Transient analysis of systems exhibiting inverse response and their control with CSTR as a case study

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Article Info	ABSTRACT
	In this paper the phenomenon of inverse response from systems was investigated and its transient response thoroughly analyzed. Inverse response is shown by non-minimum phase systems and some minimum phase systems. Transient analysis of these systems is lacking in literature A case study was done for a non-linear, non-minimum phase CSTR (Continuous Stirred Tank Reactor), which was identified. Different control schemes were also implemented on other systems as well, which exibhit inverse responses, including model reference adaptive control. The finding shows that PID goes with the inverse response, is unable to suppress it and requires large control effort which can have serious hardware limitations. Robustness is another area where PID is lacking with these systems. MRAC shemes were able to overcome all these issues. For CSTR also, these findings hold true thus points at using advance stratigies in process control industries for maximzing product yield.

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1. INTRODUCTION

The inverse response is shown by all non-minimum phase systems, if an input is applied then the system first moves in the opposite direction before moving in the right direction. Dc-Dc boost converter, hydro turbine of hydroelectric power plants, coupled tank system [1] are some of the many non-minimum phase systems showing inverse responses. An interesting phenomenon which is found in [2] is that right hand sides zeros are not necessary for system to show inverse response. Adaptive control of aircraft with non-minimum phase zeros was performed in [3], robust control for non-minimum phase boiler [4] with LQG control, design of controller for XY non-minimum phase table was performed [5], U-model design for boieng 747 altitude control [6]. As it can be seen in literature previous works are done on specific non-minimum phase plants, inverse systems are only associated with non-minimum systems which is not true as shown in [2]. The transient analysis of inverse systems is lacking in literature, which can be very crucial. In control literature the standard non-minimum phase systems which are generally dealt assume that the system is composed of two first order systems, one of them is faster than the other and the slower process is the one which reaches its steady state value sooner, this gives rise to an inverse response. But this is not the case in most processes, system can be of higher order, have complex poles and more than one right hand side zero.

CSTR is a benchmark process in industries and shows inverse response, generally arbitrary temperature feed is given to the system which yields less efficent performance in terms of time taken for

obtaining final products in chemical reactions. Its non-linearity is also a challenge, controllers were developed in [7-11] including Sliding model [9], model predictive control [11] for CSTR, but tracking was hindered by the initial inverse response. [7-11, 12] arbitrary temperature trajectories were used, PID still plays an important role in process industries but its use for these systems is not looked at carefully. Thus, there is a need to for developing an optimized trajectory for input temperature and control schemes which are able to suppress inverse response while showing good tracking bhevaiour. In addition to the CSTR case study three other systems were also looked at and performances evaluated.

A coupled tank system which is a non-minimum phase standard plant, another non-minimum phase transfer function having complex poles and higher order system which has all of its zeros in the left half plane and still shows inverse response. Three control schemes are compared for the coupled tank system namely PI inner-outer factorization controller, Feedthrough compensator, quantitative controller. For system with complex poles and higher order system, adaptive controllers namely Adaptive PID, 1-Dof MRAC and 2-Dof MRAC are used.

Quantifying the inverse response-

Figure 1(a) shows the inverse responses which has an initial undershoot. Figure 1(b) crosses zero more than once, this behavior can be predicted –

Theorem1: If G(s) has n_+ positive zeros in the ROC and zero at $s = 0$ of multiplicity $n_o (n_+, n_o \in W)$, then output has at least $(n_+ + n_o - 1)$ or n_+ number of zero crossings. [1]

