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A hybrid water cycle particle swarm optimization for solving the fuzzy underground water confined steady flow

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ABSTRACT

Groundwater sustainability is the development and use of groundwater resources to meet current and future beneficial uses without causing unacceptable environmental or socioeconomic consequences. This study is the first time to apply the hybrid optimization technique for solving of managing underground water aquifers, the confined steady flow problems, where a hybrid water cycle - particle swarm optimization WCA-PSO is proposed. In particular, we introduce a novel hybrid algorithm using water cycle algorithm (WCA) and particle swarm Optimization (PSO). The performance of the novel hybrid algorithm WCA-PSO is evaluated to solve 10 benchmark problems chosen from literature. The simulation results and comparison with pure WCA and PSO algorithms confirm the effectiveness of the proposed algorithm WCA-PSO for solving various benchmark optimization functions. Finally, we solve the problem of managing underground water aquifers by WCA, PSO and the hybrid optimization WCA-PSO. The experimental results analysis and statistical tests prove that the hybrid algorithm WCA-PSO overcomes the pure algorithms.

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1. INTRODUCTION

Many real-world optimization problems are very complex and challenging to solve, and many applications have to deal with these problems. To solve such problems, approximate optimization methods have to be used, though there is no guarantee that the optimal solution can be obtained [1]. Nature has been solving many problems for billions of years, and many kinds of biological systems have shown fascinating and remarkable efficiency in problem solving [2-4]. Over the last few decades optimization algorithms have been applied in extensive numbers of difficult problems. Several nature-inspired algorithms have been developed over the last few years by the scientific community [2-5].

Water management is a major challenge facing the different countries due to water increasing needs in all fields of life. More attention has been devoted to understanding and managing the transition from current management regimes to more adaptive regimes. So, we will manage underground water aquifers where it based on the finite difference approximation to the system as which treated through fuzziness environment. The uncertainty due to imprecise data may be come from indirect measurements, expert judgment, or subjective to the interpretation of available information. Also, the finite difference method is used to approximate the governing equation of groundwater flow, in which aquifer parameters such as transmissivity are to be considered as a fuzzy number. So, the variables in the system are fuzzy instead of its crisp values and then the dependent variable (e.g. hydraulic head) is also fuzzy. When the transmissivity is represented as a fuzzy number, the membership function of the hydraulic head outputs can be easily determined based on the analytical solution. At each level, both the transmissivity and hydraulic heads are transformed into intervals. Since there are not research studies which use the hybrid optimization technique for solving this problem. So, this work is the first time to apply the hybrid optimization technique for solving of managing underground water aquifers, the confined steady flow problems, where a hybrid water cycle - particle swarm optimization WCA-PSO is proposed.

Particle swarm optimization (PSO) algorithm is nature-inspired population-based metaheuristic algorithms originally accredited to Eberhart, Kennedy, and Russell Eberhart in 1995 [6]. This algorithm mimics the social behavior of birds flocking and fishes schooling. Starting form a randomly distributed set of particles (potential solutions), the algorithm try to improve the solutions according to a quality measure (fitness function). The improvisation is performed through moving the particles around the search space by means of a set of simple mathematical expressions which model some interparticle communications [7].

The water cycle process, also known as the hydrological or the H_2O cycle, explains the unceasing movement of water on, above, and below the surface of the earth. As we observe in nature, streams flow into rivers and rivers flow into the sea. Finally, all the rivers and/or streams end up in the sea, the most downhill (low-altitude) place in the world [8]. Therefore, similar to a metaheuristic swarm optimization algorithm, this phenomenon lends itself to finding a global optimal solution or a near-optimal solution via effective exploration and exploitation. Inspired by this observation, the water cycle algorithm (WCA) has been developed as a new metaheuristic algorithm [9].

In this work, we introduce a novel hybrid algorithm using water cycle algorithm (WCA) and particle swarm Optimization (PSO). The performance of the novel hybrid algorithm WCA-PSO is evaluated to solve 10 benchmark problems chosen from literature. The simulation results and comparison with pure WCA and PSO algorithms confirm the effectiveness of the proposed algorithm WCA-PSO for solving various benchmark optimization functions. Finally, we solve the problem of managing underground water aquifers by WCA, PSO and the hybrid optimization WCA-PSO. The experimental results analysis and statistical tests prove that the hybrid algorithm WCA-PSO overcomes the other algorithms.

The remaining of this paper is organized as follows: particle swarm optimization details and its procedure are described in Section 2. In Section 3, detailed descriptions of the water cycle algorithm (WCA) and their concepts are introduced. The proposed algorithm is discussed in Section 4. Benchmark functions accompanied with their mathematical formulations considered in this paper and the comparisons of the obtained statistical optimization results using the WCA-PSO with other traditional optimization algorithms PSO, WCA for reported problems in form of tables and figures are provided in Section 5. Section 6 describes the multiobjective fuzzy optimization model for aquifer management. Section 7 provides details of the solution and analysis results model for the aquifer management problem, also parameter settings of the algorithms and compares their results. Finally, conclusions are drawn in Section 8.

