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Online Measurement on Flatness and its Uncertainty of Small Work-piece

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Abstract

Flatness is a very important shape and position of plane surface error and has great influence on the operating function of the work-piece. The traditional measurement methods of flatness can not be satisfied with the real-time requirement, and the uncertainty of flatness is not easy to be evaluated either. So a new method is proposed based on Geometrical product specification (GPS) standard-chain. That is: the coordinate value of small work-piece is occurred by portable Coordinate Measure Machine (CMM), the flatness is calculated from these coordinates using least-square arithmetic, and the uncertainty of flatness is estimated by the experimental matrix. The qualified products are separated by the tolerance and the agreement of supply and demand sides, and all the results are shown in an interface in real-time. Random inspection shows that this method can realize to pick the rejects online, increase the measurement efficiency, and decrease the probability of misconstruction.

Keywords: *online measurement, flatness, uncertainty, geometrical product specification (GPS), least-square method*

1. Introduction

Flatness as a shape and position error of plane surface is one of the most based measured in geometric measurement. Its value has great influence on the quantity and the operating function of the work-piece. Based on the items of national standard of shape tolerance, the flatness is defined the variation that measured surface opposite to the ideal surface, while the position of the ideal surface must be in keeping with minimum requirement [1]. So the important question is how to ascertain the ideal surface. The traditional method to get the ideal surface is that: first, straight line error is confirmed, then criterion is switched and last the flatness is calculated [2, 3]. To the large work-piece, many sensors were usually used to decide the position coordinates and calculate the flatness, but this method is not suitable for small work-piece [4]. Some optical measurement methods were also reported [5, 6]. Accorded with requirement of geometrical product specification (GPS) standards, the least-square arithmetic was applied in flatness based on Coordinate Measure Machine (CMM) [7], but there is a discrepancy in assessment the uncertainty, such as in referents [8, 9]. To the authors knowledge, there is little information available in literature about the online measurement on flatness of small work-piece and possible therapeutic application. So in this paper a method is used to gain the ideal surface, calculate the flatness and its uncertainty, and pick the rejects online according to the demands of the factory.

2. Flatness Based on Least-square Method

According to GPS chain, the least-square method used in measurement flatness is composed of separation, extract, fitting and appraise. So the least-square plane is calculated from the measurement data and is considered as the ideal surface. The equation of least-square plane is supposed as follows

$$z = ax + by + c \tag{1}$$

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Where, a, b and c are the parameters of the least-square plane. If the coordinate of the small work-piece is measured by portable CMM and signed as $\left(x_i, y_i, z_i\right)$, i is from 1 to n, in which the n is the number of location spots, then, the distance from every spots to the least-square plane is

$$d_i = \frac{z_i - ax_i - by_i - c}{\sqrt{a^2 + b^2 + 1}} \quad (2)$$

So, the least-square surface must be guaranteed the square sum of all distances is minimal, that means

$$D = \sum_{i=1}^{k} d_i^2 = \sum_{i=1}^{k} \frac{\left(z_i - ax_i - by_i - c\right)^2}{a^2 + b^2 + 1} = \min \quad (3)$$

The extreme conditions satisfied Equation (3) must be

$$\frac{\partial D}{\partial a} = 0, \quad \frac{\partial D}{\partial b} = 0, \quad \frac{\partial D}{\partial c} = 0 \quad (4)$$

The a, b, c can be calculated from Equation (4), their estimators are

$$\begin{cases} b_0 = \dfrac{\left(s_{00} - s_0 s_0\right)\left(s_{12} - s_1 s_2\right) - \left(s_{01} - s_0 s_1\right)\left(s_{02} - s_0 s_2\right)}{\left(s_{00} - s_0 s_0\right)\left(s_{11} - s_1 s_1\right) - \left(s_{01} - s_0 s_1\right)\left(s_{01} - s_0 s_1\right)} \\ a_0 = \dfrac{\left(s_{02} - s_0 s_2\right) - \left(s_{01} - s_0 s_1\right) b}{\left(s_{00} - s_0 s_0\right)} \\ c_0 = s_2 - s_0 a_0 - s_1 b_0 \end{cases} \quad (5)$$

