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Network Coding-Based Communications via the Controlled Quantum Teleportation

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Abstract

Inspired by the structure of the network coding over the butterfly network, a framework of quantum network coding scheme is investigated, which transmits two unknown quantum states crossly over the butterfly quantum system with the multi-photon non-maximally entangled GHZ states. In this scheme, it contains certain number of entanglement-qubit source nodes that teleport unknown quantum states to other nodes on the small-scale network where each intermediate node can pass on its received quantum states to others via superdense coding. In order to transmit the unknown states in a deterministic way, the controlled quantum teleportation is adopted on the intermediate node. It makes legal nodes more convenient than any other previous teleportation schemes to transmit unknown quantum states to unknown participants in applications. It shows that the intrinsic efficiency of transmissions approaches 100% in principle. This scheme is secure based on the securely-shared quantum channels between all nodes and the quantum mechanical impossibility of local unitary transformations between non-maximally entangled GHZ states. Moreover, the generalized scheme is proposed for transmitting two multipartite entangled states.

Keywords: *network coding, non-maximally entangled GHZ states, controlled teleportation, butterfly network, quantum information*

1. Introduction

Quantum entanglement, which has become the essential resource for quantum information and quantum computing, is the most distinctive characteristic of quantum mechanics. Entanglement-based communication has attracted much attention since the initial work of Bennett, Brassard and Ekert. For example, in 1993 Bennet et al. [1] presented the original quantum teleportation which provides a theoretical basis for the construction of quantum channels. The reason may be that the modern technology allows it to be demonstrated in laboratory, and thus practical applications will be achieved in the future [2-4].

The conventional quantum communications are sometimes limited to the quantum system in a point to point fashion with low transmission rates. In order to achieve the multi-point communications in the practical applications, the framework of the quantum network has been introduced [5-7], however the declining transmission rate caused by the bottleneck channels becomes more and more apparent [8-9].

In order to solve the afore-mentioned problem, Hayashi et al. [10] proposed the quantum network coding (QNC) scheme while concentrating on the transmission of qubits over the butterfly quantum network similar to the classical network [11-12]. However, they found that the bottleneck can not be resolved in the quantum setting and the perfect quantum state transmission is impossible. That means one can not perform the perfect QNC in the quantum system. Fortunately, since entanglement provides several elegant approaches for enhancing efficiency of quantum communications such as superdense coding [13], entanglement swapping and teleportation [1], it is necessary to consider its applications in the QNC system. For example, Hayashi [14] suggested an improved butterfly network that allows the QNC for the transmission of one particle or two bit classical communication. Kobayashi et al. [15] proposed Constructing quantum network coding schemes from classical nonlinear protocols.

The maximal entanglement between two senders enables perfect quantum transmission though it is impossible without prior entanglement even in the modification of the

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previous QNC scheme [10]. In 2009, under the assumption that there is no prior entanglement shared among any of the parties, Kobayashi et al. [16] put forward a perfect QNC which can perfectly transfer an unknown quantum state from source subsystem to target subsystem, where both source and target are formed by the ordered sets of nodes. In 2010, Ma et al. [17] proposed an improved QNC via probabilistic teleportation based on the maximally entangled photons shared beforehand.

Recently, teleportation has been actively investigated in both theoretics and experiments. With teleportation Alice can transmit an unknown quantum state to a remote recipient Bob [18-20]. In general, a pair of maximally entangled Bell states served as a secure quantum channel should be prepared in advance, and the sender can not always teleport a single-qubit to the receiver with unit fidelity and unit probability. Consequently, there is another kind of teleportation called as probabilistic teleportation [21], from which one can achieve the fidelity with a probability less than unit. In any case, teleportation has offered a much powerful method while transmitting an unknown quantum state. Unfortunately, the universal processor based on teleportation can not work for many participants transmitting different unknown states simultaneously, and thus we are faced with a new challenge in quantum communication that extends to global quantum communication networking. This work has been devoted to quantum networking that places emphasis on basic quantum effects and on emerging technological solutions leading to practical applications in quantum communications.

In order to enhance the transmission rate of quantum network system, a novel teleportation-based QNC scheme is proposed to transmit two unknown states crossly over the butterfly network where two senders prepares two Greenberger-Horne-Zeilinger (GHZ) states in advance. While performing the controlled quantum teleportation [19-21] at intermediate node, two receivers can restore the original quantum states with probability 100%. Comparing with the previous QNC [17], the performed operation at the intermediate node over the butterfly network is for quantum operation, instead of the classical operation. At the receivers, with the help of the transmitted unitary operations, the initial quantum states can be restored completely.

The paper is organized as follows. In section 2, for the description of QNC, some basic properties of the controlled quantum teleportation are presented with simplicity. In section 3, the QNC is proposed explicitly to transmit unknown states on the basis of teleportation in terms of superdense coding at the intermediate node crossly over the butterfly network. Then the present QNC is generalized to transmit two multipartite entangled states in section 4. The security analysis is illustrated on the basis of channel detection scheme in section 5. Finally, the discussion and summary are presented in section 6.

2. Controlled Quantum Teleportation with Entanglement Analysis

The three-photon-entangled Greenberger-Horne Zeilinger states (GHZ states) have formed the basis of a very stringent test of local realistic theories. They can be used for cryptographic scenario or for multi-photon generations of super-dense coding to reduce communication complexity. It is known that GHZ states are no long a theoretical imagery since they can be experimentally implemented by single photons in the hybrid entanglement states.