2. PARTICLE SWARM ALGORITHM

Particle swarm optimization (PSO) algorithm is nature-inspired population-based metaheuristic algorithms mimic the social behavior of birds flocking and fishes schooling [6, 7]. It is considered a stochastic optimization approach based on population search. These algorithms individuals, referred to as particles, are grouped into a swarm, and each particle in the swarm represents a feasible solution to the problem in the search space. The performance of each particle is measured according to a predefined fitness function which is related to the problem being solved [10]. PSO use a population of individual particles where each particle has a position, a velocity, and memory of the location of its best fitness found during the search process. Each particle updates its velocity and memory, and then the memory of other particles is shared in its neighborhood. By updating the velocity, the particle will move to a new position in the search. The main steps of the cuckoo search algorithm are summarized in Algorithm 1. The PSO in its original form is defined by [11, 12]:

$$V_{id}^{t+1} = w.v_{id}^{t} + c_1.r_{1d}^{t}\left(P_{best,i}^{t} - x_{id}^{t}\right) + c_2.r_{2d}^{t}(G_{best}^{t} - x_{id}^{t})$$
$$X_{id}^{t+1} = X_{id}^{t} + V_{id}^{t+1}\ , \quad d = 1,2,\ldots..n \tag{1}$$

where v_{id}^t and x_{id}^t the velocity and position vectors of particle i in dimension d at time t, respectively, w is representative of the inertia weight, $P_{best,i}^t$ is the personal best position of particle i, G_{best}^t is the global best position of particle i, c_1 , c_2 are positive acceleration constants which are used to level the contribution of the cognitive and social components respectively; r_{1d}^t , r_{2d}^t are random numbers from uniform distribution U(0,1) at time.

3. WATER CYCLE ALGORITHM

The WCA mimics the flow of rivers and streams toward the sea and was derived by observing the water cycle process. Assume that there are some rain or precipitation phenomena. An initial population of design variables is randomly generated after the raining process. The best individual, classified in terms of having the minimum cost function (for minimization problems), is chosen as the sea [13].

Then, a number of good streams are chosen as rivers, whereas the remaining streams flow into the rivers and the sea. Starting the optimization algorithm requires the generation of an initial population representing a matrix of streams of size $Npop \times D$, where D is the dimension and ($Npop$) is the population size. Hence, this matrix, which is generated randomly, is given as:

$$Total\,population = \begin{bmatrix} sea \\ River_1 \\ River_2 \\ \vdots \\ Stream_{Nsr+1} \\ Stream_{Nsr+2} \\ Stream_{Nsr+3} \\ \vdots \\ Stream_{Npop} \end{bmatrix} = \begin{bmatrix} x_1^1 & x_2^1 & x_3^1 & \dots & x_D^1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^{Npop} & x_2^{Npop} & x_3^{Npop} & \dots & x_D^{Npop} \end{bmatrix} \quad (2)$$

In the first step, $Npop$ streams are created. Then, a number of best individuals Nsr (minimum values) are selected as the sea and rivers. The stream which has the minimum value among the others is considered as the sea. In fact, Nsr is the summation of the number of rivers (which is defined by the user) and a single sea. The rest of the population ($Nstream$) are considered as streams flowing into the rivers or may alternatively flow directly into the sea [14].

Depending on the magnitude of the flow, each river absorbs water from streams. Hence, the amount of water entering a river and/or the sea varies from stream to stream. In addition, rivers flow to the sea, which is the most downhill location. The designated streams for each river and the sea are calculated using the following [15]:

$$NS_n = round\left\{\left|\frac{Cost_n - Cost_{Nsr+1}}{\sum_{n=1}^{Nsr} C_n}\right| \times N_{Streams}\right\},$$

$$n = 1,2,3,\dots,N_{sr} \quad (3)$$

where NS_n is the number of streams which flow into the specific rivers and the sea. For the exploitation phase of the WCA, new positions for streams and rivers have been suggested as follows [13]:

$$X_{stream}(t+1) = X_{stream}(t) + rand \times C \times \left(X_{sea}(t) - X_{stream}(t)\right) \quad (4)$$

$$X_{stream}(t+1) = X_{stream}(t) + rand \times C \times \left(X_{river}(t) - X_{stream}(t)\right) \quad (5)$$

$$X_{river}(t+1) = X_{river}(t) + rand \times C \times (X_{sea}(t) - X_{river}(t)) \quad (6)$$

where t is an iteration index, $1 < C < 2$, and the best value for C may be chosen as 2, and rand is a uniformly distributed random number between [0,1]. In (4) And (5) are for streams which flow into the sea and their corresponding rivers, respectively. If the solution given by a stream is more optimal than that of its connecting river, the positions of the river and stream are exchanged. A similar exchange can be performed for a river and the sea. The evaporation process operator is also introduced to avoid premature (immature) convergence to local optima (exploitation phase) [13]. Basically, evaporation causes sea water to evaporate as rivers/streams flow into the sea. This leads to new precipitation. Therefore, we have to check whether the river/stream is sufficiently close to the sea to enable the evaporation process to occur. The following criterion is utilized for the evaporation condition between a river and the sea [15]:

$$\left\|X_{sea}^t - X_{river_j}^t\right\| < d_{max} \quad or \quad rand < 0.1 \;\; j = 1,2,\dots,N_{sr}-1 \quad (7)$$

where d_{max} is a small number close to zer o. After evaporation, the raining process is applied and new streams are formed in different locations. Indeed, the evaporation operator is responsible for the exploration phase in the WCA. Uniform random search is used to specify the new locations of the newly formed streams. A large value for d_{max} prevents additional searches and small values encourage the search intensity near the sea. Therefore, d_{max} controls the search intensity near the sea. The value of d_{max} adaptively decreases as follows [16]:

$$d_{max}(t+1) = d_{max}(t) - \frac{d_{max}(t)}{max.iteration} \quad t = 1,2,\dots,\text{max.iteration} \tag{8}$$

For more details about the metaheuristic approach, we can see [17, 18].