Where,

$$\begin{cases} s_0 = \dfrac{\sum_{i=1}^{n} x_i}{n}, s_1 = \dfrac{\sum_{i=1}^{n} y_i}{n}, s_2 = \dfrac{\sum_{i=1}^{n} z_i}{n}, s_{00} = \dfrac{\sum_{i=1}^{n} x_i^2}{n} \\ s_{01} = \dfrac{\sum_{i=1}^{n} x_i y_i}{n}, s_{11} = \dfrac{\sum_{i=1}^{n} y_i^2}{n}, s_{02} = \dfrac{\sum_{i=1}^{n} x_i z_i}{n}, s_{12} = \dfrac{\sum_{i=1}^{n} y_i z_i}{n} \end{cases} \quad (6)$$

In terms of the smallest containing principle, the flatness of small work-piece is the distance from peak to valley of all the measuring spots [10]. If the coordinates of peak and valley are supposed by $\left(x_{\max}, y_{\max}, z_{\max}\right)$ and $\left(x_{\min}, y_{\min}, z_{\min}\right)$ respectively, the flatness δ is obtained as below

$$\delta = \frac{\left(z_{\max} - z_{\min}\right) - a_0\left(x_{\max} - x_{\min}\right)}{\sqrt{1 + a_0^2 + b_0^2}} - \frac{b_0\left(y_{\max} - y_{\min}\right)}{\sqrt{1 + a_0^2 + b_0^2}} \quad (7)$$

3. Appraise the Uncertainty of Flatness

Obviously, the uncertainty of flatness is decided by all the uncertainty of the factors in Equation (7) and their propagation coefficients. There are two main origins which affect the flatness uncertainty. Since the factors of the uncertainty are attained from different ways and types, they should be calculated separately.

3.1. Uncertainty of the Plane Parameters

The parameters of the least-square plane a,b,c are derived from the coordinates of all the measurement points, so they are the indirect measurands. Their uncertainty should be calculated from the uncertainty of the coordinates (x_i, y_i, z_i), which are the direct measurands. So, it belongs to the appraisal of type A. We have known, a_0, b_0, c_0 is the least-square estimator of a,b,c , put a_0, b_0 to Eq. (2), the Eq. (2) can be rewritten as

$$ax_i + by_i + c = z_i - d_i\sqrt{a_0^{\ 2} + b_0^{\ 2} + 1} = l_i \tag{8}$$

A matrix adapted Equation (8) is

$$\mathbf{AX} = \mathbf{Y} \tag{9}$$

Where, $\mathbf{X}$ is the column matrix composed of a,b,c ; $\mathbf{A}$ is the coefficient matrix; $\mathbf{Y}$ is the constant matrix. They are represented as follows

$$\mathbf{X} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{pmatrix}, \quad \mathbf{Y} = \begin{pmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{pmatrix} = \begin{pmatrix} z_1 - d_1\sqrt{a_0^{\ 2} + b_0^{\ 2} + 1} \\ z_2 - d_2\sqrt{a_0^{\ 2} + b_0^{\ 2} + 1} \\ \vdots \\ z_n - d_n\sqrt{a_0^{\ 2} + b_0^{\ 2} + 1} \end{pmatrix} \tag{10}$$

According to the least-square matrix of single precision, the standard deviation of l_i is

$$\sigma_l = \sqrt{\frac{\sum_{i=1}^{n} v_i^2}{n-t}} = \sqrt{\frac{\sum_{i=1}^{n}\left(l_i - a_0x_i - b_0y_i - c_0\right)^2}{n-t}} \tag{11}$$

Where, t is the number of parameters estimator, so here $t = 3$ .There is an inverse matrix in handling, which covariance matrix is [11].

$$C = \left(A^T A\right)^{-1} = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix} \tag{12}$$