In the controlled quantum teleportation [19-21], a third participant who may take part in the process of quantum teleportation as a supervisor is included in order to achieve the transportation. Without the assistance of the control operations, the receiver cannot recover the original state from the sender.

Assuming that three participants, Alice, Bob and Charlie, prepare a non-maximal entanglement GHZ state beforehand, i.e.,

$$|GHZ\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{ABC}. \quad (1)$$

An arbitrary unknown state $|\psi\rangle_D$ on photon D is given by

$$|m\rangle = \alpha|0\rangle + \beta|1\rangle. \quad (2)$$

To accomplish the transformation,Alice performs the Bell state analysis on the particles D and A

$$|\Xi\rangle = |m\rangle \otimes |GHZ\rangle_{ABC}, \quad (3)$$

which can be rewritten as

$$|\Xi\rangle = \frac{1}{2}(|\Phi^{+}\rangle_{mA} \otimes (\alpha|00\rangle + \beta|11\rangle)_{BC} + |\Phi^{-}\rangle_{mA} \otimes (\alpha|00\rangle - \beta|11\rangle)_{BC} + |\Psi^{+}\rangle_{mA} \otimes (\beta|00\rangle + \alpha|11\rangle)_{BC} + |\Psi^{-}\rangle_{mA} \otimes (-\beta|00\rangle + \alpha|11\rangle)_{BC}), \quad (4)$$

where $|\Phi^{\pm}\rangle$ and $|\Psi^{\pm}\rangle$ denote the Bell-states given by

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle \pm |1\rangle|1\rangle), \quad |\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle \pm |1\rangle|0\rangle) \quad (5)$$

After performing the Bell-state measurement on particles *D* and *A*, the original entanglement state can be transformed into the state on the pair of particles *B* and *C*. With the help of some unitary operations, the resulting state can be transformed in the form

$$|\psi\rangle_{BC} = (\alpha|00\rangle + \beta|11\rangle)_{BC}, \quad (6)$$

from which the receiver Bob can recover the original particle $|\psi\rangle$ with the assistance of Charlie'scontrol operation performed on photon *C.* If Charlie would like to help Bob for the recovery of $|\psi\rangle$, he measure the particle *C* on the bases of $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ .The entanglement state of particles *B* and *C* can be represented as

$$|\psi\rangle_{BC} = \frac{1}{\sqrt{2}}((\alpha|0\rangle + \beta|1\rangle)_{B} \otimes |+\rangle_{C} + (\alpha|0\rangle - \beta|1\rangle)_{B} \otimes |-\rangle_{C}). \quad (7)$$

When Bob receives Charlie's measurement results via a classical channel, he performs an appropriate unitary transformation to recover the original state conveniently. This controlled teleportation over the butterfly quantum network crossly is implemented.

3. QNC via Controlled Teleportations of Two Unknown States

In this section, we elaborate on our QNC scheme in which two unknown states are transmitted from the senders A_i to the receivers $B_i (i = 1, 2)$ crossly over the butterfly quantum network. Namely receivers B_1 and B_2 can obtain two unknown states $|m_1\rangle$ and $|m_2\rangle$ with the certainty. The topology of this quantum network is similar to that of the classical butterfly network, as illustrated in Figure 1.

Two unknown states $|m_i\rangle, \forall i \in \{1, 2\}$, which will be transmitted from A_i to B_i, are denoted by

$$|m_i\rangle = \alpha_i|0\rangle + \beta_i|1\rangle, \quad (8)$$

where $|\alpha_i|^2 + |\beta_i|^2 = 1$. Two senders A_1 and A_2 prepare jointly two GHZ states given by

$$|GHZ_{A_{11},A_{21},G_1}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_{11},A_{21},G_1}, \quad (9)$$

$$|GHZ_{A_{22},A_{12},G_2}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_{22},A_{12},G_2}, \quad (10)$$

where one sender A_1 possesses photons A_{11}, A_{12} and G_1, while another A_2 keeps photons A_{21}, A_{22} and G_2. Six EPR pairs $|\Phi^{+}\rangle$ are shared between the intermediate node C and sender A_i, as well as receiver $B_{i\otimes 1}$. In addition, four EPR pairs shared by C and $B_{i\otimes 1}$ are denoted by $|\Phi_1\rangle$, $|\Phi_2\rangle$, $|\Phi_3\rangle$, and $|\Phi_4\rangle$, respectively, where $\oplus$ represents operation of plus mod 2.