4. THE PROPOSED ALGORITHM FOR OPTIMIZATION PROBLEM

In this section, we propose a new hybrid algorithm WCA-PSO is collaborative combinations of the WCA and PSO techniques. In this hybrid, firstly, WCA explores the search place in order to either isolate the most promising region of the search space. Secondly, to improve global search and avoid trapping into local optima, it is introduced PSO to explore search space (starting with the solution obtained by WCA) and find new population, which is closer to optimal solution. Further, WCA will be obtained the best model parameters vector. The structure of the hybrid WCA-PSO is shown by the following Algorithm 1.

Algorithm 1: Hybrid WCA-PSO Algorithm

Input: Objective function min or max $\boldsymbol{f}(\boldsymbol{x})$
Output: The optimal solutions for each variable and the optimal cost

Determine the initial parameters of WCA $\boldsymbol{Npop}, \boldsymbol{Nsr}$,: and Maximum Iteration.
Generate randomly initial population and Forming the initial sea, rivers and streams.
Calculate the fitness of each initial population by using $\boldsymbol{f}(\boldsymbol{x})$
Computing the corresponding flow intensity of river and sea
While ($\boldsymbol{t} <$ $\boldsymbol{Maximum\ Iteration}$)
  For i=1: Population size ($\boldsymbol{N_{pop}}$)
    Stream flows to its corresponding rivers and sea
    Calculate the objective function of the generated stream
    If F_New_Stream < F_river
      River = New_stream
      If F_New_stream < F_Sea
        Sea = New_Stream
      End
    End
    River flows to the sea
    Calculate the objective function of the generated river
    If F_New_River < F_Sea
      Sea = New_River
    End
  End
  For i=1: number of rivers ($\boldsymbol{N_{sr}}$)
    If (distance (Sea and River) < $\boldsymbol{d_{max}}$) or (rand < 0.1)
      New streams are created
    End
  End
  Reduce the $\boldsymbol{d_{max}}$
End while
Store the best solution of water cycle as the initial locations $\boldsymbol{x_i}$ of n particles
Initialize velocity $\boldsymbol{vi}$ of *n* particles.
Find $\boldsymbol{g}^*$ from objective function $\boldsymbol{f}(\boldsymbol{x})$ (at *t* = 0)
while (criterion)
  for loop over all *n* particles and all *d* dimensions
    Generate new velocity $\boldsymbol{v_{id}^{t+1}}$, Calculate new locations $\boldsymbol{X_{id}^{t+1}} = \boldsymbol{X_{id}^{t}} + \boldsymbol{V_{id}^{t+1}}$
    Evaluate objective functions at new locations $\boldsymbol{X_{id}^{t+1}}$
    Find the current best for each particle
  End for

Find the current global best g^*
Update $t = t + 1$
End while
Display result.

5. EVALUATION OF THE PROPOSED ALGORITHM WCA-PSO

The main objective of this section is the evaluation of the proposed algorithm WCA-PSO by benchmark problems. We evaluate the performance of the proposed algorithm WCA-PSO by the numerical simulation based on some Benchmark problems [17, 18] to investigate the performances of the proposed algorithms. The functions name with global optimum, search ranges and initialization ranges of the test functions are presented in Table 1. In these problems, the essential parameters of WCA are number of rivers and sea $Nsr = 4$. And the PSO constants are $C_1 = C_2 = 2$, the population size for all algorithms is 50 that are the same used for WCA-PSO algorithm. The results of all algorithms are conducted from 20 independent run for each problem. All the experiments were performed on a Windows 10 Ultimate 64-bit operating system; processor Intel Core i7 760 running at 2.40 GHz; 8 GB of RAM and code was implemented in MATLAB 2016.

From Table 2 and Figure 1, the results show that the proposed hybrid algorithm WCA-PSO overcome the traditional PSO and traditional WCA solutions. The results explain that WCA-PSO is robust and competitive with the state-of the-art well-known evolutionary algorithms. We note that the performance of WCA-PSO is significantly superior to all the present algorithms for all functions according to the experimental results. The mean and the difference between the best value and worst value of the result obtained by WCA-PSO were small compared to the results we have obtained from other algorithms in functions F03, F04, F06, F07, F08 and F10. General, the performance of WCA-PSO is highly competitive with other algorithms.