Theorem2: The system has a relative degree 1 or has at least one real pole closest to the origin, having odd number of positive zeros [1]. The magnitude of the initial undershoot can be given as $M \propto e^{-(|z_c| - |p_c|)}$, if the system satisfies Theorem 2. Where z_c and p_c are the real part of complex zero and real pole closest to the origin respectively.
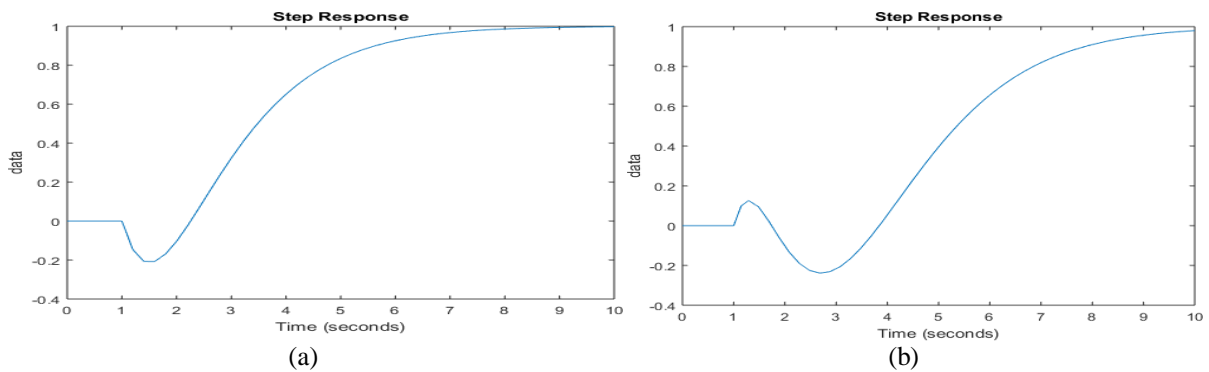

Figure 1. Inverse response of non-minimum phase system (a) one zero crossing with initial undershoot (b) multiple zero crossings

2. RESEARCH METHOD
2.1. Controller design
2.1.1. Controllers specifically for non-minimum phase systems

These controllers are for standard non-minimum phase systems such as shown in Figure 2. The tuning of the controllers has been done by taking the coupled tank system as the process/plant.
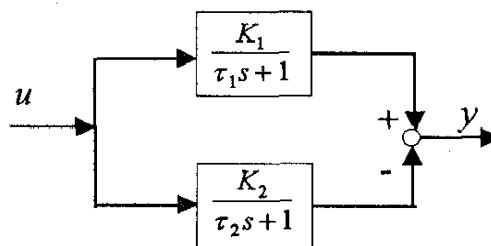

Figure 2. Standard non-minimum phase plant

a) Quantitative Controller

It is an extended version of smith's predicator based on H_∞control [13]. The controller after tuning is given by-,

$$C(s) = \frac{(25.08s^2+23.9s+1)}{(0.40552s^2+1.50552s+1)} \tag{1}$$

The plant is decomposed into two first order systems which are used in the feedback path. The compensator scheme is shown in Figure 4(a).

b) PI inner-outer factorization

This scheme is implemented to remove the non-minimum phase dynamics from the control loop [14]. It has a PI controller and two feedback compensators given by-

$$C(s) = 14.18 + \frac{3.672}{s} \tag{2}$$

$$N(s) = \frac{(-1.1s+1)}{(1.1s+1)} \tag{3}$$

$$M^{-1}(s) = \frac{1}{1+22.8s} \tag{4}$$

Where $C(s)$ is a pi controller, $N(s)$ and $M^{-1}(s)$ are part of the feedback compensation. The scheme can be seen in the Figure 4(b).

c) Feedthrough compensator

In this a compensator is chosen such that the closed loop system becomes minimum phase and with high gain the system is stabilized. The compensator chosen is $T(s) = 1/(s^2 + 5s + 2)$ and the gain K=74. The scheme [15] is shown in Figure 5(a).

2.1.2. Controllers for non-minimum phase system with complex poles and minimum phase higher order system showing inverse response
a) Adaptive PID

The approximate version of the adaptive PID is implemented which does not require any plant knowledge [16]. The adaptive law is given as-,

$$\dot{K}_P = -\gamma ey_1 \tag{5}$$

$$\dot{K}_I = -\gamma ey_2 \tag{6}$$

$$\dot{K}_D = -\gamma ey_3 \tag{7}$$

γ is the adaptive gain, e is the error signal and $y_1,$ y_2, y_3 are outputs from first three subsystems. The scheme is shown in Figure 5(b).

b) Dof model reference adaptive control

The adaptive law is given as-,

$$\dot{K}_c = -\gamma ey_m(t) \tag{8}$$

$$e = y_p - y_m \tag{9}$$

where γ the adaptive gain, $y_m(t)$ is the output from the reference model and e is the error between the plant and reference model output [17]. The controller is shown in Figure 6(a).

c) Dof model reference adaptive control

This a 2-Dof implementation of MRAC [18],

$$\dot{\theta}_1 = -\gamma_1 y_p G_m e \tag{10}$$

$$\dot{\theta}_2 = -\gamma_2 r G_m e \tag{11}$$

Where γ_1, γ_2 are the adaptive gains. y_p, r, e, G_m are the plant output, reference input, error and reference model respectively. The scheme can be seen in Figure 6(b).