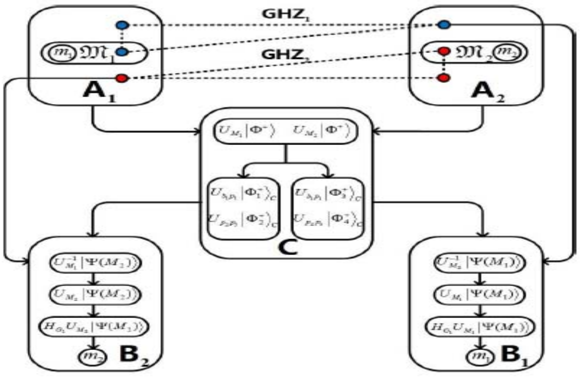
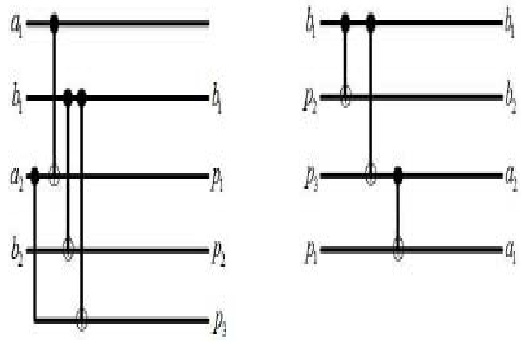


Figure 1. The protocol for transmit two quantum states. m_i is the particle which ought to be sent. The coding operation will be performed on the relay node C.



Encoding circuit Decoding circuit

Figure 2. The encoding circuit and the decoding circuit at the relay node C

Table 1. The result of the Operation $U_{M_{i\oplus1}}|\Psi(M_{i\oplus1})\rangle$

$U_{M_{i\oplus1}}\|\Psi(M_{i\oplus1})\rangle$	Operation result
$U_{01}\|\Psi(01)\rangle$	$(\alpha_i\|00\rangle+\beta_i\|11\rangle)_{A_{i\oplus1,i}G_i}$
$U_{00}\|\Psi(00)\rangle$	$(-\beta_i\|01\rangle+\alpha_i\|10\rangle)_{A_{i\oplus1,i}G_i}$
$U_{10}\|\Psi(10)\rangle$	$(\beta_i\|00\rangle-\alpha_i\|11\rangle)_{A_{i\oplus1,i}G_i}$
$U_{11}\|\Psi(11)\rangle$	$(\alpha_i\|01\rangle+\beta_i\|10\rangle)_{A_{i\oplus1,i}G_i}$

Table 2.The result of the operation $H_{G_{i\oplus1}}U_{M_{i\oplus1}}|\Psi(M_{i\oplus1})\rangle$.

$H_{G_{i\oplus1}}U_{M_{i\oplus1}}\|\Psi(M_{i\oplus1})\rangle$	Operation result
$H_{G_{i\oplus1}}U_{00}\|\Psi(00)\rangle$	$\frac{1}{\sqrt{2}}[(\alpha_{i\oplus1}\|0\rangle+\beta_{i\oplus1}\|1\rangle)\|0\rangle$ $+(\alpha_{i\oplus1}\|0\rangle-\beta_{i\oplus1}\|1\rangle)\|1\rangle]_{A_{i,i\oplus1}G_{i\oplus1}}$
$H_{G_{i\oplus1}}U_{01}\|\Psi(01)\rangle$	$\frac{1}{\sqrt{2}}[(-\beta_{i\oplus1}\|0\rangle+\alpha_{i\oplus1}\|1\rangle)\|0\rangle$ $+(\beta_{i\oplus1}\|0\rangle+\alpha_{i\oplus1}\|1\rangle)\|1\rangle]_{A_{i,i\oplus1}G_{i\oplus1}}$
$H_{G_{i\oplus1}}U_{10}\|\Psi(10)\rangle$	$\frac{1}{\sqrt{2}}[(\beta_{i\oplus1}\|0\rangle-\alpha_{i\oplus1}\|1\rangle)\|0\rangle$ $+(\beta_{i\oplus1}\|0\rangle+\alpha_{i\oplus1}\|1\rangle)\|1\rangle]_{A_{i,i\oplus1}G_{i\oplus1}}$
$H_{G_{i\oplus1}}U_{11}\|\Psi(11)\rangle$	$\frac{1}{\sqrt{2}}[(\alpha_{i\oplus1}\|0\rangle+\beta_{i\oplus1}\|1\rangle)\|0\rangle$ $+(-\alpha_{i\oplus1}\|0\rangle+\beta_{i\oplus1}\|1\rangle)\|1\rangle]_{A_{i,i\oplus1}G_{i\oplus1}}$

Table 3. The Unitary Operation Which be performed on the particle $A_{i,i\oplus1}$ here $M_i\in(00,01,10,11)$

	00	01	10	11
V_{M_i}	$\begin{pmatrix}1&0\\0&1\end{pmatrix}$	$\begin{pmatrix}0&1\\-1&0\end{pmatrix}$	$\begin{pmatrix}0&-1\\1&0\end{pmatrix}$	$\begin{pmatrix}1&0\\0&1\end{pmatrix}$
W_{M_i}	$\begin{pmatrix}1&0\\0&-1\end{pmatrix}$	$\begin{pmatrix}0&1\\1&0\end{pmatrix}$	$\begin{pmatrix}0&1\\1&0\end{pmatrix}$	$\begin{pmatrix}-1&0\\0&1\end{pmatrix}$

Then the present QNC goes as follows.

