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Robust Control of Urban Industrial Water Mismatching Uncertain System

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Abstract

Urban industrial water system parameter fluctuation producing uncertainty may not occur in a control input channel, can be applied mismatching uncertain system to describe. Based on Lyapunov direct method and linear matrix inequality, design the urban industrial water mismatching uncertain system feedback stabilization robust control scheme. Avoid the defects that the feedback stabilization control method based on the matrix Riccati equation need to preset equation parameters, easier to solve and can reduce the conservative.

Keywords*: industrial water, uncertainty, robust control*

1. Introduction

In China, the industry is the important force supporting a regional economic and social development. Industrial water accounts for sizeable portion in the regional water, generally after agriculture. So industrial water efficiency considerably influences on a regional water resources use efficiency [1]. Research on urban industrial water system effective control, both are meaningful for the urban industry and the water resources system sustainable development.

Control, decision and evaluation theory have been researched in water management. Such as the automated real-time online water quality monitoring program based on wireless sensor network [2], fuzzy temperature control for heating and cooling water system [3], four types of rural drinking water project management model based on project scale [4], the multiple attribute decision making problems for evaluating the construction projects of wastewater treatment in the paper making factory with uncertain linguistic variables [5], the closed loop control strategy linear regulator to solve water supply system [6], the complex mixture model combinating the deterministic model and machine learning model to the water supply system fault detection [7], the water pump outlet flow adaptive controller based on Gaussian linear quadratic form [8], model predictive control of canals running process [9], the urban water supply investment optimal control [10] , and pole assignment robust control [11].

For the urban industrial water system, as the parameter may fluctuate, can produce uncertainty. And the uncertainties may not occur in a control channel, so it can be used mismatching uncertain system to describe. For this kind of system, based on Lyapunov direct method and linear matrix inequality, we design the feedback stabilization robust control scheme of urban industrial water mismatching uncertain systems. This method can avoid the defects that the feedback stabilization control method based on the matrix Riccati equation need to preset equation parameters, and increase the opportunity of obtaining the feasible solution, reduce the results conservative.

2. Nominal Model

Let urban industrial water demand function is $D(t)$, its state equation is

$$\dot{D}(t) = -\alpha D(t) + \beta R(t) \tag{1}$$

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In which, $R(t)$ is industrial added value, α is industrial water reuse rate, β is the water consumption per unit industrial added value. Due to the impact of many factors, such as economic structure, water conservation level, climate factors and water resources condition, these parameters are greatly different. In China's industrial sector, in 2006, α was about 40%, and the developed countries reached 75-85% [1]. In 2010, China's α although has increased, still only 60% - 65%. Compared with the international water advanced level, China still has a big gap [12]. The value of β has also been differences throughout the country, such as Tianjin is 0.0125 m^3 / yuan, and Tibet is 1.7811 m^3 / yuan, the latter is the former 142 times [12]. According to the "eleventh five-year " planning, for industrial departments, especially water intensive industry, such as power plants, mining, steel mills, particular emphasis will be placed on water saving technology development and innovation, its binding force goal is to reduce β 30% [1].

Let urban industrial water supply function be $S(t)$, owing to this industry as highly capital intensive [13], let its equation be

$$S(t) = \theta K(t) \tag{2}$$

In which, $K(t)$ is fixed capital stock of urban industrial water supply industry, θ is output-capital ratio of industrial water supply.

By investment theory [14], the state equation of $K(t)$ is

$$\dot{K}(t) = I(t) - \delta K(t) \tag{3}$$

In which, $I(t)$ is fixed capital investment flow of industrial water supply industry, δ is fixed capital depreciation rate of industrial water supply industry. Depreciation calculation method contain straight-line depreciation method, workload method and accelerated depreciation method, etc. The principle of straight-line depreciation method and workload method principle are similar, the former is average share of depreciation of total fixed assets in use period, and the latter is average share of depreciation of total fixed assets in the actual working hours or work within the machine-team. And accelerated depreciation method refers to all kinds of depreciation calculation method of accelerating the investment recovery rate, namely early amortization is more, later period amortization less [15]. So the selection of the depreciation method will make depreciation rate be different, produce fluctuation.

For the relationship $I(t)$ with $R(t)$, suppose water supply industry investment level to be a certain proportion of urban industrial added value, have

$$I(t) = \gamma R(t) \tag{4}$$

In which, γ is industrial water supply investment ratio. In the urban total investment of industrial construction, water supply investment is usually accounted for 3% - 5% [16].

