, the i-th rotating direction; $\alpha(S(i)) = \arctan(2^{-S(i)})$, the corresponding basic Angle elements when the i-th micro-rotated, in which $S(i) \in \{0,1,2,\cdots N-1\}$.)

This paper use MVR - CORDIC algorithm vector model [13], in which $\mu(i)$ is a symbol function, its value determined by $Y(i)$. Iterative equations can be written as follows:

$$\begin{cases} X(i+1) = X(i) - \mu(i) * Y(i) * 2^{-s(i)} \\ Y(i+1) = Y(i) + \mu(i) * X(i) * 2^{-s(i)} \\ Z(i+1) = Z(i) - \mu(i) * \alpha[S(i)] \end{cases} \tag{3}$$

$X(0), Y(0)$ (X_0, Y_0 hereafter) Corresponding request Angle cosine and sinusoidal component, $Z(0) = 0$; In the beginning of iteration ,select the appropriate S(i), to make Y(i+1) minimum; take $\mu(i), S(i)$ into the type X(i+1) of iterative equations to get corresponding X(i+1); look-up table to get $\alpha(S(i))$, and thus get Z(i+1),so far complete a iterative; After R_m iteration, results of output as follow:

$$\begin{cases} X(i+1) = P\sqrt{X_0^2 + Y_0^2} \\ Y(i+1) = 0 \\ Z(i+1) = \arctan(Y_0 / X_0) \end{cases} \quad \left(P = \prod_{i=0}^{R_m-1} \sqrt{1 + 2^{-2s(i)}} \right) \tag{4}$$

From equations (4), we can see that, due to the existence of the scaling-factor P and its uncertainty, so scale-factor calculation and vector module compensation are needed, that is, $K = 1/P$,we can write as.

$$X(i+1)' = K * X(i+1) \tag{5}$$

In the actual use, various improvement of MVR--CORDIC algorithm is made to improve the efficiency and speed of the algorithm; For example, according to the hardware structure of FPGA, proposed the use of check ROM table instead of the original algorithm scale-factor calculation method [13].In this paper, MVR - CORDIC algorithm, which had been modified and improved, is applied to calculate angle based on 51-MCU.

2.2 Algorithm Improvement

In the realization of algorithm, the basic MVR - CORDIC pre-rotated Angle is modified to find out the effective bits, calculation formula of basic Angle and the number of element, and employed adaptive iteration times and the method of improved-rotation sequence proposed. The improved algorithm flow chart as Figure.1:

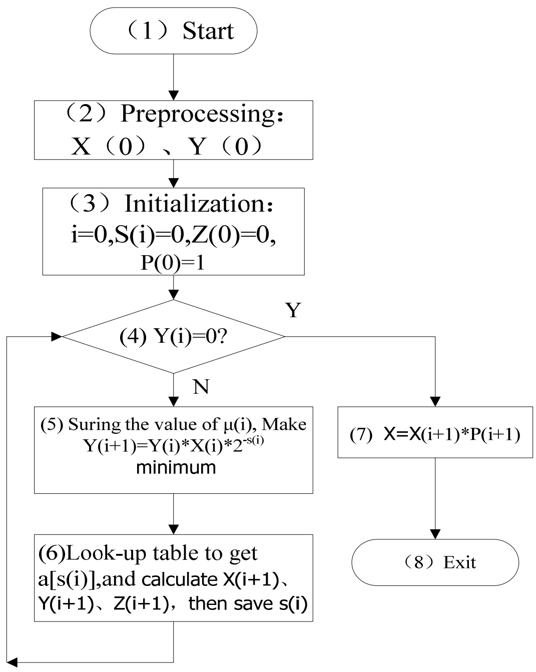


Figure 1. MVR-CORDIC flow chart

First of all, in order to make the algorithm convergence in the whole plane, and achieve higher accuracy, the pre-rotation of the initial Angle needed carrying out, i.e. The step 2 of figure 1, and to get $Z(i+1) \in [0, \pi/4)$,Correspondingly, at the same time, X_0 , Y_0 should be pretreated to make $\theta = \arctan(Y_0 / X_0) \in [0, \pi/4)$.

This paper pretreated X_0 , Y_0 to make $Y_0 \geq 0, X_0 > 0, X_0 \geq Y_0$, and coded the θ Result (as table 1). After the iteration, the iteration results are reversely processed to get the real Angle value according to the code value.