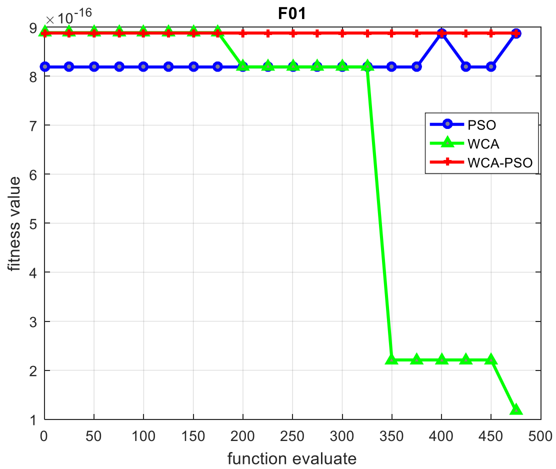
Table 1. The benchmark functions

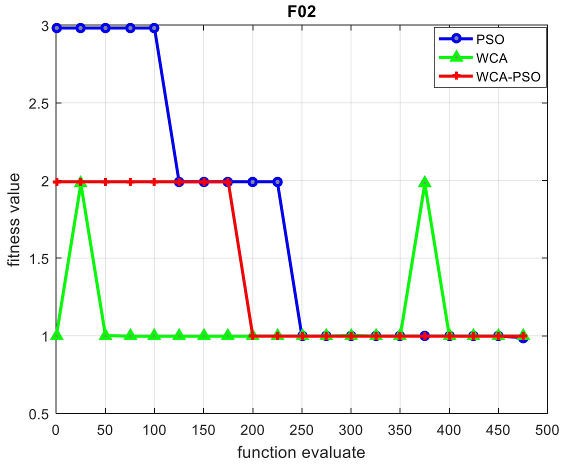
ID.	FUNCTION	DOMAIN	FORMULATION	G.M
F01	Ackley	[-32,32]	$-20\exp\left(-0.2\sqrt{\frac{1}{d}\sum_{i-1}^{N}x_i^2}\right)-\exp\left(\frac{1}{n}\sum_{i=1}^{N}\cos 2\pi x_i\right)+20+e$	0
F02	De Jongfunction N.5	[-65.54,65.54]	$\left(0.002+\sum_{i=1}^{25}\frac{1}{i+(x_1-a_{1i})^6+(x_1-a_{2i})^6}\right)^{-1}$ $a=\begin{pmatrix}-32 & -16 & 0 & 16 & 32 & -32 & \dots & 16\\ -32 & -32 & -32 & -32 & -32 & -16 & \cdots & 32\end{pmatrix}$	1
F03	Drop-wave	[-5.12,5.12]	$-\frac{1+\cos(12\sqrt{x_1^2+x_2^2})}{2+0.5(x_1^2+x_2^2)}$	-1
F04	Goldstein and Price	[-2,2]	$[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)]\times$ $\times[30+(2x_1-3x_2)^2(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)$	3
F05	Griewank	[-600,600]	$\frac{1}{4000}\sum_{i=1}^{N}x_i^2-\prod_{i=1}^{N}\cos(\frac{x_i}{\sqrt{i}})+1$	0
F06	Himmelblau	[-6,6]	$(x_1^2+x_2-11)^2+(x_2^2+x_1-7)^2$	0
F07	Rastrigrin	[-5.12,5.12]	$\sum_{i=1}^{d}[x_i^2-10\cos(2\pi x_i)+10]$	0
F08	Rotated hyper-ellipsoid	[-69.54,69.54]	$\sum_{i}^{N}(\sum_{j=1}^{i}x_j)^2$	0
F09	Schwefel	[-500,500]	$418.9829N-\sum_{i=1}^{N}(x_i\sin(\sqrt{\lvert x_i\rvert}))$	0
F10	sphere	[-5.12,5.12]	$\sum_{i=1}^{N}x_i^2$	0

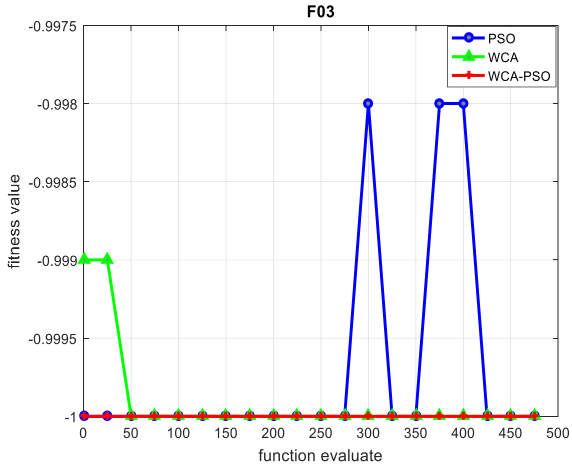
Table 2. The optimal solution results of proposed algorithm and other algorithms

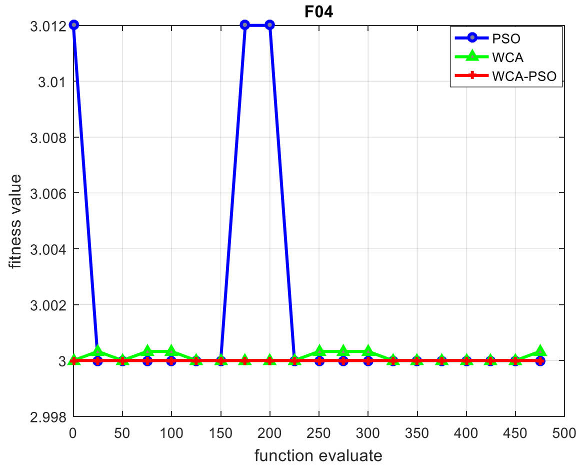
ID.	Algorithm	Min	Max	Mean	Stander Deviation
	PSO	8.19E-16	8.88E-16	8.26E-16	2.1327E-17
F01	WCA	1.17E-16	8.89E-16	8.19E-16	3.10E-16
	WCA-PSO	8.88E-16	8.88E-16	8.88E-16	1.01169E-31
	PSO	9.88E-01	2.98E+00	1.74E+00	0.844551579
F02	WCA	9.98E-01	1.98E+00	9.98E-01	0.302118
	WCA-PSO	9.98E-01	1.99E+00	9.98E-01	0.499609

