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Position control of ball and beam system using robust H_∞ Loop shaping controller

Shahad Sami Ali
Department of Computer Engineering, Al-Nahrain University, Iraq

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ABSTRACT

Laboratory Ball and Beam prototype (B&B) is a system designed to implement the controlling of space application studies such as aircraft flight and land. In this paper, to control the position of the rolling ball on the beam, MATLAB program will be used to design and implement PID and robust H_∞ Loop Shaping controllers. The open loop response of the system is unstable, because the ball continuously rolling on the beam when a constant input applied. To stabilize the system, a PID controller used first to achieve the desired position. Then, robust H_∞ Loop Shaping controller was used to achieve performance requirement for system with uncertainties. Results for the step response shows that robust H_∞ Loop Shaping controller response have no over shoot, faster about 80 times when compared to step response of PID controller, it's more effective and had better performance compared to other controllers in the control of B&B system.

Corresponding Author:

Shahad Sami Ali,
Department of Computer Engineering,
Al-Nahrain University, Iraq.
Email: shahad.sami@eng.nahrainuniv.edu.iq

1. INTRODUCTION

Ball and Beam system (BBS) is one of the most important laboratory model for control, it is a highly nonlinear, unstable open loop system [1-3]. BBS consist of beam connected from one end to a motor, and a ball rolling freely on it. By controlling the angular position of the motor, the ball position will be controlled. The system is a prototype to represent many of the complex dynamics associated with unstable system [4], such as horizontally stabilizing an airplane during landing and in turbulent airflow.

BBS is a typical under actuated system with two degree of freedom (ball position and beam angle), and one degree of actuation. The under actuated systems are low energy consuming, light weight, but they are more complicated than fully actuated systems [5]. Because of the system behavior, the ball and beam system considered as a complex system to be controlled. Many researchers consider it as an ideal prototype to learn different control strategies (classical and modern). Different techniques and controllers had been used to control the BBS, ranging from conventional controllers to intelligent controllers.

Mahmud and Rini [1] designed a two degree of freedom controller which is developed based on algebraic method. Abdulgani A. [2] in his paper presented an optimization technique (PSO algorithm) to optimize PID controller parameters to control linearized BBS, while A. I. I. et.al. [3] Applied an intelligent hybrid fuzzy controller based PID controller for different testing conditions. Anjali T. and Shyju S. M. [4] used Genetic algorithm to optimize the PID controller parameters. Adaptive dynamic surface control based on T-S Fuzzy model was presented in [5] by Yeong-Hwa. N. S. et.al. [6] Present a Fuzzy PID controller, while Reza [7] designed a PID controller, Mamdani Fuzzy Logic controller and Sugeno Fuzzy Logic Controller to control the non-linear model of the ball and beam system. M. K. and G. N. [8] used ESO based LQR controller, while

R. S. and Dr. S. [9] used Sliding mode controller. Back propagation neural network was presented by G. L. and L. Y [10].

Because of the high effectiveness of H_∞ loop shaping controller, many applications had been controlled using it. It achieves a good performance in controlling a pneumatic servosystem [11]. K. T. and E. M. [12, 13] confirm the effects of H_∞ loop shaping controller under different kinds of disturbance and prove the powerful of H_∞ loop shaping controller on the Hezarfen UAV compared to classical PID. A. Iqbal and et.al. Designed H_∞ loop shaping controller for a 2DoF Stabilized Platform [14]. M. and et.al. [15] used the controller to control 3 dof helicopter, while I. T. and et.al. Used it for autonomous helicopter hovering control [16]. Sheng and et.al. [17] used H_∞ loop shaping for route tracking control of tractor semitrailer It had been used also for positioning a pneumatic servo actuator by H. I. [18]. R. E. use it for flexible beam controlling [19]. M., R. and T. Used the controller for aircraft landing [20, 21], S. and H. used the controller for haptic System [22].