Table 1. Rotation vector initial value pretreatment and coding

Coding	θ Initial value	Pre-rotation	θ Result
0	[0,π/4)	$\begin{bmatrix} X_0' \\ Y_0' \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix}$	0
1	[π/4,π/2)	$\begin{bmatrix} X_0' \\ Y_0' \end{bmatrix} = \begin{bmatrix} Y_0 \\ X_0 \end{bmatrix}$	π/2-θ
2	[π/2,3π/4)	$\begin{bmatrix} X_0' \\ Y_0' \end{bmatrix} = \begin{bmatrix} Y_0 \\ -X_0 \end{bmatrix}$	θ-π/2
3	[3π/4,π)	$\begin{bmatrix} X_0' \\ Y_0' \end{bmatrix} = \begin{bmatrix} -X_0 \\ Y_0 \end{bmatrix}$	π-θ

Secondly, in the improved algorithm, data effective length is determined by accuracy of angle which is determined by the system requirements. Calculating formula as follows:

$$\theta = \arctan(x) = \arctan(2^N) \Rightarrow \theta' = \frac{1}{1+2^N} \tag{6}$$

In which $x = 2^N$, N=data significant bit length. Furthermore, the number of the elements of basic Angle set of improved algorithm is determined by effective bit length. Formula is as follows:

$$\alpha(m) = \arctan(2^{-m}) \le \frac{1}{2^N} \tag{7}$$

In which m = the minimum number of elements within the basic Angle set. In the basic MVR-CORDIC algorithm, R_m , a constant, see type (2), is defined the minimum of iterations to satisfy the performance of algorithm. in most cases, the actual iterations are less than R_m in the actual calculation.

Adaptive iteration method did not provide constant iterations, but automatically determined by iterative process itself. When the algorithm met the precision requirement, namely, end iteration process, to avoid unnecessary iterative process and thus improve the efficiency. In the applications, $R_m^{'} \le R_m$ is given to ensure the process of algorithm in a controlled way.

The improved algorithm provided the way to search the rotary sequence. The step 5 of figure.1, the right $\mu(i)$ and $S(i)$ are needed to minimize $Y(i+1)$. According to the principle, the value of $\mu(i)$ can be directly determined by $Y(i)$, So the step 5 is main to determine the value of $S(i)$ .So, Quickly finding suitable $S(i)$ is an effective way to improve the speed of iteration. From type (2), we can see $S(i) \in \{0,1,2,\cdots,N-1\}$ is an increasing sequence; From the iterative equations (3) and iterative process, we can find out:

$$|X(i+1)| \ge |X(i)| \text{ ; } |Y(i+1)| < |Y(i)| \text{ ; } \Rightarrow S(i+1) > S(i) \tag{8}$$

Improved search method characterized in that, using the $S(i)$, which is saved by the step 6, determine the range of $S(i+1)$ on step 5 of the next iteration process ; so, searching range of $S(i)$ would decrease with i increasing, which speeded up the searching speed and shorten the step 5 operation time ; time of single iteration also reduced correspondingly, improving the speed of execution.

The scale-factor is produced during the iteration, see type (4). The Scale-factor is also changing because of rotary sequence and iteration times of uncertainty. The algorithm always direct calculate scale factor in the iteration process, and then product the previous storage factor and storage. see step (7) in Figure1.

3. Research Method

3.1. Angle Measurement Principle

Acceleration sensor can be used to determine changes or constant acceleration. On the earth any position objects are subject to the effect of gravity and produce a acceleration and acceleration of gravity is a special case of constant acceleration.

According to the rigid body dynamics principle, it is known that the Angle between the reference coordinate system bound together with moving object and fixed reference coordinate system can be called the inclined state (namely attitude).see Figure 2.

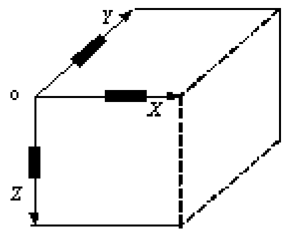


Figure 2. Coordinate system

At this time, yaw angle, pitch angle and roll angle in the space can be expressed as follows:

Yaw angle (ϕ): angle between OX shaft's the projection line in horizontal plane (O $X_d Y_d$) and OX_d shaft.

Pitch angle (θ): angle between OX shaft and the horizontal plane ($OX_d Y_d$)

Roll angle (γ): angle between the symmetry plane of the object and the vertical plane which contains X_d shaft.