To differentiate formula (2), and substituting formula (3), formula (4) into, we get $S(t)$ state equation as

$$\dot{S}(t) = -\delta S(t) + \theta\gamma R(t) \tag{5}$$

Simultaneous combination formula (1) and formula (5), we get urban industrial water system nominal model as

$$\begin{pmatrix} \dot{D}(t) \\ \dot{S}(t) \end{pmatrix} = \begin{pmatrix} -\alpha & 0 \\ 0 & -\delta \end{pmatrix} \begin{pmatrix} D(t) \\ S(t) \end{pmatrix} + \begin{pmatrix} \beta \\ \theta\gamma \end{pmatrix} R(t) \tag{6}$$

Let $x(t)=\begin{pmatrix} D(t) \\ S(t) \end{pmatrix}, u(t)=R(t), A=\begin{pmatrix} -\alpha & 0 \\ 0 & -\delta \end{pmatrix}, B=\begin{pmatrix} \beta \\ \theta\gamma \end{pmatrix},$ then $x(t)\in R^2, u(t)\in R^1$ is respectively state variable and control input, $A\in R^{2\times2}, B\in R^{2\times1}$ is respectively system matrix and input matrix. A,B are known constant matrix, being the nominal value of the coefficient matrix.

3. Model Uncertainty

For the urban industrial water system, due to the system parameters are possible fluctuation, can produce uncertainty. To design urban industrial water robust control scheme, need to describe the uncertainty of the system.

With $\Delta A\in R^{2\times2}$ expressed system matrix uncertainty, $\Delta B\in R^{2\times1}$ expressed input matrix uncertainty, $\Delta A, \Delta B$ is respectively bounded time-varying uncertainty. And with additive uncertainty to describe the uncertainty of the system, i.e., the uncertain system is

$$\dot{x}(t)=\left(A+\Delta A\right)x(t)+\left(B+\Delta B\right)u(t) \tag{7}$$

The system structural uncertainty problems are generally divided into two classes: matching uncertain problems and mismatching uncertain problems. Matching uncertain problem refers to uncertainty happened in control input channel, $\Delta A, \Delta B$ can be linear expressed by matrix B in mathematics. This expression makes the process be simplified, and can give some perfect control strategy. But in engineering, not all of the uncertainty occurs in the control input channel. For the control systems with actuator, uncertainties caused by system or external disturbance not occured in the control input channels, are known as mismatching uncertain system. They have more extensive practical significance and theory significance [17]. As for the urban industrial water system, the parameter α is influenced by the economic structure and the water saving level, uncertainty not necessarily occuring in the control input channel, so urban industrial water system can be used mismatching uncertain system to describe.

For mismatching uncertain system, can be used the structural uncertainty to describe the model uncertainty, that is

$$\Delta A(t)=E_1\Sigma_1(t)F_1, \Delta B(t)=E_2\Sigma_2(t)F_2, \Sigma_i^T(t)\Sigma_i(t)\le I, i=1,2 \tag{8}$$

In which, E_i, F_i are known constant matrix, expressed the structure of uncertainty. $\Sigma_i(t)$ is unknown bounded uncertain time-varying parameter or time-varying matrix, expressed the bounded time-varying perturbation in the uncertainty structure. I is the unit matrix.

To formula (7) mismatching uncertain system which meet the formula (8) structural uncertainty, the literature [18] researches of the system robust stabilization problem, this paper presents the state feedback stabilization control law related with solving matrix Riccati equation as follows:

Suppose matrix $\left(A,B\right)$ of formula (7) system can be completely controled, and satisfy the formula (8). If there are positive number $\gamma_1, \gamma_2, \varepsilon > 0$, to make the matrix Riccati equation

$$PA+A^TP+P\left(2BB^T+\gamma_1^{-1}E_1E_1^T+\gamma_2^{-1}BF_2^TF_2B^T+\gamma_2E_2E_2^T\right)P+\gamma_1F_1^TF_1+\varepsilon I=0 \tag{9}$$

have positive definite symmetric solutions $P>0$, then formula (7) system can be state feedback stabilization. The feedback control law is

$$u=B^TPx \tag{10}$$

From the above solving process, to get formula (10) feedback control law, we must solve formula (9) matrix Riccati equation, and to solve formula (9) need to preset $\gamma_1, \gamma_2, \varepsilon$ parameter value. So make the results conservative using of this method be quite large. Worst case, may even have the result: the positive definite symmetric solutions P of the problem actually existing, just because of parameter $\gamma_1, \gamma_2, \varepsilon$ improper setting, to the extent that we can't solve positive definite symmetric solutions P which meet formula (9).