So the uncertainty of the parameters a,b and ab are separately as follows,

$$u_a = \sqrt{d_{11}}\sigma_l,\ u_b = \sqrt{d_{22}}\sigma_l,\ u_{ab} = \sqrt{d_{12}}\sigma_l \tag{13}$$

The error transmits coefficients corresponding with u_a and u_b are the partial differential of Equation (7)

$$\begin{cases} a_1 = \frac{\partial \delta}{\partial a} = \frac{-(x_{max} - x_{min})}{(1+a_0^2+b_0^2)^{\frac{1}{2}}} - \frac{b_0\left[(z_{max}-z_{min}) - a_0(x_{max}-x_{min}) - b_0(y_{max}-y_{min})\right]}{(1+a_0^2+b_0^2)^{\frac{3}{2}}} \\ a_2 = \frac{\partial \delta}{\partial b} = \frac{-(y_{max} - y_{min})}{(1+a_0^2+b_0^2)^{\frac{1}{2}}} - \frac{b_0\left[(z_{max}-z_{min}) - a_0(x_{max}-x_{min}) - b_0(y_{max}-y_{min})\right]}{(1+a_0^2+b_0^2)^{\frac{3}{2}}} \end{cases} \quad (14)$$

3.2. Uncertainty of the Coordinates

The coordinates of the peak and valley are attained by portable CMM, so their uncertainty is decided by the measurement precision of CMM and belongs to the appraisal of type B. By the handbook or demarcation of portable CMM, its measurement uncertainty is $\pm 3u$ and equals to three times of standard deviation u . So the uncertainty of coordinates is the same u .

$$u_{x_{max}} = u_{x_{min}} = u_{y_{max}} = u_{y_{min}} = u_{z_{max}} = u_{z_{min}} = u \quad (15)$$

Their corresponding error transmits coefficients are also the partial differential of Equation (7)

$$\begin{cases} a_3 = \frac{\partial \delta}{\partial x_{max}} = \frac{-a_0}{\sqrt{1+a_0^2+b_0^2}}, a_4 = \frac{\partial \delta}{\partial x_{min}} = \frac{a_0}{\sqrt{1+a_0^2+b_0^2}} \\ a_5 = \frac{\partial \delta}{\partial y_{max}} = \frac{-b_0}{\sqrt{1+a_0^2+b_0^2}}, a_6 = \frac{\partial \delta}{\partial y_{min}} = \frac{b_0}{\sqrt{1+a_0^2+b_0^2}} \\ a_7 = \frac{\partial \delta}{\partial z_{max}} = \frac{1}{\sqrt{1+a_0^2+b_0^2}}, a_8 = \frac{\partial \delta}{\partial z_{min}} = \frac{-1}{\sqrt{1+a_0^2+b_0^2}} \end{cases} \quad (16)$$

Other uncertainty caused by random factors can be ignored. Therefore the uncertainty of flatness is the compound of all the branches as follows

$$u_\delta = \sqrt{\sum_{i=1}^{8} (a_i u_i)^2 + 2a_1 a_2 \rho_{ab} u_{ab}} \quad (17)$$

where, ρ_{ab} is the coefficient of a, b correlation, it is decided by the related degree of the reality.

4. Work-piece and Program

The outline of the small metal plate is shown in figure 1. Theoretical, the more the number of the measurement points is, the more precise of flatness is. But considering the efficiency and the memory capacity, the number of sampling point is not suitable too much. The stars are the measurement points (in Figure 1). The measurement steps are: 1) coordinates of sampling points are collected by the portable CMM; 2) the flatness δ and its uncertainty u_δ are calculated by Equation (7) and Equation (17) individually; 3) the accepted products are separated by the tolerance and the agreement of supply and demand sides; 4) all the results are shown in an interface. The technological process is shown in Figure 2.