Three rotations corresponding with three matrixes, which can be written as:

$$R_\gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix} \quad R_\theta = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \quad R_\phi = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In the ideal conditions, the Measurement value of angle sensor or triaxial accelerometer can be normalized as:

$$G_m = R_\gamma R_\theta R_\phi G = [-\sin\theta \quad -\sin\gamma\cos\theta \quad \cos\gamma\cos\theta]^T.$$

Where: $G = [0 \quad 0 \quad 1]^T$ the normalized expression Gravity field (namely g) in definite reference coordinate system. From G_m, we can deduce Pitch angle and roll angle expression as follows:

$$\theta = \arctan\left(\frac{-G_{mx}}{\sqrt{G_{my}^2 + G_{mz}^2}}\right) \qquad \gamma = \arctan\frac{G_{my}}{G_{mz}} \tag{9}$$

3.2. Data Measurement Principle

Angle sensor was installed on a horizontal rotating table free of magnet during Experiment; in order to validate the data, output signal from the sensor, through serial transmission, to computer was transmitted after A/D sampling. the serial given orders, single-chip gathered the encoder and transmitted to the computer; after that rotation level turntable was rotated. Operation above duplicated, level turntable was rotated from the 80 degrees in Angle of pitch and 280 degrees in angle of roll, respectively to gather a group of data every 10 degrees, and then recorded the accelerometer output value and the corresponding Angle of pitch and roll; a total of 16 x16 group data was measured.

Ideally, Angle of pitch and roll from the turntable should be consistent with that displayed on the compute, however, due to all kinds of error in the actual measurement. Error equally existed between the measurement results and standard ones. According to our acquisition data, correction coefficient was utilized to do the calculation, and then applied to the system, Angle of pitch and roll measured repeatedly was compared with standard turntable comparison; the data showed that data vibrated among [- 80°, 80°] range, with error below 0.4°, which met the requirement of experiment.

Because of the circuit itself and the outside environment factors, it is inevitable to commit errors in angle measurement system. According to the nature and form of expression of the error, Errors can be divided into system error, random error and gross error. The systematic error keeps constant or changes with a certain law, which can be compensated and corrected; random error is unpredictable and uncorrected, but statistically estimable in its implications; gross error is caused by human factors, with the error value generally more obvious and avoided. The chapter deals with analysis of systematic error, the error compensation and its verification.

3.3. Error Analysis

Error factors of acceleration sensor in the field of inertial navigation have been studied, it is generally accepted that the error of acceleration sensor lies mainly in the installation error, sensitivity error, cross axis sensitivity, cross coupling error and random error; for the low precision of micro mechanical acceleration sensor, its error source was negligible in terms of order two or more error coefficient.

A tilt sensor structure shows us, when the inclination sensor being used to do static attitude measurement, with no lateral displacement effect, the output signal of the sensor is not affected by acceleration interference [14]. The main signal error components:

(1) Zero error with temperature: zero error driven by changes in the temperature value and temperature coefficient, temperature can be changed by instrument calibration to reduce, temperature coefficient effect can be lessened by temperature compensation.

(2) Error with sine function: the single axis accelerometer signal output is proportional to **sin(x)**, where x is a single axis accelerometer tilt angle. In the measurement of large amplitude angle, this effect is seen as a nonlinear added value.

(3) Error with vibration: such an error with vibration may be caused by slight vibration of experiment platform in experiment environment, as opposed to the required stationary signal.

(4) Error with voltage changes: for best results, the voltage should be maintained at 5V; if the voltage is changed, output singal will range within 2% in error.

(5) Error with installation: during installation, three sensor measuring shafts are not parallel to carrier in longitudinal, transverse, vertical vector of three axis, respectively.

3.4. Error Compensation Method

3.4.1. Nonlinear Error Compensation Method

Single axis accelerometer output signal is proportional to the sine value of inclination angle, therefore, we adopt the linearization method of sine value and acceleration, each axle acceleration compensated. Specific methods are as follows:

(1): first calculate corresponding angle X with each sine value, then determine the axis of calibration accelerometers, after that rotate axis to angle X in calibration, record output acceleration value of calibration shaft in the Table 2.