The adaptive law is given by-,

$$K_c = r\theta_1 - y_p\theta_2 \tag{12}$$

2.2. System Identification of non-linear, non-minimum phase CSTR

The input signal used for the identification of real time CSTR is APRBS, as shown in Figure 3 and a transfer function was approximated using system identification toolbox in matlab.
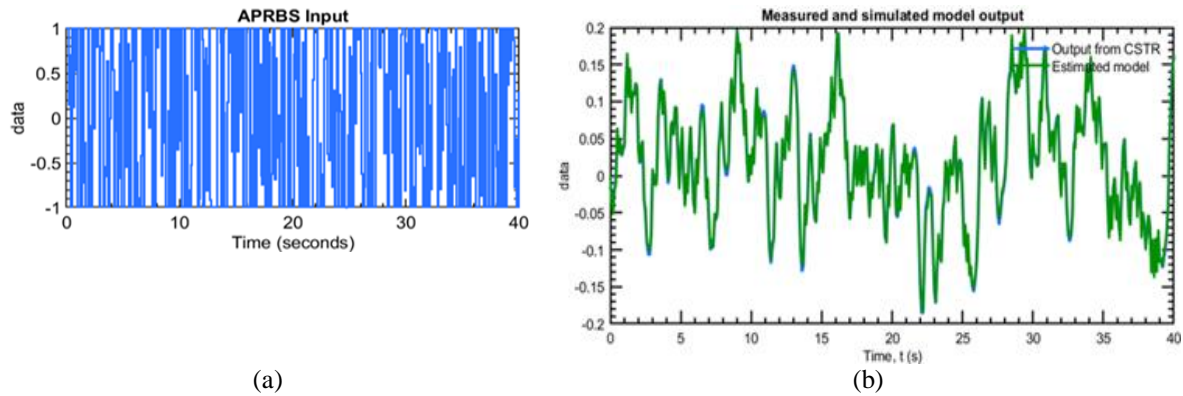

Figure 3. (a) Input APRBS signal to CSTR (b) Output measured and estimated output

The estimated transfer function is given by,

$$Gp(s) = \frac{0.4(-0.4s+1)}{(0.6s+1)(0.5s+1)} \tag{13}$$

2.3. Novel optimized temperature trajectory for CSTR using acado

An optimized trajectory is important in industries as by using this the final product in any chemical reaction can be maximized in a fixed time interval which increases efficieny as well as profits. The chemical reaction and model parameters for the differential equations for the reactor, which can be used for this CSTR system as well are given in [19].

The differential equation used for the optimization is given by-

$$\frac{dC_A}{dt} = -koe^{-\frac{ER}{T}}C_A{}^2 \tag{14}$$

$$\frac{dT}{dt} = -\frac{H}{Mcp}e^{-\frac{ER}{T}}C_A{}^2 - \frac{UA}{Mcp}(Tc - T) \tag{15}$$

This becomes a non-linear constrained dynamic optimization problem and the constraints are initial temperature, bounds in jacket temperature. The jacket temperature which is the manipulated variable in this case. But for code generation is taken as control varibable, having a different context in optimization. An overview of acado and optimization can be found in [20] which is left to the interested reader.