Step 1 Two senders A_i prepare unknown states $|m_i\rangle$ and GHZ states $|GHZ\rangle_{A_{1i},A_{i\oplus 1,i},G_i}$ in Eq.(9). Then the combined quantum systems can be denoted as

$$|\Phi\rangle_{m_i,A_{i,i},A_{i\oplus 1,i},G_i}=|m_i\rangle\otimes|GHZ\rangle_{A_{i,i},A_{i\oplus 1,i},G_i}, \quad (11)$$

which can be rewritten as

$$\begin{aligned}|\Phi\rangle_{m_iA_{i,i}A_{i\oplus 1,i}G_i}&=\frac{1}{2}(|\Phi^+\rangle_{m_iA_{i,i}}\otimes(\alpha_1|00\rangle+\beta_1|11\rangle)_{A_{i\oplus 1,i}G_i}+|\Phi^-\rangle_{m_iA_{i,i}}\otimes(\alpha_1|00\rangle-\beta_1|11\rangle)_{A_{i\oplus 1,i}G_i}\\&+|\Psi^+\rangle_{m_iA_{i,i}}\otimes(\beta_1|00\rangle+\alpha_1|11\rangle)_{A_{i\oplus 1,i}G_i}+|\Psi^-\rangle_{m_iA_{i,i}}\otimes(-\beta_1|00\rangle+\alpha_1|11\rangle)_{A_{i\oplus 1,i}G_i}).\end{aligned} \quad (12)$$

Step 2 Each sende A_i performs the Bell-state measurement on particles $|m_i\rangle$ and $A_{i,i}$ with four respective Bell states $\{|\Phi^{\pm}\rangle,|\Psi^{\pm}\rangle\}$ corresponding to results given by $M=\{\phi_{00},\phi_{01},\phi_{10},\phi_{11}\}$. The entangled state of the remaining particles $A_{i\oplus 1,i}$, and G_i can be denoted as $\{|\Psi(M_i)\rangle : M_i\in\{00,01,10,11\}\}$. Subsequently, the inverse unitary operation $U_{M_i}^{-1}$ will be applied on particle $A_{i,i\oplus 1}$ of the resulting state $|\Psi(M_{i\oplus 1})\rangle$, where unitary operations U_{M_i} are given as

$$\begin{aligned}&U_{00}=|0\rangle\langle 0|+|1\rangle\langle 1|, U_{01}=|1\rangle\langle 0|+|0\rangle\langle 1|,\\&U_{10}=|0\rangle\langle 0|-|1\rangle\langle 1|, U_{01}=|0\rangle\langle 1|-|1\rangle\langle 0|\end{aligned} \quad (13)$$

It is easy to prove that

$$U_{M_i}U_{M_{i\oplus 1}}=\pm U_{M_i\oplus M_{i\oplus 1}} \quad (14)$$

Then the particle $A_{i,i\oplus 1}$ is sent to receiver $B_{i\oplus 1}$ via the edge $A_iB_{i\oplus 1}$. At the same time, A_i performs the same unitary operation U_{M_i} on one particle of the EPR pair $|\Phi^+\rangle$ shared beforehand between A_i and *C*. As soon as the operation being accomplished, the resulting particle is also delivered to *C*.

Step 3 At node *C*, as soon as it receives the particles from A_i, the Bell-state analysis will be performed on each pair of the particles with results corresponding to $A_i=a_ib_i\in\{00,01,10,11\}$. After that, an encoding operation is employed on a_1b_1 and a_2b_2 in succession, as is illustrated in Figure 2(1). Here, the CNOT-gate is deployed to accomplish the encoding operation, i.e., $p_1=C_{a_1,a_2}a_2$, $p_2=C_{b_1,b_2}b_2$ and $p_2=C_{b_1,a_2}a_2$. After that the unitary operations $U_{b_1p_1}$ and $U_{p_2p_3}$ are respectively applied on the particles of four EPR pairs which are shared between *C* and $B_{i\oplus 1}$, i.e., $U_{b_1p_1}|\Phi_1\rangle_C$, $U_{P_2p_3}|\Phi_2\rangle_C$, $U_{b_1p_1}|\Phi_3\rangle_C$, and $U_{P_2p_3}|\Phi_4\rangle_C$. Finally, the resulting particles corresponding to $U_{b_1p_1}|\Phi_1\rangle_C$ and $U_{P_2p_3}|\Phi_2\rangle_C$ are delivered to B_2, while the particles of $U_{b_1p_1}|\Phi_3\rangle_C$ and $U_{P_2p_3}|\Phi_4\rangle_C$ will be sent to B_1.

Step 4 When $B_{i\oplus 1}$ receives the particles from *C*, the Bell-state analysis are applied on the particles of the resulting EPR pairs. Consequently, $B_{i\oplus 1}$ recovers the classical bit string $b_1p_1p_2p_3$. After the operation, $B_{i\oplus 1}$ obtains a_1b_1 and a_2b_2 with the decoding circuit in Figure 2(2). Then the unitary operation $U_{M_i\oplus M_{i\oplus 1}}$ is performed on the yielded state $U_{M_i}^{-1}|\Psi(M_{i\oplus 1})\rangle$, i.e., $U_{M_i\oplus M_{i\oplus 1}}U_{M_i}^{-1}|\Psi(M_{i\oplus 1})\rangle$, According to Eq. (14), obtaining state $U_{M_{i\oplus 1}}|\Psi(M_{i\oplus 1})\rangle$, the result is shown in Table 1.

Step 5 The sender A_{i+1} performs Hadamard operation on the particle $G_{i\oplus 1}$ to restore the initial state $|m_{i\oplus 1}\rangle$, i.e., $H_{G_{i\oplus 1}}U_{M_{i\oplus 1}}|\Psi(M_{i\oplus 1})\rangle$. The result is shown in Table 2.