Since the ball roll on the beam, it has an acceleration, by adjusting the acceleration, the position of the ball could be controlled. The goal here is to stabilize the ball on a desired position for the longest possible time, and reach that position in shortest time by tuning the angle of the beam. PID controller will be used first to stabilize the system, then H_∞ loop shaping controller will be used to achieve better requirement and to overcome the uncertainty in ball and beam system.

2. RESEARCH METHOD

In this section, system description and mathematical model of Ball and Beam system will be present, followed by the method used to control B&B system.

2.1. System description and mathematical modelling

BBS consist of two main systems, as shown in Figure 1, the electromechanical system (DC servomotor), and mechanical system (ball and beam). The beam is connected in one end to the motor by lever arm so it could move up and down, while the other end of the beam is pinned. When the motor rotate, the lever moves up and down so that the ball roll on the beam. The motor rotate when it receive an electrical signal from controller, that lead the beam to oscillate [2]. A linear potentiometric sensor is used to get the current position of the ball [8], and feed back this position to be compared with the desired position to examine if the ball reach the desired point or not. The beam will swing even if it is nearly horizontal, without active feedback that lead to roll the ball till reach to the end of the beam [8].

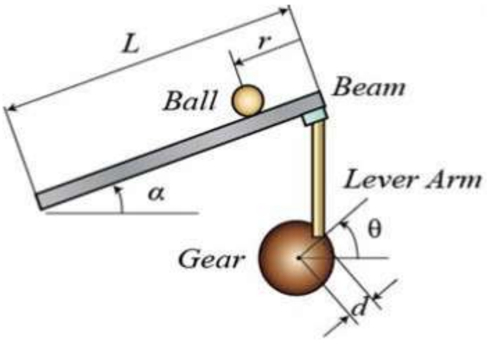

Figure 1. Illustration of ball and beam system [7]

To control the position of the ball and stop it at that desired position, the motor rotation should be controlled by adjusting the angle of the gear [9] which effect directly the beam inclination angle (α). Two different methods were used to derive the mathematical model of ball and beam system, Newton [2, 3, 9], and Lagrangian [4, 6, 7]. In this paper, a Newton's second low of motion will be considered to derive the system dynamic equation, It was assumed that the friction between ball and beam is negligible. The mathematical expression between inclination angle (α) and angular position of beam (θ) can be described as:

$$\alpha = \frac{d}{L}\theta \tag{1}$$

the movement of the ball on the beam can be expressed as:

$$\left(\frac{J_b}{R_b^2}+M\right)\ddot{r}+\ Mg\sin\alpha = Mr(\alpha)^2 \tag{2}$$

by simplification and taking Laplace to (2), the transfer function of system is:

$$\frac{R(s)}{\theta(s)} = -\frac{0.7}{s^2} \tag{3}$$

where, L is the length of the beam (cm), θ is the angular position of the beam, d is the distance between contact point and the mid point (cm), J_b is the rotational inertia of the ball (kg.m^2), R_b is the radius of the ball (m), M is the mass of the ball (kg), r is the ball position, and g is the gravity constant (m/s^2).

The final state space representation of the ball and beam system after linearizing the non-liner model around equilibrium point ($r = 0, \dot{r} = 0, \theta = 0, \dot{\theta} = 0$) is as follows [9]:

$$\dot{X} = AX + BU\ , Y = CX + DU \tag{4}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & -7.0071 & 0\\ 0 & 0 & 0 & 1\\ -24.5225 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ 49.9950\end{bmatrix}u \tag{5}$$

$$y = \begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \tag{6}$$

2.2. System behavior and controller design

BBS is unstable, controllable, observable, and nonlinear. The open loop behavior of the system is completely unstable as shown in Figure 2 for time response and Figure 3 for frequency response. It is clear that the system needs to be stabilized first, and meet a reasonable performance specification for the controlled Ball & Beam system.