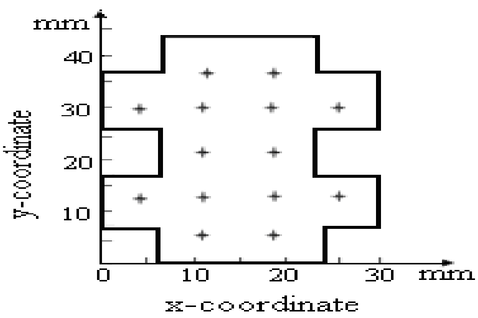

Figure 1. Sketch of Work Piece and Measuring Spots

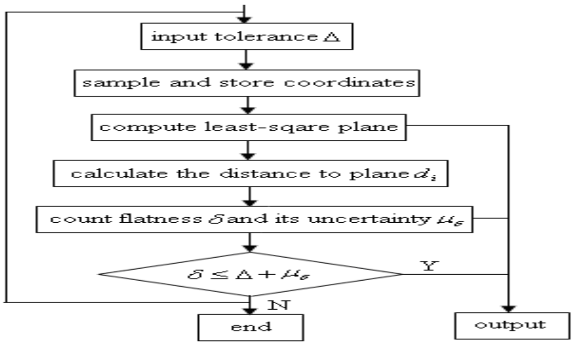

Figure 2. Chart of Program

5. Application Example

Through the above method and some software and hardware, the flatness and its uncertainty can be shown in real-time. Some specimens were carried out to experiment in random. The contract agreed on that the tolerant of flatness was $\Delta = 0.020mm$, only when the real experimental flatness $\delta \le \Delta + \mu_\delta$, the product can be qualified. The coordinates of one piece of product was exhibited in Table 1.

Table 1. Coordinates of the Random Sample

Experiment point	Coordinates x (mm)	Coordinates y (mm)	Coordinates z (mm)
1	11.500	6.999	1.024
2	18.499	6.998	1.025
3	4.500	13.001	1.019
4	11.501	12.999	1.020
5	18.499	13.000	1.026
6	25.500	12.998	1.023
7	11.499	22.001	1.022
8	18.501	22.001	1.027
9	4.501	29.999	1.022
10	11.502	30.000	1.019
11	18.498	29.999	1.011
12	25.500	29.998	1.032
13	11. 499	37.001	1.019
14	18. 500	37.000	1.024

According to the programming result, the output interface is shown in Figure 3.

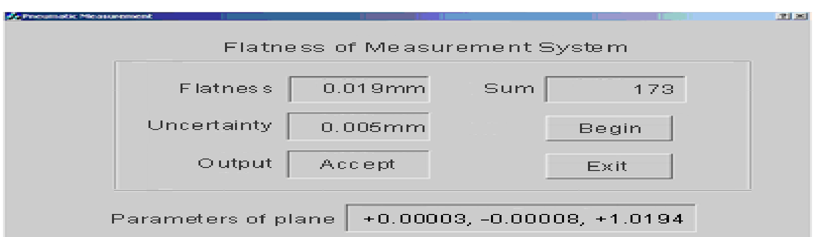

Figure 3. Man-machine Interface

Therefore, the final equation of least-square plane is $z = 0.00003x - 0.00008y + 1.0194$

The flatness of small work-piece is $\delta = 0.019\text{mm}$,and its uncertainty is $\mu_\delta = 0.005\text{mm}$, by the agreement the product is accepted.

From the above theory and real experiment, all the result information is decided by this time measurement data, but not relied on the random vectors or matrix, so reduced the disagreement between supply and demand. Obviously this method can realize to measure the flatness in real-time, be easy to control, and fit in with the judge norm of ISO14253. Owing to many measurement points and considering the influence of flatness uncertainty, the result is more rational, and the probability of misapprehend is decreased. This method is also suitable for measuring flatness of other small work-piece, such as small fixture of riveting.

6. Conclusion

In accordance with the demands of business, a method to measuring flatness online of small work-piece was proposed in this paper. Based on portable CMM and least-squares arithmetic, the flatness is calculated. According to the error transfer function and least-squares measure matrix, the uncertainty of flatness is appraised. The result of checking a random sampling indicated that the method can measure the flatness of work-piece online, realize to separate the accepted products and rejects, and decrease the probability of misapprehend.

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