As soon as the above-mentioned transformation is accomplished, the particle $G_{i\oplus 1}$ will be measured by the sender A_{i+1} .Then the initial state of $|m_{i\oplus 1}\rangle$ can be recovered by the receiver B_{i+1} after performing a suiTable unitary operation, as shown in Table III. For example, if G_i is measured with $|0\rangle$, the resulting state of $A_{i+1,i}$ can be expressed as one of the four states

$$\alpha_i|0\rangle+\beta_i|1\rangle,-\beta_i|0\rangle+\alpha_i|1\rangle,\beta_i|0\rangle-\alpha_i|1\rangle,\beta_i|1\rangle+\alpha_i|0\rangle. \tag{15}$$

Otherwise, the state of $A_{i+1,i}$ can be obtained as

$$\alpha_i|0\rangle-\beta_i|1\rangle,\beta_i|0\rangle+\alpha_i|1\rangle,\beta_i|0\rangle+\alpha_i|1\rangle,-\alpha_i|0\rangle+\beta_i|1\rangle. \tag{16}$$

To accomplish the transformation, four recovery unitary operation are introduced in Table 3. Here, if G_i is measured with $|0\rangle$, the operation V_{M_i} will be applied on $A_{i+1,i}$. Otherwise, the operation W_{M_i} is applied. After the unitary operation, no matter what result is obtained, the receiver B_{i+1} can resume $|m_i\rangle$ from $H_{G_i}U_{M_i}|\Psi(M_i)\rangle$.

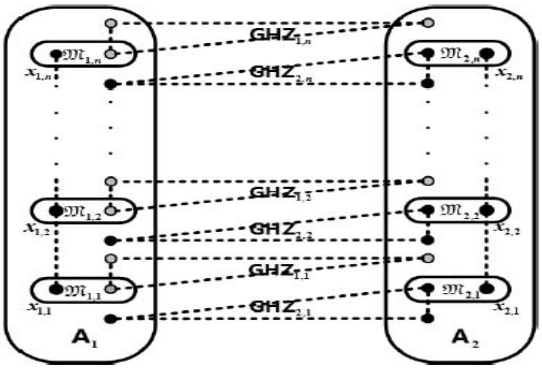


Figure 3. The protocol for transmission two multipartite entangled states. The particles $x_{i,k}$ are the multipartite entangled states which ought to be sent to the receiver B_i .

4. QNC with multipartite entangled states

In this section, with the assistance of the d-dimensional controlled quantum teleportation scheme [22], we generalize the protocol in section 3 to one protocol for transmitting two multipartite n-qudit entangled states $|m\rangle_{x_{i,1},\cdots,x_{i,n}}$ crossly over the butterfly network, where

$$|m\rangle_{x_{i,1},\cdots,x_{i,n}} = \sum_{x_{i,1},\cdots,x_{i,n}=0}^{d-1} \alpha_{x_{i,1},\cdots,x_{i,n}} |x_{i,1},\cdots,x_{i,n}\rangle. \tag{17}$$

The complex coefficients $\alpha_{x_{i,1},\cdots,x_{i,n}}$ satisfy the following constraints

$$\sum_{x_{i,1},\cdots,x_{i,n}}^{d-1} \left|\alpha_{x_{i,1},\cdots,x_{i,n}}\right|^2 = 1. \tag{18}$$

Assuming that two senders A_1 and A_2 share $n+1$ generalized GHZ states beforehand

$$|GHZ\rangle_{A_{11,k},A_{21,k},G_{1,k}} = \frac{1}{\sqrt{d}}\sum_{l=0}^{d-1}|lll\rangle_{A_{11,k},A_{21,k},G_{1,k}}, \tag{19}$$

$$|GHZ\rangle_{A_{12,k},A_{22,k},G_{2,k}} = \frac{1}{\sqrt{d}}\sum_{l=0}^{d-1}|lll\rangle_{A_{12,k},A_{22,k},G_{2,k}}, \tag{20}$$

where particles $A_{11,k}$, $A_{12,k}$ and $G_{1,k}$ are held in A_1's hands while particles $A_{21,k}$, $A_{22,k}$ and $G_{2,k}$,are kept by A_2 , $\forall k \in \{1,2,\cdots,n\}$, illustrated in Figure 3. In addition $6n$ generalized EPR states

$$\left|\Phi_k\right\rangle^{(d)} = \frac{1}{\sqrt{d}}\sum_{t=1}^{d}\left|tt\right\rangle \tag{21}$$

are prepared by C and A_i as well as B_i . In order to facilitate description, $4n$ generalized EPR pairs between C and B_i are denoted by $\left|\Phi_{1,k}\right\rangle$, $\left|\Phi_{2,k}\right\rangle$,$\left|\Phi_{3,k}\right\rangle$ and $\left|\Phi_{4,k}\right\rangle$, respectively.