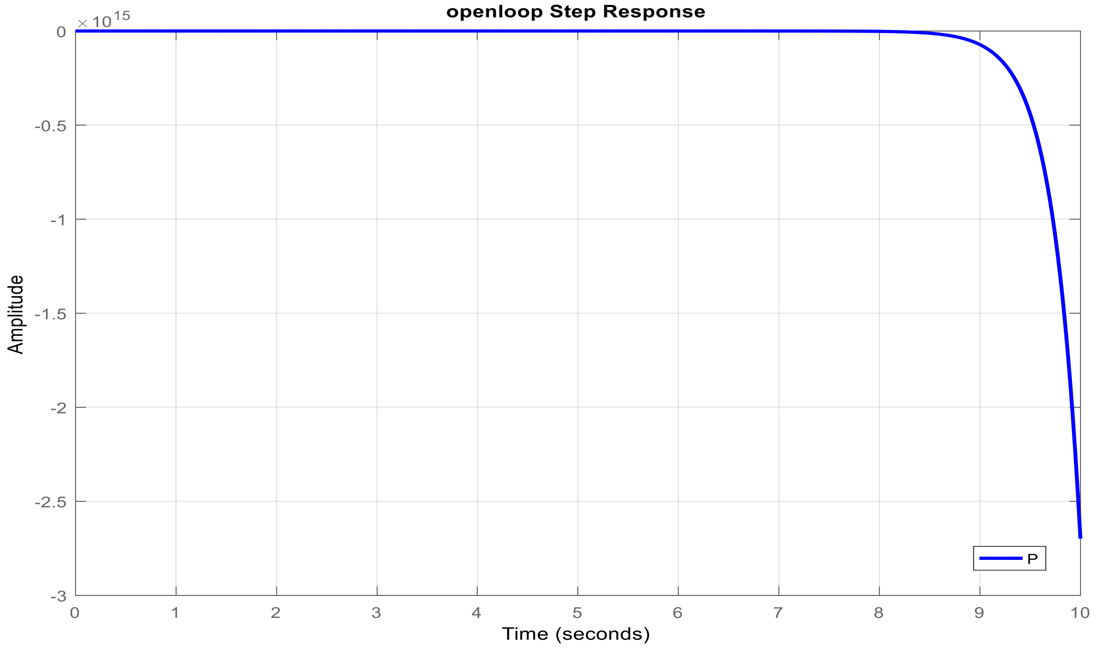

Figure 2. Open loop time domain response of the Ball and Beam system model

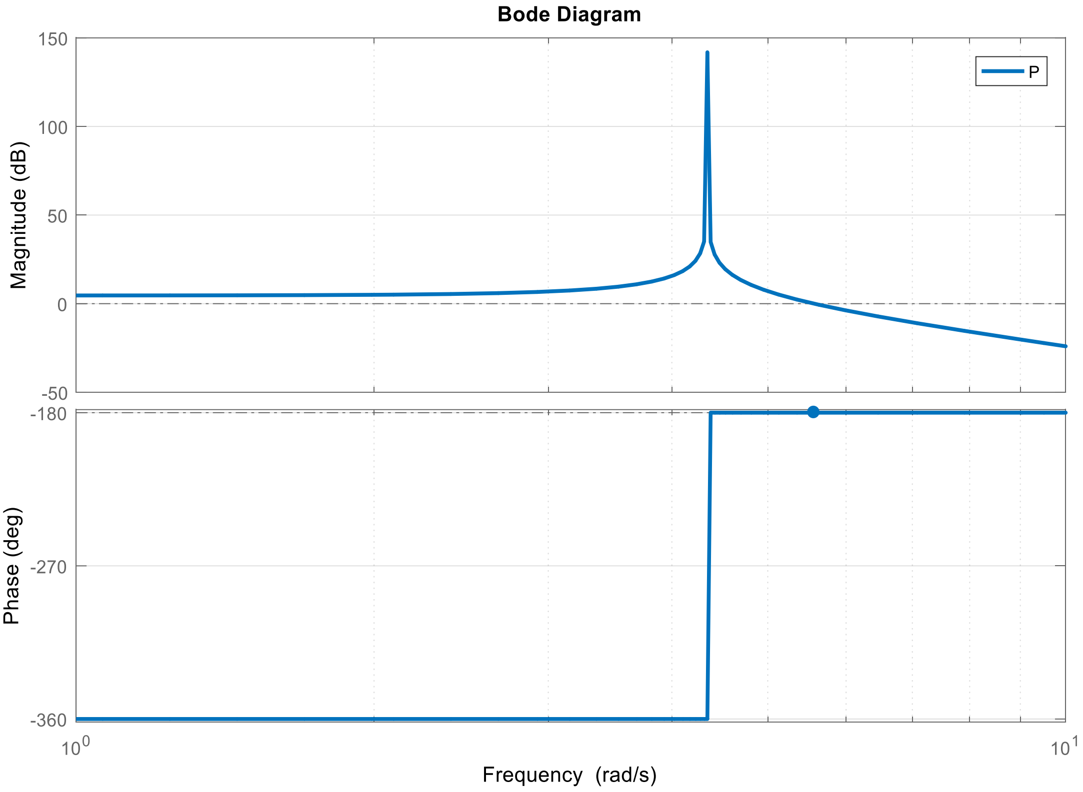

Figure 3. Open loop frequency domain response of the Ball and Beam system model

2.3. H∞ loop shaping control design

This controller combines classical loop-shaping ideas with an effective method for robustly stabilizing the feedback loop [23]. The process of designing this controller required two stages, proposed by McFarlane and Glover [23]: First, determining the pre and post compensators (W_1 and W_2) to give the desired shape to the singular value of the system frequency response, and second: robustly stabilize the resulting shaped plant with respect to coprime factore uncertainty. Figure 4 shows the block diagram of the controller designed, where W_1 and W_2 are the weights, while G is the transfer function of the system.

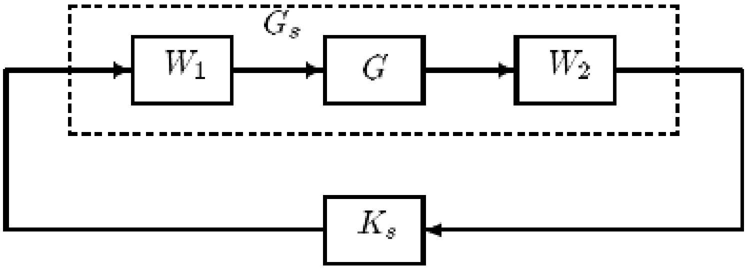

Figure 4. The shaped plant and controller [23]

To robustly stabilize the shaped plant, the normalized coprime factorization method of robust controller design should be applied [24, 25], it is important to find K(s) and γ_{min} such that:

$$\left\| \begin{pmatrix} I \\ K \end{pmatrix} (1 - GK)^{-1} \, (IW_2GW_1) \right\|_\infty = \gamma_{min} = \varepsilon_{max}^{-1} \tag{7}$$

where the minimal value is given by:

$$\gamma_{min} = \varepsilon_{max}^{-1} = \sqrt{1 + \lambda_{sup}(XY)} \tag{8}$$

The procedure of designing pre- and post-filters (W_1 and W_2) involves [24]:

a) Scaling the plant inputs and outputs to improve the design problem condition.
b) Selecting proper pre- and post- filters in such a way that the shaped plant has high gain at low frequencies.

MATLAB was used to determine the weighting functions W_1 and W_2, their values were:

$$W_1 = \frac{10(s+0.5)}{s(s+0.4)} \quad , \qquad W_2 = \frac{20(s+0.03)(s+0.5)}{s(s+1)}$$

Maximum stability margin is estimated as performance criterion and is defined as $e_{max} = 1/\gamma_{min}$, where γ_{min} has been described as H_∞ optimal cost [13]. The maximum stability margin could be found by using the (ncfsyn) command in MATLAB, e_{max} is obtained as 0.7071.