	PSO	-1.00E+00	-9.98E-01	-1.00E+00	0.000732695
F03	WCA	-1.00E+00	-9.99E-01	-1.00E+00	0.00030779
	WCA-PSO	-1.00E+00	-1.00E+00	-1.00E+00	0.00E+00
	PSO	3.00E+00	3.01E+00	3.00E+00	0.004396171
F04	WCA	3.00E+00	3.00E+00	3.00E+00	0.000158
	WCA-PSO	3.00E+00	3.00E+00	3.00E+00	0.00E00
	PSO	0.00E00	7.40E-03	1.85E-03	0.003285759
F05	WCA	-7.39E-03	4.00E-01	4.74E-03	0.01976451
	WCA-PSO	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	PSO	0.00E+00	7.89E-31	1.58E-31	3.23635E-31
F06	WCA	0.00E+00	7.89E-31	0.00E+00	1.88793E-31
	WCA-PSO	0.00E+00	7.89E-31	0.00E+00	3.5E-31
	PSO	0.00 E+00	1.78E-14	2.66E-15	6.50633E-15
F07	WCA	0.00E+00	3.55E-15	1.78E-15	1.04E-15
	WCA-PSO	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	PSO	1.64E-220	9.50E-114	4.75E-115	2.1242E-114
F08	WCA	3.97E-223	4.37E-118	3.35E-218	9.65E-119
	WCA-PSO	6.43E-225	8.74E-206	4.94E-217	0.00E00
	PSO	2.55E-05	1.18E+02	4.74E+01	59.53005127
F09	WCA	2.55E-05	1.18E+02	2.55E-05	43.38946163
	WCA-PSO	2.55E-05	1.18E+02	2.55E-05	60.44877
	PSO	1.05E-228	4.08E-112	2.07E-113	9.1E-113
F10	WCA	1.29E-224	3.78E-117	2.59E-220	8.4051E-118
	WCA-PSO	4.31E-227	7.78E-209	3.58E-220	0.00 E+00









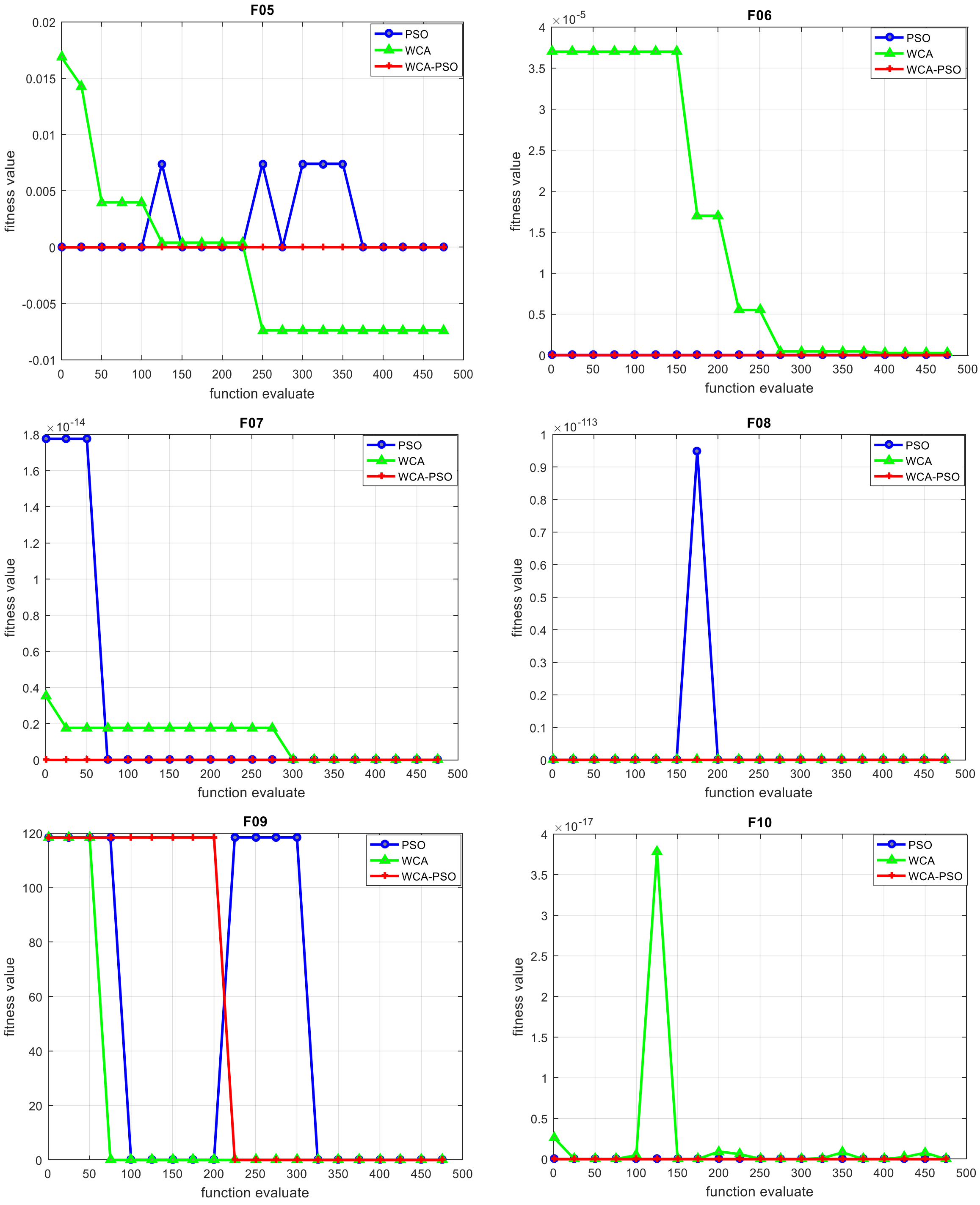


Figure 1. F01:F10 the convergence rate of the function error values on 10 function (continue)

6. MULTIOBJECTIVE FUZZY OPTIMIZATION MODEL FOR AQUIFER MANAGEMENT

O. M. Saad *et al.* [19] formulated the fuzzy multiobjective optimization model for the aquifer management, in three dimensions as follows:
(FMOM):

$$\max \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} \tilde{L}_{ijk} \tag{9a}$$

$$max \ \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} \tilde{W}_{ijk} \tag{9b}$$

$$\min \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} [\theta(\tilde{L}_{ijk})^{\delta} + \gamma + \beta] \tag{9c}$$