The process of our protocol is as follows:

Step 1 Sender A_i prepares n-qudit unknown state $\left|m\right\rangle_{x_{i,1},\cdots,x_{i,n}}$ and n generalized GHZ states. The whole quantum system is described as

$$\left|\Upsilon\right\rangle = \left|m\right\rangle_{x_{i,1},\cdots,x_{i,n}} \otimes_{k=1}^{n} \left|GHZ\right\rangle_{A_{i,i,k},A_{i\oplus 1,i,k},G_{i,k}} . \tag{22}$$

Then the n generalized Bell-state measurement (GBSM) with the basis of

$$U_{a_{i,k},b_{i,k}} = \frac{1}{\sqrt{d}}\sum_{l=0}^{d-1} e^{\frac{2\pi i}{d} l a_{i,k}} \left|l\right\rangle \otimes \left|\lfloor l \oplus b_{i,k} \rfloor_d\right\rangle \tag{23}$$

is performed on the particles $x_{i,k}$ and $A_{i,k}, \forall k \in \{1,2,\cdots,n\}$, where $a_{i,k}, b_{i,k} \in \{0,1,\cdots,d-1\}$. Then state of the remaining particles $A_{i\oplus 1,i,k}$ and $G_{i,k}$ can be written as:

$$\begin{aligned}\left|\Psi(M_{i,k})\right\rangle = \frac{1}{d^n}\sum_{l_1,l_2,\cdots l_n=0}^{d-1} e^{-\frac{2\pi i}{d}\sum_{k=1}^{n} l_k a_{i,k}} a_{l_1,l_2,\cdots l_n} \\ \prod_{k=1}^{n}\left|\lfloor l_k \oplus b_{i,k}\rfloor_d, \lfloor l_k \oplus b_{i,k}\rfloor_d\right\rangle G_{i,k}, A_{i\oplus 1,i,k}.\end{aligned} \tag{24}$$

After that, the corresponding unitary operation $U^{-1}_{M_{i,k}}$ is performed on particle $A_{i,i\oplus 1,k}$ in state $\left|\Psi(M_{i\oplus 1,k})\right\rangle$. After that the resulting particles is delivered to receiver $B_{i\oplus 1}$ via the edge $A_i B_{i\oplus 1}$. Meanwhile, the unitary operation $U_{M_{i,k}}$ is applied on particles of generalized EPR pairs $\left|\Phi\right\rangle^{(d)}$ of A_i for superdense coding. After that, the n particles are sent to C.

Step 2 At C, after it obtains the particles from A_i , the GBSM is jointly performed on the particles that have been received from A_i and the particles in it's hands. After that, k classical bit string pairs $a_{i,k}$, $b_{i,k}$ and $a_{i\oplus 1,k}$, $b_{i\oplus 1,k}$ are both created. Then an encoding operation similar to that of operation in section 3 is applied on $a_{i,k}$, $b_{i,k}$ and $a_{i\oplus 1,k}$, $b_{i\oplus 1,k}$. After that, the unitary operations $U_{b_{1,k},p_{1,k}}$ and $U_{p_{2,k},p_{3,k}}$ will be performed on the particles of $4k$ generalized EPR pairs shared between C and $B_{i\oplus 1}$, i.e., $U_{b_{1,k},p_{1,k}}\left|\Phi_{1,k}\right\rangle$, $U_{p_{2,k},p_{3,k}}\left|\Phi_{2,k}\right\rangle$,$U_{b_{1,k},p_{1,k}}\left|\Phi_{3,k}\right\rangle$ and $U_{p_{2,k},p_{3,k}}\left|\Phi_{4,k}\right\rangle$. The resulting particles will be delivered to $B_{i\oplus 1}$ respectively.

Step 3 At $B_{i\oplus 1}$, after applying the GBSM on the pairs of corresponding EPR particles, the strings $b_{1,k}, p_{1,k}, p_{2,k}, p_{3,k}$ are restored, subsequently. Then the unitary operation $U_{p_{1,k},p_{2,k}}$ is performed on the received particles from A_i , i.e., $U_{p_{1,k},p_{2,k}} U^{-1}_{M_{i,k}} \left|\Psi(M_{i\oplus 1,k})\right\rangle$ .On the basis of the following property

$$U_{M_{i,k}} U_{M_{i\oplus 1,k}} = U_{M_{i,k} \oplus M_{i\oplus 1,k}} \tag{25}$$

in the d-dimensional Hilbert space, one obtains

$$U_{M_{i\oplus 1,k}}\left|\Psi(M_{i\oplus 1,k})\right\rangle=\frac{1}{d^n}\sum_{l=0}^{d-1}e^{\frac{2\pi i}{d}la_{i\oplus 1,k}}\left|\lfloor l\oplus b_{i\oplus 1,k}\rfloor_d\right\rangle\otimes\langle l|\otimes\sum_{l_1,\cdots,l_n=0}^{d-1}e^{-\frac{2\pi i}{d}\sum_{k=1}^{n}l_k a_{i\oplus 1,k}}a_{l_1,\cdots,l_n}\otimes\prod_{k=1}^{n}\left|\lfloor l_k\oplus b_{i\oplus 1,k}\rfloor_d,\lfloor l_k\oplus b_{i\oplus 1,k}\rfloor_d\right\rangle_{G_{i\oplus 1,k},A_{i,i\oplus 1,k}} \tag{26}$$

Then the decoding operation is applied on the strings $b_{1,k}, p_{1,k}, p_{2,k}, p_{3,k}$. After that, $a_{i,k}, b_{i,k}$ and $a_{i\oplus 1,k}$, $b_{i\oplus 1,k}$ can be achieved by $B_{i\oplus 1}$.