4. Control System Design

In order to solve the above problems, based on the Lyapunov direct method and linear matrix inequality, we design feedback stabilization robust control scheme of urban industrial water mismatching uncertain system.

Theorem 1 For formula (7) mismatching uncertain system which meet formula (8), suppose system matrix (A, B) can be completely controled. If the linear matrix inequality about matrix Q and parameters γ_2, γ_3:

$$\begin{pmatrix} AQ + QA^T + 2BB^T + \gamma_3 E_1 E_1^T + \gamma_2 E_2 E_2^T & BF_2^T & QF_1^T \\ F_2 B^T & -\gamma_2 I & 0 \\ F_1 Q & 0 & -\gamma_3 I \end{pmatrix} < 0 \quad (11)$$

have positive definite symmetric solutions $Q > 0$ and $\gamma_2 > 0, \gamma_3 > 0$, then formula (7) system can state feedback stabilization. The feedback control law is

$$u = B^T Q^{-1} x \quad (12)$$

In which, the Q of formula (12) just is the P^{-1} of formula (10).

Proof: let $V(x) = x^T P x$ be a Lyapunov function, $u = B^T P x$, P is a positive definite symmetric matrix, then the time derivative of $V(x)$ along formula (7) system trajectory is

$$\begin{aligned} \dot{V}(x) &= x^T P \dot{x} + \dot{x}^T P x \\ &= x^T P\left[(A + \Delta A)x + (B + \Delta B)u\right] + \left[(A + \Delta A)x + (B + \Delta B)u\right]^T P x \\ &= x^T \left[P\left(A + E_1 \Sigma_1(t) F_1\right) + \left(A + E_1 \Sigma_1(t) F_1\right)^T P + P\left(B + E_2 \Sigma_2(t) F_2\right) B^T P + PB\left(B + E_2 \Sigma_2(t) F_2\right)^T P \right] x \\ &= x^T \left[PA + A^T P + 2PBB^T P + PE_1 \Sigma_1(t) F_1 + \left(E_1 \Sigma_1(t) F_1\right)^T P + PE_2 \Sigma_2(t) F_2 B^T P + PB\left(E_2 \Sigma_2(t) F_2\right)^T P \right] x \\ &\le x^T \left[PA + A^T P + P\left(2BB^T + \gamma_1^{-1} E_1 E_1^T + \gamma_2^{-1} BF_2^T F_2 B^T + \gamma_2 E_2 E_2^T\right) P + \gamma_1 F_1^T F_1 \right] x \end{aligned} \quad (13)$$

In which, γ_1, γ_2 are arbitrary positive number.

By formula (13), if

$$PA + A^T P + P\left(2BB^T + \gamma_1^{-1} E_1 E_1^T + \gamma_2^{-1} BF_2^T F_2 B^T + \gamma_2 E_2 E_2^T\right) P + \gamma_1 F_1^T F_1 < 0 \quad (14)$$

Then have $\dot{V}(x) < 0$. Formula (14) is a matrix inequality about matrix P and the parameter γ_1, γ_2. But as it contains secondary or three items multiplication of γ_1, γ_2, P and $\gamma_1^{-1}, \gamma_2^{-1}, P$, it is not the linear matrix inequality about matrix P and the parameter γ_1, γ_2, with the inconvenience directly to solve it. Yet linear matrix inequality is essentially a convex optimization problem, relatively convenient to solve using interior point method. Therefore the following try to transform formula (14) into a linear matrix inequality equation.