$$\max_u J = -g(x, u) \tag{16}$$

$$\dot{X} = f(x) \tag{17}$$

$$20°C < Tjacket < 50°C \tag{18}$$

$$T = 25°C \tag{19}$$

In (16)-(19) represents the lagrange cost function, which is to minimize the reactant A in a fixed time interval with constraints on initial temperature and allowable jacket temperature. The acado matlab code is given as,

```
BEGIN_ACADO;
acadoSet('problemname', 'CSTR');
DifferentialState ca T % defining differential states
Control Tj % control variable
c2=0.02881;
c1= -5.79; % constants
E=13550;
ko=4.7*10^19;
f = acado.DifferentialEquation();
f.add(dot(ca) == -ko*exp(-E/T)*ca*ca); % differential equations
f.add(dot(T) == -c1*ko*exp(-E/T)*ca*ca- c2*(T-Tj));
ocp = acado.OCP(0,100,100); % number of steps and final time
ocp.minimizeLagrangeTerm(ca); % minimizing the Lagrange term, final concentration of reactant A
ocp.subjectTo( f );
ocp.subjectTo( 'AT_START', ca == 0.9 ); % initial concentration of A
ocp.subjectTo( 'AT_START', T == 298.16); % initial temperature
ocp.subjectTo( 293.15 <= Tj <= 323.15 ); % jacket temperature constraint
algo = acado.OptimizationAlgorithm(ocp);
algo.set('INTEGRATOR_TOLERANCE', 1e-1 ); % tolerance for convergence
-END_ACADO;
out = CSTR_RUN(); % run command
plot(out.STATES(:,1), out.STATES(:,3))
```

2.4. Control algorithm

All control schemes are implemented using Matlab Simulink environment. The following shows the control schemes with their respective plant transfer functions,
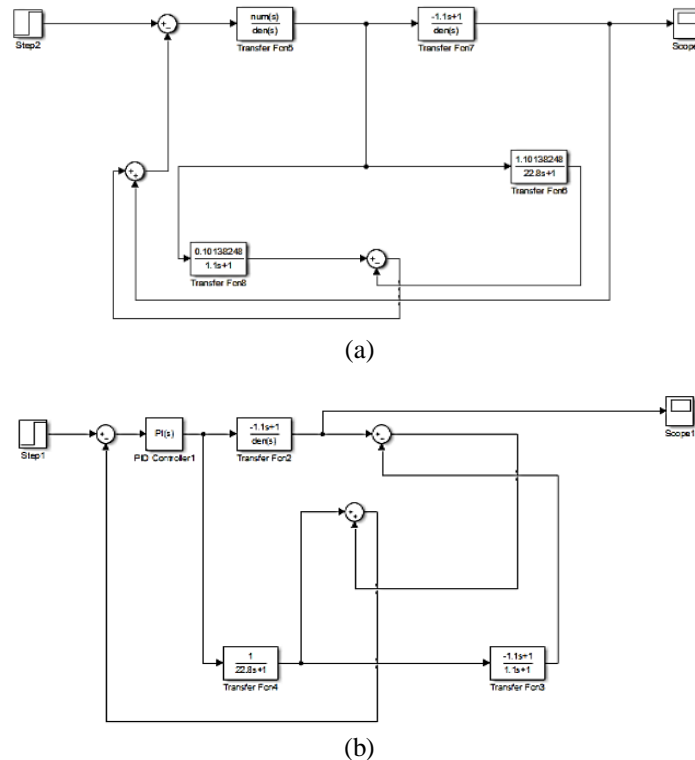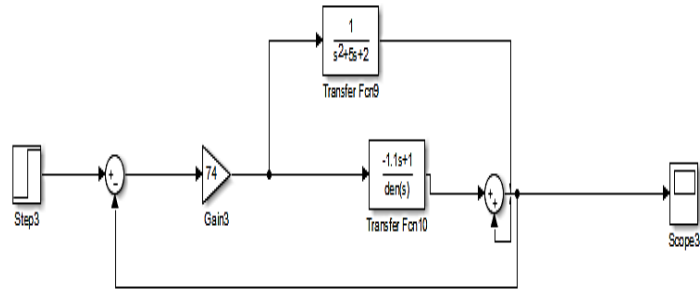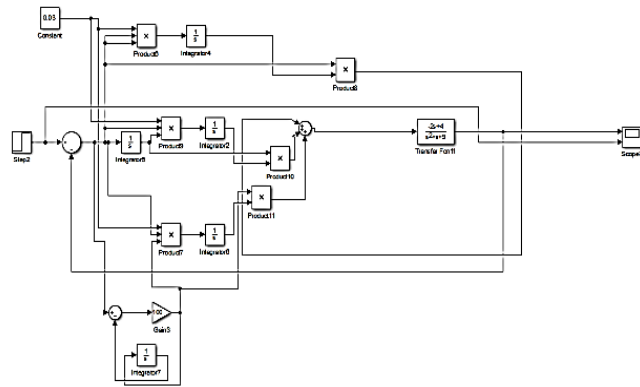