3. RESULTS AND DISCUSSION

A conventional simple PID controller is designed first to stabilize the system, it's block diagram is shown in Figure 5. We assume it's parameters as: proportional *P=-0.716*, derivative *D=-2.164*, and integral *I=-0.05*. Figure 6 represent the step response which indicates the position of ball controlled by PID controller. Although there is no steady state error, it can be noted that there is an overshoot (14.3%) which is not preferable in such systems, and the response is somewhat slow (t_s = 8 seconds). The closed loop step response for ball position of H_∞ loop shaping controlled system is shown in Figure 7.

It is clear from Figure 7, there is no over shoot and the response is very fast (settling time less than 0.1 sec.). The resulted magnitude and phase for controlled system is shown in Figure 8, it is clear that the right choice of shaping functions guaranteed the stability of the system. To test the strength of the controller, 20% uncertainty was added to the plant using the same values of W_1 and W_2. The step response for ball position after adding the uncertainty is shown in Figure 9. It can be concluded from the results shown in Table 1 as a comparison of H_∞ loop shaping controller with other controllers that the settling time for the H_∞ loop shaping controller was improved 80 times than in PID controller, at the same time the overshoot was eliminated.

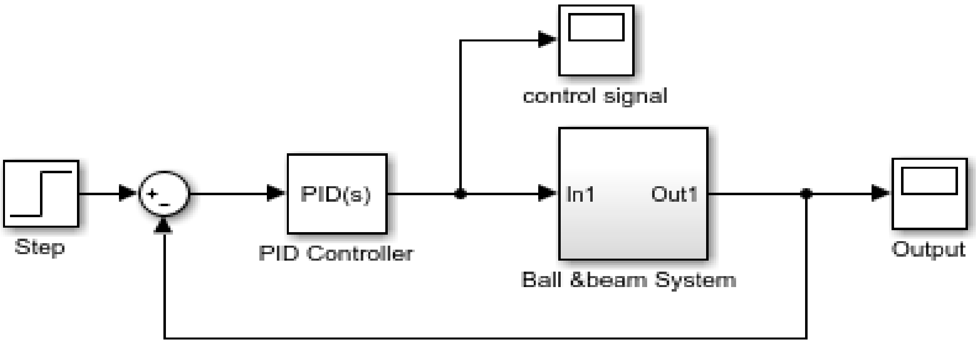

Figure 5. PID control system block

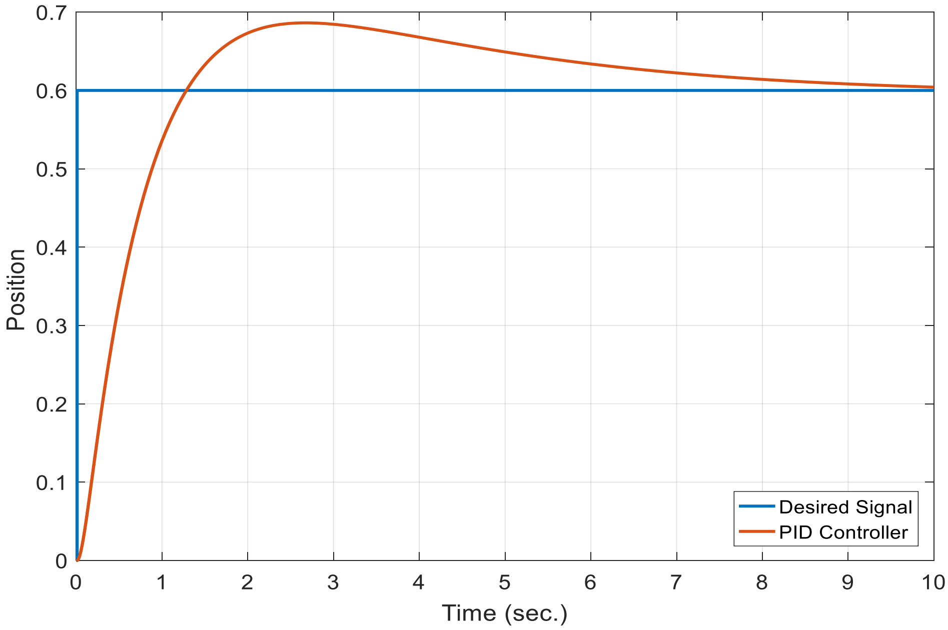

Figure 6. Step response of PID control system

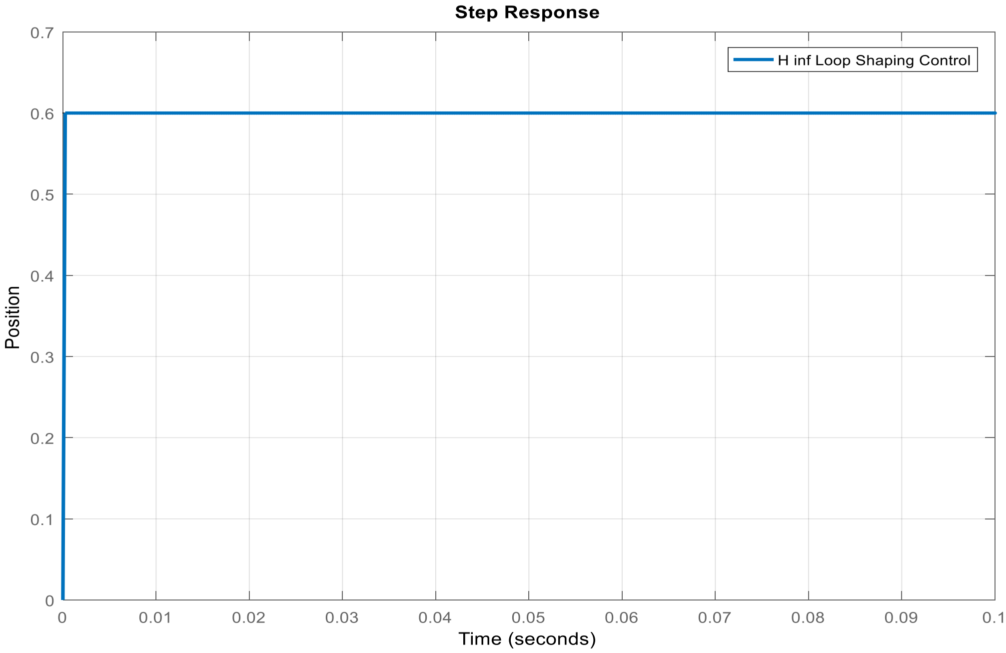

Figure 7. Step response of H_∞ loop shaping robustly controlled system

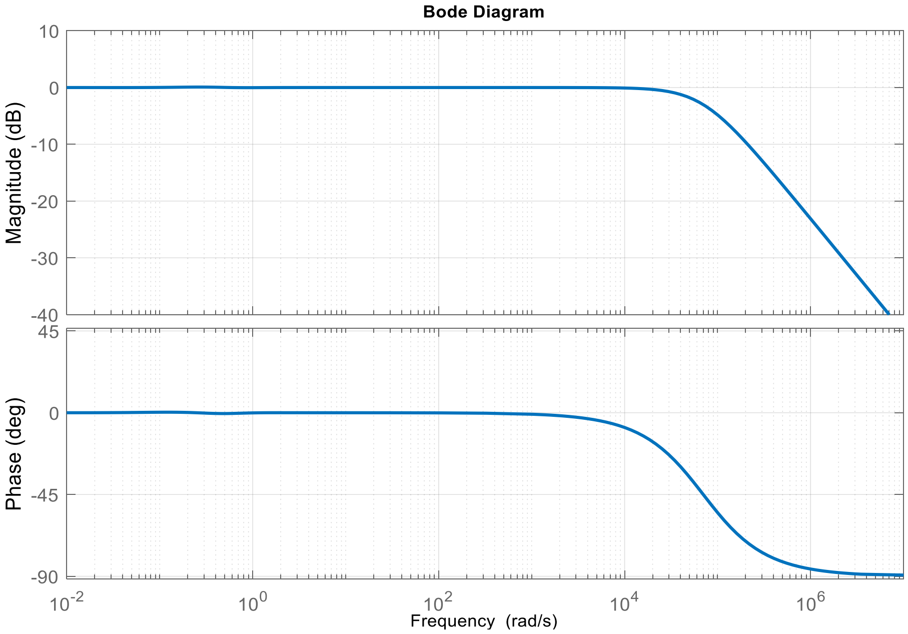

Figure 8. Bode diagram of H_∞ loop shaping robustly controlled system

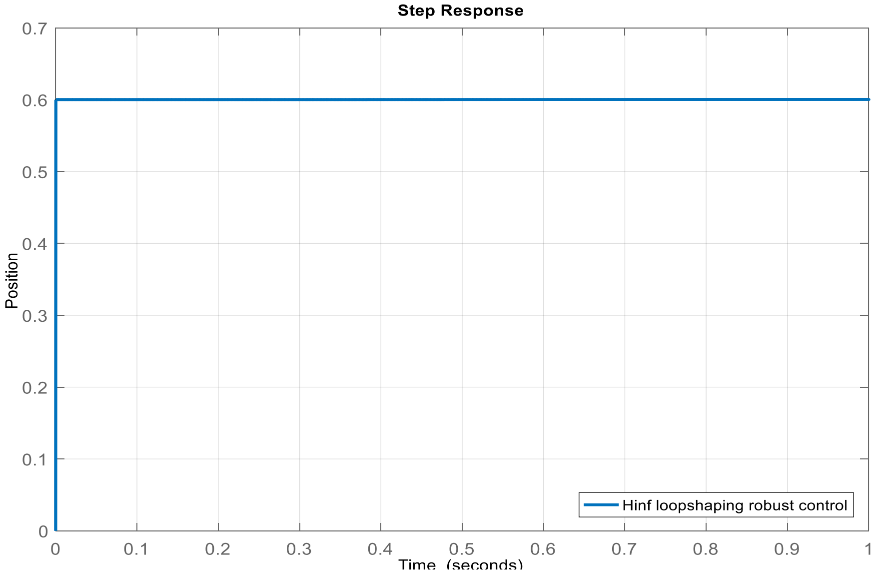

Figure 9. Step response of H_∞ loop shaping robustly controlled system with uncertainty 20%

Table 1. Step response results for different controllers

Performance Index	2 DOF Controller [1]	PSO [2]	Hybrid Fuzzy Controller [3]	Fuzzy PID [6]	PID Controller	H_∞ loop shaping controller
Settling Time (sec.)	0.19	3.7	2.2664	5.34	8	Less than 0.1
Overshoot	1.8%	3.67%	23.11%	0	14.3	0

4. CONCLUSION

(a) Although there is no error steady state, the designed PID controller did not meet the requirement of settling time and overshot. (b) The designed H_∞ loop shaping controller achieve all requirements (no over shoot and very fast response). (c) Using H_∞ loop shaping controller robustly overcome the uncertainty added to the system. (d) Robust H_∞ loop shaping controller gives better performance than different types of controllers.

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BIOGRAPHY OF AUTHORS

Shahad S. Ali, received her the B.Eng. and M.Sc. degrees in Control and System Engineering from the University of Technology-Baghdad. She is currently an Assistence lecturer in computer engineering department at Al-Nahrain university-Baghdad. Shahad has publication in Extremum seeking control and her main research interests include automation and robotics, optimization and optimal control, and robust control.