Formula (14) with multiplied by P^{-1} on both sides, and let

$$Q = P^{-1}, \gamma_3 = \gamma_1^{-1} \quad (15)$$

we get

$$AQ+QA^T+2BB^T+\gamma_3E_1E_1^T+\gamma_2^{-1}BF_2^TF_2B^T+\gamma_2E_2E_2^T+\gamma_3^{-1}QF_1^TF_1Q<0 \quad (16)$$

For formula (16), by the matrix Schur complement properties, we have

$$\begin{aligned}
&AQ+QA^T+2BB^T+\gamma_3E_1E_1^T+\gamma_2^{-1}BF_2^TF_2B^T+\gamma_2E_2E_2^T+\gamma_3^{-1}QF_1^TF_1Q<0\\
\Leftrightarrow &\begin{cases}\begin{pmatrix} AQ+QA^T+2BB^T+\gamma_3E_1E_1^T+\gamma_2E_2E_2^T+\gamma_3^{-1}QF_1^TF_1Q & BF_2^T\\ F_2B^T & -\gamma_2I\end{pmatrix}<0\\ -\gamma_2I<0\end{cases}\\
\Leftrightarrow &\begin{cases}\begin{pmatrix} AQ+QA^T+2BB^T+\gamma_3E_1E_1^T+\gamma_2E_2E_2^T & BF_2^T\\ F_2B^T & -\gamma_2I\end{pmatrix}+\gamma_3^{-1}\begin{pmatrix}QF_1^T\\0\end{pmatrix}\begin{pmatrix}F_1Q & 0\end{pmatrix}<0\\ -\gamma_2I<0\end{cases}\\
\Leftrightarrow &\begin{cases}\begin{pmatrix} AQ+QA^T+2BB^T+\gamma_3E_1E_1^T+\gamma_2E_2E_2^T & BF_2^T & QF_1^T\\ F_2B^T & -\gamma_2I & 0\\ F_1Q & 0 & -\gamma_3I\end{pmatrix}<0\\ -\gamma_2I<0,-\gamma_3I<0\end{cases}
\end{aligned} \quad (17)$$

Formula (17) is the linear matrix inequality group about matrix Q and parameters γ_2,γ_3. If formula (17), namely formula (11) have positive definite symmetric solutions $Q>0$ and $\gamma_2>0,\gamma_3>0$, then formula (12) feedback control law can make the formula (7) system robust stabilization. The proof is completed.

This control scheme does not need to preset any parameters, increases the opportunity of work out the feasible solution, easier to solve and can reduce the conservative.

5. Examples

An urban industrial water mismatching uncertain system conduct robust control according to the above methods. System parameter are as follows: (1) The nominal model: Industrial water reuse rate $\alpha=75\%$, the water consumption per unit industrial added value $\beta=0.50\,\mathrm{m}^3$ / yuan, output-capital ratio $\theta=0.70\,\mathrm{m}^3$ / yuan, industrial water supply investment ratio $\gamma=4\%$, capital depreciation rate $\delta=4\%$. Then we have $A=\begin{pmatrix}-0.75 & 0\\ 0 & -0.04\end{pmatrix}, B=\begin{pmatrix}0.5\\0.028\end{pmatrix}$.
(2) Model uncertainty: $\Delta A(t)=\begin{pmatrix}-0.10r_1(t) & 0\\ 0 & -0.02r_2(t)\end{pmatrix}, \Delta B(t)=\begin{pmatrix}-0.15s_1(t)\\ 0.01s_2(t)\end{pmatrix}$, In which, $r_i(t),s_i(t)$ are bounded function, have $|r_i(t)|\le 1, |s_i(t)|\le 1, i=1,2,\forall t\ge 0.$
Conducting matrix decomposition of $\Delta A(t),\Delta B(t)$, we can get

$$\Delta A(t)=\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}r_1(t)&0\\0&r_2(t)\end{pmatrix}\begin{pmatrix}-0.10&0\\0&-0.02\end{pmatrix}, \Delta B(t)=\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}s_1(t)&0\\0&s_2(t)\end{pmatrix}\begin{pmatrix}-0.15\\0.01\end{pmatrix}$$

Comparing the above formula with formula (8), we can let

$$E_1=\begin{pmatrix}1&0\\0&1\end{pmatrix},\Sigma_1(t)=\begin{pmatrix}r_1(t)&0\\0&r_2(t)\end{pmatrix},F_1=\begin{pmatrix}-0.10&0\\0&-0.02\end{pmatrix},E_2=\begin{pmatrix}1&0\\0&1\end{pmatrix},\Sigma_2(t)=\begin{pmatrix}s_1(t)&0\\0&s_2(t)\end{pmatrix},F_2=\begin{pmatrix}-0.15\\0.01\end{pmatrix}$$

and have $\Sigma_1^T(t)\Sigma_1(t)=\begin{pmatrix}r_1^2(t)&0\\0&r_2^2(t)\end{pmatrix}\le I, \Sigma_2^T(t)\Sigma_2(t)=\begin{pmatrix}s_1^2(t)&0\\0&s_2^2(t)\end{pmatrix}\le I,$ meet the condition of formula (8).