(a)

(b)

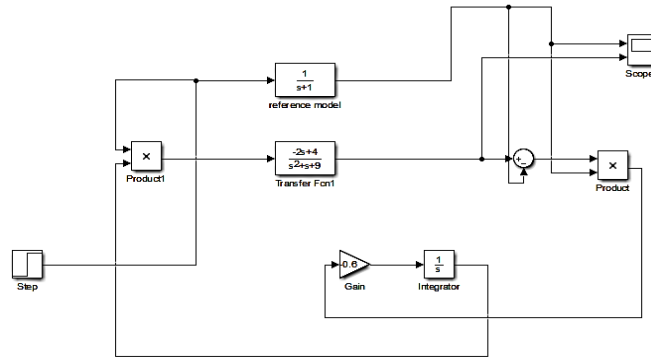Figure 4. (a) Quantitative controller (b) PI inner-outer factorization
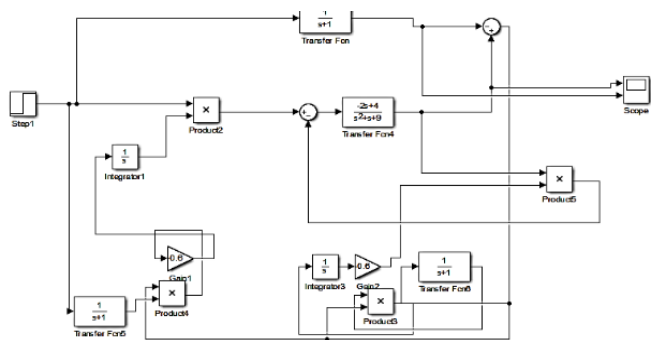
(a)

(b)

Figure 5. (a) Feedthrough compensator (b) Adaptive PID

(a)

(b)

Figure 6. (a) 1-Dof MRAC (b) 2-Dof MRAC

3. RESULTS AND DISCUSSION
3.1. Standard non-minimum phase system
The coupled tank system $G_1(s)$ [1] which is a non-minimum phase system with real roots is considered. The Transfer function is given as-,

$$G_1(s) = \frac{(1-1.1s)}{(1+1.1s)(1+22.8s)} \tag{20}$$

The controllers were tuned using the plant parameters and the PID was tuned using Ziegler–Nichols, the PID parameters are P=14.18, I=3.672, D=13.69.

3.1.1. Step response
It can be seen in Figure 7, that the feedthrough compensator gives the least initial undershoot, but due to its oscillatory behavior and overshoot, PI input-output factorization controller is chosen for this plant as a compromise is achieved between initial undershoot and system performance. PID shows a large initial undershoot in Figure 7, the performances are compared in Tables 1 and 2.
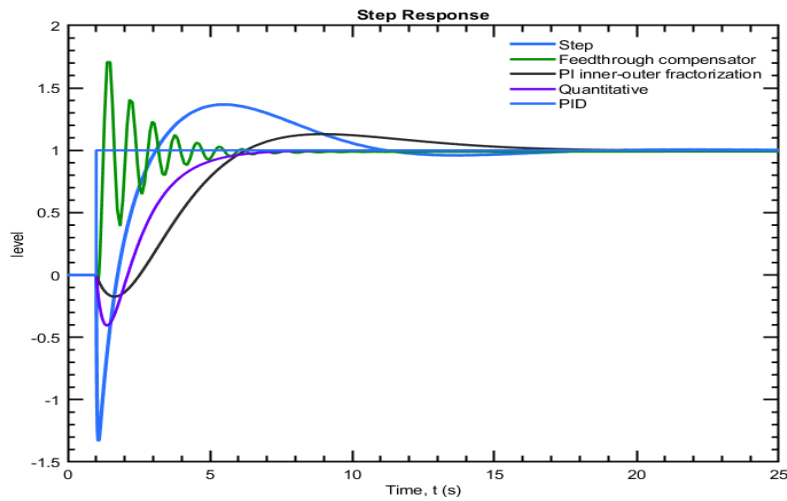