Subject to:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} \widetilde{W}_{ijk} \geq \text{Demand} \tag{10a}$$

$$A_{n\times n}(\widetilde{T})\tilde{L}_{n\times 1} \leq \tilde{b}_{n\times 1} + \widetilde{W}_{n\times 1} \tag{10b}$$

$$W_l \leq W^{\alpha} \leq W_u \\ \text{T, L, b, W} \geq 0 \tag{10c}$$

where $A_{n\times n}(\widetilde{T})$ is the matrix of fuzzy head coefficients which is a function of the transmissivity, $\tilde{L}_{n\times 1}$ is a fuzzy vector of unknown head values at each node, $\tilde{b}_{n\times 1}$is a fuzzy vector containing the boundary head conditions, $\widetilde{W}_{n\times 1}$ is a fuzzy vector which associated with the pumping rate, $\theta = 5543$, $\delta = 0.299$, γ is the per-well drilling cost ($/well), and β is the pump cost ($/pump), ~ represents the presence of fuzzy numbers within the matrices or vectors. Thus, model output will be expressed by membership functions that describe the head values as fuzzy variables.

Definition 1

The ∝-cut (alpha cut) is a method to generate a crisp interval corresponding to a given membership value. The crisp set contains all elements of the universal set are greater than or equal to the specified value. A ∝-cut set of triangular fuzzy number $\tilde{a} = (a^l, a^m, a^r)$ is defined as [20].

$$\tilde{a}(x) = \{x: \mu_{\tilde{a}}(x) \geq \alpha\}$$

Thus, for any $\alpha \in [0,1]$, we can obtain a ∝-cut set of triangular fuzzy number $\tilde{a}$, which is an interval, denoted by:

$$\tilde{a}(\alpha) = [a^l(\alpha), a^R(\alpha)]\ ; \\ a^l(\alpha) = \alpha a^m + (1-\alpha)a^l \\ a^R(\alpha) = \alpha a^m + (1-\alpha)a^R \tag{11}$$

where $a^l(\alpha)$ is a left number, $a^R(\alpha)$ is a right number and a^m is a mean of a^l and a^R, as shown at Figure 2.

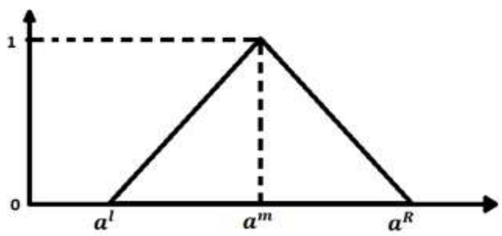


Figure 2. Triangle Membership function of fuzzy number

For certain values α_T^*, α_h^*, α_b^*, α_q^* to be in the interval [0, 1], the problem (FMOM) (9a) and (10c) can be reformulated as the following fuzzy multiobjective fuzzy optimization model for the aquifer management, in three-dimensions as follows:

$(\alpha - FMOM)$:

$$\mathbf{max} \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} \boldsymbol{L_{ijk}} \tag{12a}$$

$$\mathbf{max} \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} \boldsymbol{W_{ijk}} \tag{12b}$$

$$\mathbf{min} \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} \left[\boldsymbol{\theta}(\boldsymbol{L_{ijk}})^{\boldsymbol{\delta}} + \boldsymbol{\gamma} + \boldsymbol{\beta}\right] \tag{12c}$$

Subject to:

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l}\boldsymbol{W_{ijk}} \geq \text{Demand} \tag{13a}$$

$$\boldsymbol{A_{n\times n}(T)L_{n\times 1}} \leq \boldsymbol{b_{n\times 1}} + \boldsymbol{W_{n\times 1}} \tag{13b}$$

$$\boldsymbol{T_l} \leq \boldsymbol{T^{\alpha}} \leq \boldsymbol{T_u} \tag{13c}$$

$$\boldsymbol{L_l} \leq \boldsymbol{L^{\alpha}} \leq \boldsymbol{L_u} \tag{13d}$$

$$\boldsymbol{b_l} \leq \boldsymbol{b^{\alpha}} \leq \boldsymbol{b_u} \tag{13e}$$

$$\boldsymbol{W_l} \leq \boldsymbol{W^{\alpha}} \leq \boldsymbol{W_u} \tag{13f}$$

T, L, b, W$\geq \boldsymbol{0}$

where $T_l, T_u, L_l, L_u, b_l, b_u, W_l$ and W_u are lower and upper bounds on T, L, b and W, respectively.

In (13b) can be calculated from the partial differential equation describing the system of interest in three dimensions as follows:

$$\frac{\partial}{\partial x}\left(T\frac{\partial L}{\partial x}\right) + \frac{\partial}{\partial y}\left(T\frac{\partial L}{\partial y}\right) + \frac{\partial}{\partial z}\left(T\frac{\partial L}{\partial z}\right) = W \tag{14}$$

this can be decoded on the following equation:

$$L_{i+1,j,k} + L_{i-1,j,k} + L_{i,j+1,k} + L_{i,j-1,k} + L_{i,j,k+1} + L_{i,j,k-1} - 6L_{i,j,k} = \frac{(\Delta x)^2}{T} W_{i,j,k} \tag{15}$$

the lower bound L_l and the upper bound L_u can be calculate using the following nonlinear programming problems [19].