Substituting the above correlated parameter into formula (17), using Matlab calculation software, solving the linear matrix inequality equation about matrix Q and parameters γ_2,γ_3, we get a feasible solution: $Q=\begin{pmatrix}2.7772 & 0.0349\\ 0.0349 & 41.1606\end{pmatrix}, \gamma_2=1.1844>0, \gamma_3=1.1844>0$. The eigenvalue of Q

is respectively 2.7772 and 41.1606, so Q is a positive definite symmetric matrix. Substituting B,Q into formula (12), we get

$$R(t) = \begin{pmatrix} 0.1800 & 0.0005 \end{pmatrix} \begin{pmatrix} D(t) \\ S(t) \end{pmatrix} \tag{17}$$

This is known as the system state feedback control law, can make the urban industrial water mismatching uncertain system robust stabilization.

6. Conclusion

For the urban industrial water mismatching uncertain system which uncertainty do not occur in the control input channel, based on the Lyapunov direct method and linear matrix inequality, can design feedback control law, to realize the system feedback stabilization robust control. This method can avoid the defects that the feedback stabilization control method based on the matrix Riccati equation need to preset equation parameters, can increase the obtaining feasible solution opportunity and reduce the results conservative.

References

[1] Xie J. Addressing China's Water Scarcity: Recommendations for Selected Water Resources Management Issues. Beijing: China CITIC Press. 2010.

[2] Zhou Y, Wen D, Yuan F, Li JY, Li MW. Research of online water quality monitoring system based on zigbee network. *Advances in Information Sciences and Service Sciences*. 2012; 4(5): 255-261.

[3] Abduljabar ZA. Simulation and design of fuzzy temperature control for heating and cooling water system. *International Journal of Advancements in Computing Technology*. 2011; 3(4): 41-48.

[4] Liu Z. Research on rural drinking water project management. *International Journal of Advancements in Computing Tethnology*. 2012; 4(15): 387-394.

[5] Jian X, Li Y, Jiang z, Li LT. An approach to evaluating the construction projects of wastewater treatment in the paper making factory with uncertain linguistic variables. *International Journal of Digital Content Technology and its Applications*. 2012; 6(4): 95-101.

[6] Zelezik J. The closed-loop control for the water supplying system. *Annual Review in Automatic Programming*. 1985; 12(Part 2): 271-274.

[7] Izquierdo J, López PA, Martínez FJ, Pérez R. Fault detection in water supply systems using hybrid (theory and data-driven) modelling. *Mathematical and Computer Modelling*. 2007; 46(3-4): 341-350.

[8] Mourad E, Lachhab A, Limouri M, Dahhou B, Essaid A. Adaptive control of a water supply system. *Control Engineering Practice*. 2001; 9(3): 343-349.

[9] Cui W, Wang CD, Guan GH, Fan J. Model Predictive Control for Automatic Operation of Canals. *Journal of Hydraulic Engineering*. 2005; 36(8): 1000-1006.

[10] Li KB, Chen SF. Control and Simulation of Urban Water Supply and Demand Discrete System's Investment. *Journal of System Simulation*. 2010; 22(7): 1746-1751.

[11] Li KB, Chen SF. Robust Control of Urban Water Resources Effective Supply. *System Engineering*. 2007; 25(8): 58-62.

[12] Xu XY, Wang HR, Liu HJ, LI JQ, Pang B, Xu F. China Water Resources Use Efficiency Evaluation Report. Beijing: Beijing Normal University Press. 2010.

[13] Baumann DD, Boland JJ, Hanemann WM. Urban Water Demand Management and Planning. Beijing: Chemical Industry Press. 2005.

[14] Takayama A. Analytic Methods in Economics. Beijing: China Renmin University Press. 2001.

[15] Zhang ZY, Cai SH. Water Conservancy Engineering Economics. Beijing: China Waterpower Press. 2005.

[16] Liu JL, Shi XG. Water Engineering Technical and Economic Instance Analysis and Application. Beijing: Chemical Industry Press. 2004.

[17] Yang YS, Jia XL. Uncertain System Robust Control and Its Application. Dalian: Dalian Maritime University Press. 2003.

[18] Zhang SY, Gao LQ. Modern Control Theory. Beijing: Tsinghua University Press. 2010.