Figure 7. Step response of all control schemes with PID-ZN as a benchmark

Table 1. Comparison of system performance index with different control schemes

	PID	Feedthrough compensator	Quantitative Controller	PI inner-outer factorization
Settling time(sec)	16.2	11.6	6.8	15.8
Rise time(sec)	2.85	1.24	4.8	5.6
Overshoot(%)	36	77	0	13

Table 2. Comparison of inverse response form different controllers

	Initial undershoot	Number of zero crossing
PID	-1.34	1
Feedthrough Compensator	-0.075	1
Quantitative Controller	-0.4	1
PI inner-outer factorization	-0.174	1

3.1.2. Robustness analysis
Figure 8 shows response of the system to input disturbances for a train of pulses, PID gives the maximum deviation of 0.25 and the other controllers have negligible deviation. The poor robustness of PID can be explained due to the systems low gain margin, a problem with non-minimum phase system [21]. As the closed loop gain increases system poles move towards the righthand side zeros.
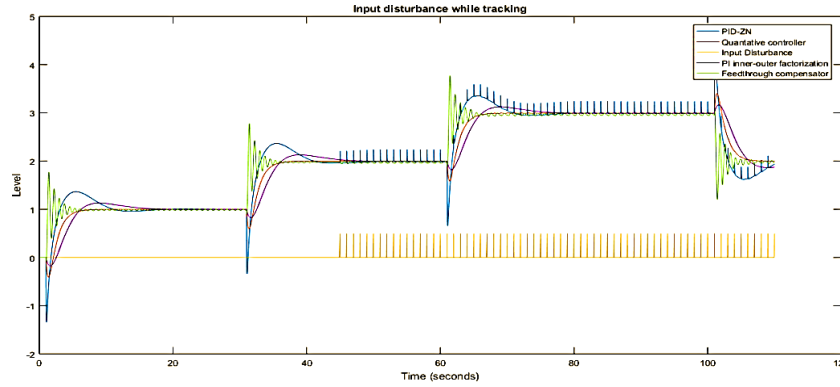
Figure 8. Input disturbance response while tracking

3.2. Non-minimum phase system with complex roots

The controllers implemented before, work on the assumption that the system is a standard system having two real poles, which is popular in control literature.

But system can be of higher order with complex poles thus adaptive schemes have been used for controlling this system. The system $G_2(s)$ is given as in [22] having complex roots-,

$$G_2(s) = \frac{(-2s+4)}{(s^2+s+9)} \tag{21}$$

The PID-ZN tuning rules for this system were P=0.6749, I=0.539, D=0.2113.

3.2.1. Step response

Adaptive controller parameters i.e. adaptive gains are given in Table 3, the reference model chosen for 1-Dof and 2-Dof MRAC is $1/(s+1)$.

Table 3. Adaptation gain values of controllers

	γ_1	γ_2
Adaptive PID	0.03	NA
1-Dof MRAC	0.6	NA
2-Dof MRAC	0.62	0.62

Step response is shown in Figure 9 and performances are compared in Table 4 and Table 5, it can be seen that there are multiple zero crossings for PID and Adaptive PID controller. PID also gives the largest initial undershoot. The best performance is obtained by 2-Dof MRAC controller.
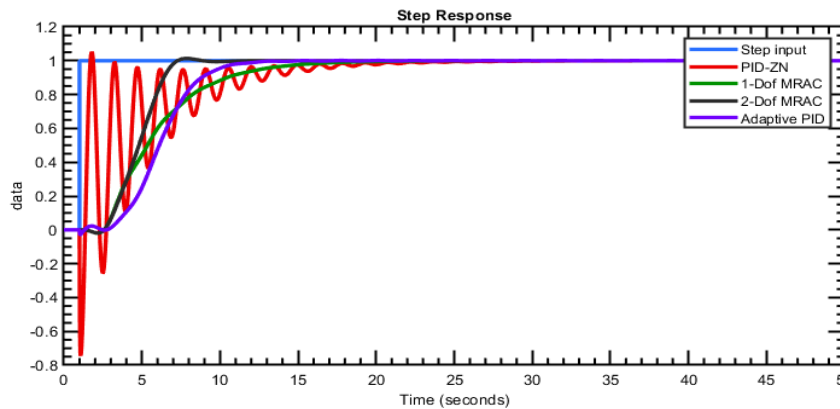