L_l^*:

$$\min \quad L_{i,j,k}^{\alpha}$$

Subject to

$$\begin{aligned} &A\,(T^{\alpha})\;(L)^{\alpha} = b^{\alpha} \\ &\underline{T_{i,j,k}^{\alpha}} \leq T^{\alpha} \leq \overline{T_{i,j,k}^{\alpha}} \\ &\underline{b_{i,j,k}^{\alpha}} \leq b^{\alpha} \leq \overline{b_{i,j,k}^{\alpha}} \end{aligned} \tag{16}$$

L_u^*:

$$max \quad L_{i,j,k}^{\alpha}$$

Subject to

$$\begin{aligned} &A\,(T^{\alpha})\;(L)^{\alpha} = b^{\alpha} \\ &\underline{T_{i,j,k}^{\alpha}} \leq T^{\alpha} \leq \overline{T_{i,j,k}^{\alpha}} \\ &\underline{b_{i,j,k}^{\alpha}} \leq b^{\alpha} \leq \overline{b_{i,j,k}^{\alpha}} \end{aligned} \tag{17}$$

The lower bound W_l and the upper bound W_u can be calculated using the following nonlinear programming problems:

W_l^*:

$$min \quad W_{i,j,k}^{\alpha}$$

Subject to

$$\begin{aligned} &A\,(T^{\alpha})\;(L)^{\alpha} = b^{\alpha} \\ &\underline{T_{i,j,k}^{\alpha}} \leq T^{\alpha} \leq \overline{T_{i,j,k}^{\alpha}} \end{aligned} \tag{18}$$

$$\underline{b^{\alpha}_{i,j,k}} \leq b^{\alpha} \leq \overline{b^{\alpha}_{i,j,k}}$$

W_u^*:

$$\max \quad W^{\alpha}_{i,j,k}$$

Subject to

$$\begin{aligned} &A\,(T^{\alpha})\ (L)^{\alpha} = b^{\alpha} \\ &\underline{T^{\alpha}_{i,j,k}} \leq T^{\alpha} \leq \overline{T^{\alpha}_{i,j,k}} \\ &\underline{b^{\alpha}_{i,j,k}} \leq b^{\alpha} \leq \overline{b^{\alpha}_{i,j,k}} \end{aligned} \tag{19}$$

where $\underline{T^{\alpha}_{i,j,k}}$, $\overline{T^{\alpha}_{i,j,k}}$, $\underline{b^{\alpha}_{i,j,k}}$, $\overline{b^{\alpha}_{i,j,k}}$, are the lower and upper bounds on $T^{\alpha}_{i,j,k}$,$b^{\alpha}_{i,j,k}$ respectively , T^{α} is the vector of transmissivities at the specified α-cut level, $A\,(T^{\alpha})$ is the matrix of head coefficients which is a function of T^{α}, b^{α} is the right hand side vector containing the boundary conditions and source/sink terms and L^{α} is the vector of unknown heads at the specified α- level cut. Thus, to calculate fuzzy head at a specific node two nonlinear programming problems are considered "the lower and upper bound of the unknown head can be calculated by optimization the two models mathematical, and then we find the optimal solutions using any suitable software, is obtained. For more details about the multiobjective linear and non-linear programming approach, we can see [21- 26].

7. RESULTS AND DISCUSSIONS

Supposing the leakage of flux into or out of aquifer and the well diameters are to be negligible, well losses are negligible, and the head in the well is measured from the surface of the producing layer which is considered as a horizontal datum. Input data for the simulation model includes fuzzy transmissivity values at each node, fuzzy number head boundary conditions, transmissivity of boundary nodes, the discharge rate of the well and the basic simulation parameters. The heads on boundaries are fuzzy number values of 50 to 60 m. The demand is 500 m/day and the upper bound of the total water production is 2000.

$$\Delta x = \Delta y = \Delta z = 10m,, T \in [200{,}300]\ \text{m2/day},$$

$$\gamma = 13.511\ (\$/well), \beta = 3832\ (\$/pump), \alpha \in [0{,}1]$$

Case 1

Triangle membership function $(n = m = l = 2)$,

We compute the lower and upper bound of head and pump rate for each node using the (16-19), and Set α = 0.4, using the triangle membership function and (11) to get the lower and upper bound of head and pump rate for each node, and 220 ≤ T ≤ 280, 52 ≤ b ≤ 58, Table 3 and Table 4 show the solution of the model.

Table 3. Results of water head and pumping rate for each wall

Water head (L)	Rang of water head	Rang of water head at α = 0.4	Optimal water head	Pump rate (W)	Rang of pumping rate	Rang of pumping rate at α = 0.4	Optimal pump rate
L_{111}	[0, 50.49]	[10.10, 40.40]	40.40	W_{111}	0	0	0
L_{112}	[0, 50.48]	[10.10, 40.39]	40.39	W_{112}	[0, 528.76]	[105.7,423.24]	423.24
L_{121}	[0, 50.48]	[10.10, 40.39]	40.24	W_{121}	[0, 221.25]	[44.25, 177]	177
L_{122}	[0, 50.31]	[10.06, 40.2]	40.39	W_{122}	0	0	0
L_{211}	[0, 51.40]	[10.28, 41.12]	41.12	W_{211}	[0, 400.44]	[420.35, 480]	480
L_{212}	[0, 50.76]	[10.15, 40.61]	40.61	W_{212}	0	0	0
L_{221}	[0, 50.76]	[10.15, 40.61]	40.61	W_{221}	[0, 384.95]	[76.99,307.96]	307.96
L_{222}	[0, 51.06]	[10.21,40.85]	40.85	W_{222}	[0, 464.59]	[92.9,371.68]	371.68

Table 4. Comparison among PSO, WCA and WCA-PSO on multiobjective optimization model for aquifer management

	PSO	WCA	WCA-PSO
Max sum of water head	324.61	324.61	324.61
Max total water production	1760	1760	1760
Minimum cost	165265.88	165265.88	165265.88
Number of iterations	500	100	50
CPU time (sec)	2.51	0.21	0.35

Case 2

Triangle membership function $(n = m = l = 3)$

We compute the lower and upper bound of head and pump rate for each node using the (16-19), and Set α = 0.4, using the triangle membership function and (11) to get 220 ≤ T ≤ 280 and 52 ≤ b ≤ 58. Table 5 and Table 6 show the solution of the model. Tables 3-6 show that the WCA-PSO hybrid algorithm overcomes the other optimization algorithms according to number of iterations and the CPU time for the two cases. It is also proving that increasing the CPU time when the size of the issue is greater and it will be occurs when choosing n, m and l larger.