Table 2. i-axis nonlinear compensation sampling data

Sinx	-1	-0.9	-0.8	-0.7	-0.6	…	0.6	0.7	0.8	0.9	1
a_i	…	…	…	…	…	…	…	…	…	…	…

(2): curve fitting with least square method

Curve fitting was designed to use a simple function to approximate a complex or unknown function, with function value of finite sampling points in an interval as target [25]. Let f (x) be the interpolation function, P (x) a polynomial function, when the sum of square of the difference of the two functions was minimal, we may call P (x) as the best square approximation of f (x) [15].

Curve fitting of least squares is to seek a coefficient vector, which is a polynomial coefficient. Linear equation of each axis in the design is: Sin (x) = k*ai + b, i.e. to figure out the value k and b, where x is a single axis accelerometer tilt angle.

3.4.2. Installation Error Compensation Method

According to the angle measure principal, we may know that, ideally, the output signal of three axis meets the equation groups in 10:

$$\begin{cases} Ax = -g\sin\theta \\ Ay = g\sin r\cos\theta \\ Az = g\cos r\cos\theta \end{cases} \tag{10}$$

In a nonideal environment, due to the such inconsistencies of the three output signal from three axis accelerometers and presence of error after the input signal being the systematically magnified, sampled, acceleration sensor calibration process, based on the analysis of errors, can be equivalent to 12 coefficients in formula (11)

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \tag{11}$$

Compensation matrix coefficients are stored in a Flash in advance, power MCU, loaded into the on-chip RAM. Input signal after compensation calculation and get component signal free of error.

The calibration parameters can be directly calculated based on three axis acceleration sensor measuring values. The correction process is as follows: Adjust the turntable to the level state, and angle measurement system of the three axis acceleration sensor is placed on a turntable, rotating table located at a different pitch angle and roll angle integrated attitude, recording three axis sensor output and the corresponding pitch angle and roll angle, every measured value of the acceleration shall be satisfied:

$$K_a\hat{G}_m + G_{offi} = [-\sin(\theta_i), \sin(\gamma_i)*\cos(\theta_i), \cos(\theta_i)*\cos(\gamma_i)]^T \tag{12}$$

Where: $\hat{G}_m = [g_{mxi}, g_{myi}, g_{mzi}]^T$ is each measurement value of accelerator of three axis.

$G_{offi} = [g_{oxi}, g_{oyi}, g_{ozi}]^T$ is bias of accelerator of three axis.

Therefore, theoretically, as long as measuring points under the four postures fixed, correction coefficient Ka and Goff can be calculated. In order to improve the calibration precision, fitting correction can be achieved by the use of multiple inclined posture measuring point (more than 4).

Where:

$$Ka = \begin{bmatrix} C_{11}C_{12}C_{13} \\ C_{21}C_{22}C_{23} \\ C_{31}C_{32}C_{33} \end{bmatrix} \qquad G_{off} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

The measuring system of attitude was tested on the turntable, the results as follows:

Table 3. Output roll angle γ (θ= 0°)

code	Theoretical γ	Output γ	Error
1	0°	-0.13°	0.13°
2	10°	10.22°	0.22°
3	20°	19.88°	0.12°
4	30°	29.79°	0.21°
5	40°	40.32°	0.32°

Table 4. Output roll angle γ (θ= 15°)

code	Theoretical γ	Output γ	Error
1	0°	0.14°	0.14°
2	10°	9.89°	0.11°
3	20°	19.79°	0.21°
4	30°	30.38°	0.38°
5	40°	40.29°	0.29°

Table 5. Output Pitch angle θ (γ= 0°)

code	Theoretical θ	Output θ	Error
1	0°	-0.31°	0.31°
2	10°	10.19°	0.19°
3	20°	19.76°	0.24°
4	30°	29.73°	0.27°
5	40°	39.64°	0.36°

Table6. Output Pitch angle θ (γ= 15°)

code	Theoretical θ	Output θ	Error
1	0°	0.21°	0.21°
2	10°	9.73°	0.27°
3	20°	19.78°	0.22°
4	30°	30.29°	0.29°
5	40°	39.73°	0.27°

4. Conclusion

This paper put forward the method of adaptive iteration times and improved rotation sequence search method by employing the improved MVR - CORDIC algorithm in 51 MCU. The experimental results show that the Angle indicator system has high resolution; accuracy of pitch Angle within the ±0.4°; the accuracy of roll within the ±0.4° when pitch angle is not more than 80°, which meet the measuring system accuracy. The experimental results show the algorithm efficiency was greatly improved .But more further research were needed to optimize its theory and application.

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