Figure 9. Step response of all adaptive control schemes with PID-ZN as a benchmark

Table 4. Comparison of inverse response form different controllers

	Maximum initial undershoot	Number of zero crossing
PID	-0.75	3
1-Dof MRAC	-0.02	1
Adaptive PID	-0.028	3
2-Dof MRAC	-0.02	1

Table 5. Comparison of system performance index with different control schemes

	PID	1-Dof MRAC	2-Dof MRAC	Adaptive PID
Settling time(sec)	35	15.4	7	11
Rise time(sec)	1.64	10.5	6.4	8.8
Overshoot(%)	5	0	1	0

3.2.2. Controller effort

The controller effort is shown in Figure 10, conventional PID requires high control effort which can be a serious limitation in hardware. The initial high control output of PID can be attributed for trying to suppress the system inverse response.
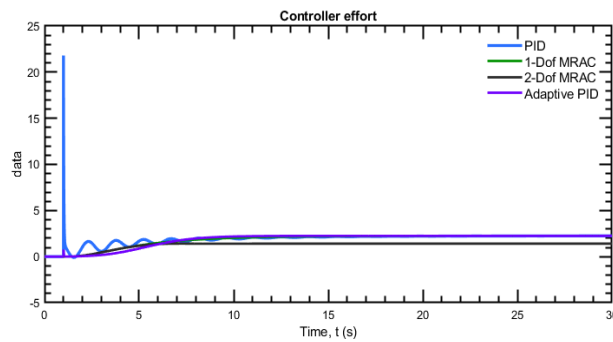

Figure 10. Control effort

3.2.3. Higher order system with zeros on the left half plane

In Figure 11(a) it can be seen that the system $G_3(s)$ shows an inverse response even though it has all zeros on the left half plane. For an inverse response the system does not necessarily have to be non-minimum phase. The system transfer function $G_3(s)$ is given as in [2]. Performance of the adaptive controllers is shown in Figure 11(b), all adaptive controllers were able to eliminate the inverse response and have zero initial undershoots.1-Dof MRAC gives the optimal performance.

$$G_3(s) = \frac{(7.688s^5+6.043s^4+9.882s^3+5.594s^2+2.088s+0.7047)}{(0.1896s^6+2.078s^5+9.12s^4+20.07s^3+22.21s^2+10.17s+0.2639)} \tag{22}$$

(a)   (b)
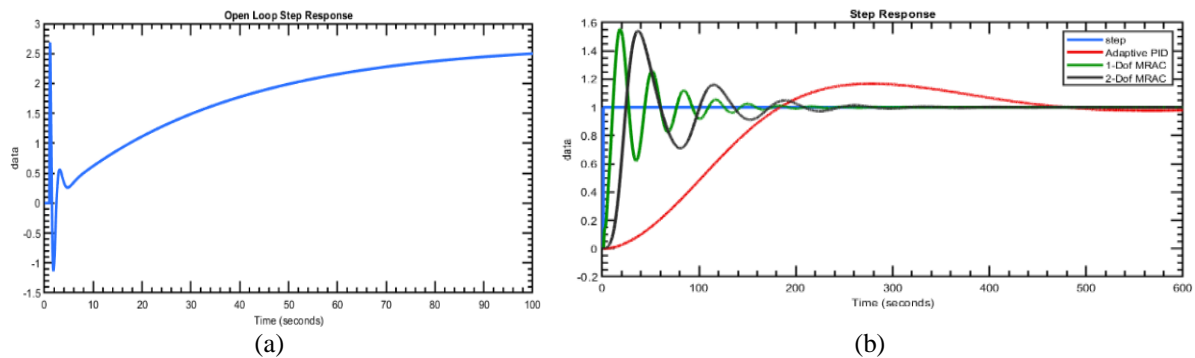
Figure 11. (a) Open loop step response of higher order system (b) Step response of higher order system with adaptive controller

Transient analysis of systems exhibiting inverse response and their control with CSTR as… (Rohan Lakhani)

3.3. A continious stired tank reactor case study, with novel optimized trajectory for temperature

Here a non-minimum phase CSTR system is identified and the optimized trajectory is used for reference tracking. The significace of an optimium trajectory can be seen in [23], thus it becomes interesting to see how a popular system in process industries showing inverse response behaves with an optimum feed, shown in Figure 12 as it has impications in terms of the time, profits and product quality. A desired trajectory was used in [24] and an optimized trajectory was formed in [25].