Table 5. Results of water head and pumping rate for each wall

Water head (L)	Rang of water head	Rang of water head at α = 0.4	Optimal water head	Pump rate (W)	Rang of pumping rate	Rang of pumping rate at α = 0.4	Optimal pumprate
L_{111}	[0,57.80]	[11.57,46.28]	46.28	W_{111}	[0,487.20]	[97.44,389.7]	389.76
L_{112}	[0,57.70]	[11.54,46.16]	46.16	W_{112}	[0,324.80]	[64.96,259.8]	259.84
L_{113}	[0,57.66]	[11.53,46.13]	46.13	W_{113}	[0,487.20]	[97.44,389.7]	389.76
L_{121}	[0,57.69]	[11.54,46.15]	46.15	W_{121}	[0,324.80]	[64.96,259.8]	259.84
L_{122}	[0,57.33]	[11.47,45.86]	45.86	W_{122}	[0,162.40]	[32.48,129.9]	129.92
L_{123}	[0,57.15]	[11.43,45.72]	45.72	W_{123}	[0,324.80]	[64.96,259.8]	259.84
L_{131}	[0,57.66]	[11.53,46.13]	46.13	W_{131}	[0,487.20]	[97.44,389.7]	389.76
L_{132}	[0, 57.15]	[11.43,45.72]	45.72	W_{132}	[0,324.80]	[64.96,259.8]	259.84
L_{133}	[0, 56.63]	[11.33,45.30]	45.30	W_{133}	[0,487.20]	[97.44,389.7]	389.76
L_{211}	[0,57.70]	[11.54,46.16]	46.16	W_{211}	[0,324.80]	[64.96,259.8]	259.84
L_{212}	[0,57.3]	[11.47,45.86]	45.86	W_{212}	[0,162.40]	[32.48,129.9]	129.92
L_{213}	[0, 57.15]	[11.43,45.72]	45.72	W_{213}	[0,324.80]	[64.96,259.8]	259.84
L_{221}	[0,57.33]	[11.47,45.86]	45.86	W_{221}	[0,162.40]	[32.48,129.9]	129.92
L_{222}	[0,56.29]	[11.26,45.03]	45.03	W_{222}	0	0	0
L_{223}	[0,55.26]	[11.05,44.21]	44.21	W_{223}	[0,162.40]	[32.48,129.9]	129.92
L_{231}	[0,57.15]	[11.43,45.72]	45.72	W_{231}	[0,324.80]	[64.96,259.8]	259.84
L_{232}	[0,55.26]	[11.05,44.21]	44.21	W_{232}	[0,162.40]	[32.48,129.9]	129.92
L_{233}	[0,51.48]	[10.30,41.18]	41.18	W_{233}	[0,324.80]	[64.96,259.8]	259.84
L_{311}	[0,57.66]	[11.53,46.13]	46.13	W_{311}	[0,487.20]	[97.44,389.7]	389.76
L_{312}	[0,57.15]	[11.43,45.72]	45.72	W_{312}	[0,324.80]	[64.96,259.8]	259.84
L_{313}	[0,56.6]	[11.33,45.30]	45.30	W_{313}	[0,487.20]	[97.44,389.7]	389.76
L_{321}	[0,57.15]	[11.43,45.72]	45.72	W_{321}	[0,324.80]	[64.96,259.8]	259.84
L_{322}	[0,55.26]	[11.05,44,21]	44. 21	W_{322}	[0,162.40]	[32.48,129.9]	129.92
L_{323}	[0, 51.48]	[10.30,41.18]	41.18	W_{323}	[0,324.80]	[64.96,259.8]	259.84
L_{331}	[0, 56.63]	[11.33,45.30]	45.30	W_{331}	[0,487.20]	[97.44,389.7]	389.76
L_{332}	[0,51.48]	[10.30,41.18]	41.18	W_{332}	[0,324.80]	[64.96,259.8]	259.84
L_{333}	[0,58.00]	[11.60,46.40]	46.40	W_{333}	[0,486.20]	[97.44,389.7]	389.76

Table 6. Comparison among PSO, WCA and WCA-PSO on multiobjective optimization model for aquifer management

	PSO	WCA	WCA-PSO
Max sum of water heads (m^3)	1218.57	1218.57	1218.57
Max sum of water pumping	7015.68	7015.68	7015.68
Minimum cost ($)	571291.1	571291.1	571291.1
Number of iterations	20000	500	100
CPU time (sec.)	50.32	1.20	0.69

8. CONCLUSION

In this work, we introduced a novel hybrid algorithm using water cycle algorithm (WCA) and particle swarm optimization (PSO). The performance of the novel hybrid algorithm WCA-PSO we evaluated to solve 10 benchmark problems chosen from literature. The simulation results and comparison with pure WCA and PSO algorithms confirmed the effectiveness of the proposed algorithm WCA-PSO for solving various benchmark optimization functions. Finally, we solved the problem of managing underground water aquifers by WCA, PSO and the hybrid optimization WCA-PSO. The experimental results analysis and statistical tests proved that the hybrid algorithm WCA-PSO overcomes the other algorithms. In future work, we can improve this work by using the different metaheuristic algorithms with other mathematical models.

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