Step 4 In order to recover the initial states in $|m\rangle_{x_{i\oplus 1,1},\cdots,x_{i\oplus 1,n}}$, $A_{i\oplus 1}$ performs the generalized d-dimension Hadamard gate operation on particles $G_{i,i\oplus 1,k}$

$$H_{G_{i\oplus 1,k}}=\frac{1}{\sqrt{d}}\sum_{l,j=0}^{d-1}e^{2\pi ijl/d}\,|j\rangle\langle l| \tag{27}$$

i.e., $H_{G_{i\oplus 1,k}}U_{M_{i\oplus 1,k}}\left|\Psi(M_{i\oplus 1,k})\right\rangle$, which can be rewritten as

$$\frac{1}{d^{\frac{2n+1}{2}}}\sum_{l,j=0}^{d-1}e^{\frac{2\pi il}{d}(\lfloor j\oplus a_{i\oplus 1,k}\rfloor_d}\,|j\rangle\langle l|\otimes\left|\lfloor l\oplus b_{i\oplus 1,k}\rfloor_d\right\rangle\otimes\langle l|\otimes\sum_{l_1,\cdots,l_n=0}^{d-1}e^{-\frac{2\pi i}{d}\sum_{k=1}^{n}l_k a_{i\oplus 1,k}}a_{l_1,\cdots,l_n}\otimes\prod_{k=1}^{n}\left|\lfloor l_k\oplus b_{i\oplus 1,k}\rfloor_d,\lfloor l_k\oplus b_{i\oplus 1,k}\rfloor_d\right\rangle_{G_{i\oplus 1,k},A_{i,i\oplus 1,k}} \tag{28}$$

After the measurement of particles $G_{i\oplus 1,k}$ at $A_{i\oplus 1}$ on the base of $\{|0\rangle,|1\rangle,\cdots,|d-1\rangle\}$, the initial state $|m\rangle_{x_{i,1},\cdots,x_{i,n}}$

5. Security analysis

So far we have consider the design of the QNC based on the multi-photon entangled states via teleportation. In this section, we investigate the security of the present QNC scheme.

5.1. The security analysis of the channel $A_iB_{i\oplus 1}$

In order to detect the channel's security, an EPR pair $|\psi\rangle$ which is shared by A_i and $B_{i\oplus 1}$ is introduced as the detection channel. Before particle $A_{i,i\oplus 1}$ is delivered, the CNOT-gate operation C_{al} will be performed on the detection particle

$$|\iota\rangle=\cos\theta|0\rangle+e^{-i\varphi}\sin\theta|1\rangle. \tag{29}$$

Here subscript a denotes the particle of the EPR pair in A_i's hands. And the result can be expressed as

$$\left|\iota^C\right\rangle=C_{a,\iota}|\psi\rangle|\iota\rangle=\sum_{i=0,1}\frac{(\cos\theta\delta_{l,0}+\sin\theta^{-i\varphi}\delta_{l,1})}{\sqrt{2}}(|0_a0_bi_\iota\rangle+|0_a0_b(i\oplus 1)_\iota\rangle), \tag{30}$$

here $\delta_{i,j}$ is the Kronecker symbol. After the operation, the detection particle $|\iota\rangle$ will be entangled with the detection channel $|\psi\rangle$, and the three-particle entanglement particles are generated correspondingly.

Assuming that Eve's strategy is attack on the channel C with a probe particle $|\varepsilon\rangle$

$$|\psi_{a,b,\varepsilon}\rangle=|\psi\rangle\otimes|\varepsilon\rangle, \tag{31}$$

after the unitary operation U, the entanglement state can be denoted as

$$U\left|\psi_{a,b,\varepsilon}\right\rangle=\left|0_a0_b\right\rangle\otimes\left|E_1\right\rangle+\left|0_a1_b\right\rangle\otimes\left|E_2\right\rangle+\left|1_a0_b\right\rangle\otimes\left|\tilde{E}_1\right\rangle+\left|1_a1_b\right\rangle\otimes\left|\tilde{E}_2\right\rangle. \quad (32)$$

Here $\left|E_1\right\rangle\perp\left|E_2\right\rangle,\left|\tilde{E}_1\right\rangle\perp\left|\tilde{E}_2\right\rangle$ and $\left\langle E_1\middle|\tilde{E}_2\right\rangle+\left\langle E_2\middle|\tilde{E}_1\right\rangle=0$. Eq. (30) can be rewritten as

$$\begin{aligned}\left|\iota^{C'}\right\rangle&=C_{a,\iota}(U\left|\psi_{a,b,\varepsilon}\right\rangle)\left|\iota\right\rangle=\frac{1}{2}[\cos\theta\left|0_a0_b0_l\right\rangle+e^{-i\varphi}\sin\theta\left|0_a0_b1_l\right\rangle]\otimes\left|E_2\right\rangle\\&+\frac{1}{2}[\cos\theta\left|0_a0_b0_l\right\rangle+e^{-i\varphi}\sin\theta\left|0_a1_b1_l\right\rangle]\otimes\left|E_1\right\rangle+\frac{1}{2}[\cos\theta\left|1_a0_b1_l\right\rangle\\&+e^{-i\varphi}\sin\theta\left|1_a0_b1_l\right\rangle]\otimes\left|\tilde{E}_1\right\rangle+\frac{1}{2}[\cos\theta\left|1_a1_b1_l\right\rangle+e^{-i\varphi}\sin\theta\left|1_a1_b0_l\right\rangle]\otimes\left|\tilde{E}_2\right\rangle.\end{aligned} \quad (33)$$