The values of temperature in the figure has been scaled down for representation. The performance can be seen in Figure 13, PID is unable to suppress the inverse response and tracks the reference with oscillations but other advanced schemes are giving a good response. Figure 14 and 15 shows the conversion profile of reactant A to product B, it can be seen that when optimum feed is used the conversion is much faster when compared to the feed which uses step input. The results were obtained for Figures 14 and 15 using ode45 solver in matlab.
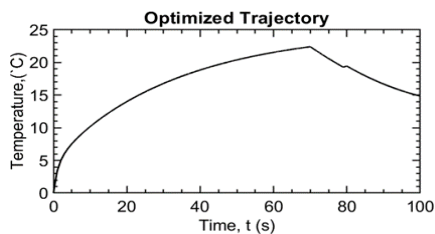

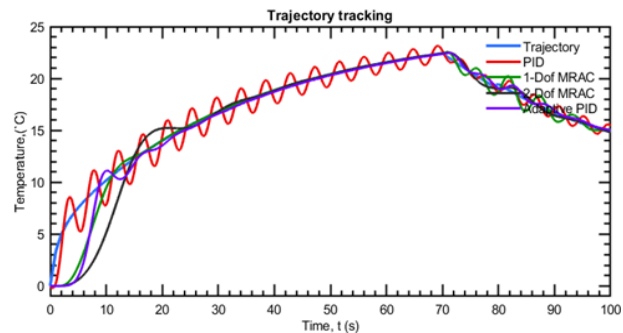Figure 12. Optimized Trajectory for non-minimum phase, non-linear CSTR

Figure 13. CSTR performance with input oprimized trajectory for tracking with different control schemes
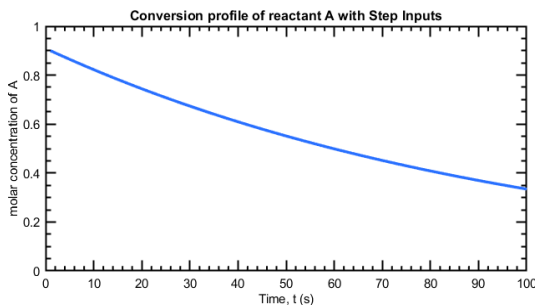

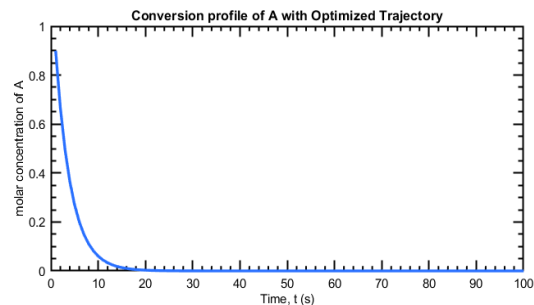Figure 14. Conversion of A during the reaction with step input

Figure 15. Conversion of A during the reaction with optimized trajectory

4. CONCLUSION

Analysis of three systems were done and performance compared, PI inner-outer factorization controller gives the best performance in terms of minimum initial undershoot and tracking performance for standard non-minimum phase coupled tank system. In adaptive control, 2-Dof MRAC gives superior performance for general non-minimum phase systems.

The novel optimum trajectory for CSTR was formed by solving a non-linear dynamic constrained optimization problem using acado and it was shown that, for a particular chemical reaction the reactant is converted to the product in a much shorter time, adaptive control was used for CSTR which suppressed the inverse response and gave good tracking performance for the trajectory. PID which is the workhorse in process industries has limitations when systems exhibiting inverse response is concerned as shown for CSTR. Robustness, high control effort and large initial undershoot are problems found with the conventional controllers and other previous schemes in laterature which were overcome using Adaptive control schemes. Tracking was not hindered by using MRAC techniques, inverse response was suppressed, maximum product was achieved in minimum amount of time with low control effort.

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