At $B_{i\oplus1}$, as soon as it receives the particle $\left|\iota\right\rangle$, the CNOT-gate operation $C_{b,l}$ will be performed on the particle of the EPR pair which already in it's hands and the particle $\left|\iota\right\rangle$. The result can be expressed as:

$$\begin{aligned}C_{b,\iota}\left|\iota^{C'}\right\rangle&=\frac{1}{2}[\cos\theta\left|0_a0_b0_\iota\right\rangle+e^{-i\varphi}\sin\theta\left|0_a0_b1_\iota\right\rangle]\otimes\left|E_1\right\rangle+\frac{1}{2}[\cos\theta\left|0_a1_b1_\iota\right\rangle+e^{-i\varphi}\sin\theta\left|0_a1_b1_\iota\right\rangle]\otimes\left|E_2\right\rangle\\&+\frac{1}{2}[\cos\theta\left|1_a0_b1_\iota\right\rangle+e^{-i\varphi}\sin\theta\left|1_a0_b0_\iota\right\rangle]\otimes\left|\tilde{E}_1\right\rangle+\frac{1}{2}[\cos\theta\left|1_a1_b0_\iota\right\rangle+e^{-i\varphi}\sin\theta\left|1_a1_b1_\iota\right\rangle]\otimes\left|\tilde{E}_2\right\rangle.\end{aligned} \quad (34)$$

Obviously, $C_{b,\iota}\left|\iota^{C'}\right\rangle\neq\left|\psi\right\rangle\otimes\left|\iota\right\rangle$. Therefore, $\left|\iota\right\rangle$ can not be extracted from $\left|\iota^{C'}\right\rangle$, the eavesdropper Eve can be detected. With the assistance of the detection operation, the quantum channel $A_iB_{i\oplus1}$'s security is guaranteed.

5.2. The security analysis of the cross-channel A_iB_i

To ensure the security of the cross-channel, the secondary detection will be introduced. For clarity, we focus on the channel A_1C to illustrate the details of the detection. The rest of the cross-channel's security analysis is similar to it.

The details of the detection on the channel A_1C can be illustrated as follows: At A_1, in addition to the EPR pair $\left|\psi\right\rangle$ which encoded the information with a local operation on a single particle, A_1 prepares an ordered N-1 pair in state $\left|\psi\right\rangle$. After that, the N ordered EPR pairs will be divided into two parts, namely, $P_i(L)$ and $P_i(M)$. A_1 sends the L sequence to the relay node C, and the eavesdropper Eve will be checked by the following procedure: Firstly, at the node C, a number of the particles from the L sequence will be chosen randomly and it will be perceived by A_1. After that, two sets of MBs, says, σ_z and σ_x will be chosen to measure the particles. When the operation is accomplished, C tells A_1 which MB it has chosen for each particle and the outcomes of it's measurement. After A_1 uses the same measuring basis as which be used at the relay node C to measure the corresponding photons in the M sequence and checks with the results of C, if there is no eavesdropping exists, their results should be completely opposite.

Supposing that an attacker Eve may access to the quantum channel to acquire some information, owing to the particles are entanglement states, the most possible attack is the attack n the entangled state $\left|\psi\right\rangle$, namely,

$$\left|\psi_{a,b,\varepsilon}\right\rangle=\sum_{a,b\in0,1}\left|\varepsilon_{a,b}\right\rangle\left|a\right\rangle\left|b\right\rangle, \quad (35)$$

where $\left|\psi_{a,b}\right\rangle$ describes Eve's probe state, $\left|a\right\rangle$ and $\left|b\right\rangle$ are single-photon states of A_1 and C in each EPR pair respectively. We describe Eve's effect on the system as

$$\tilde{E}\left|0,E\right\rangle\equiv\tilde{E}\left|0\right\rangle_C\left|E\right\rangle=\alpha\left|0,\varepsilon_{00}\right\rangle+\beta\left|1,\varepsilon_{01}\right\rangle, \quad (36)$$

$$\tilde{E}|1,E\rangle \equiv \tilde{E}|1\rangle_C |E\rangle = \beta'|0,\varepsilon_{10}\rangle + \alpha'|1,\varepsilon_{11}\rangle. \quad (37)$$

Here Eve's probe can be modeled by

$$\tilde{E} = \begin{pmatrix} \alpha & \beta' \\ \beta & \alpha' \end{pmatrix}, \quad (38)$$

the complex numbers must satisfy

$$|\alpha|^2 + |\beta|^2 = 1, \quad (39)$$

$$\alpha\beta^* + \alpha'^*\beta'^* = 0. \quad (40)$$

The Eve's eavesdropping will introduce an error rate

$$e = |\beta|^2 = |\beta'|^2 = 1 - |\alpha|^2 = 1 - |\alpha'|^2. \quad (41)$$

Here the probability of the information which Eve can maximally gain will be calculated as follows. Assuming that at A_1, the measurement result of the EPR particle is $|1\rangle$, then the state of the system composed of the relay node C's particle and Eve's probe can be described by

$$|\psi'\rangle = \hat{E}|0,E\rangle \equiv \alpha|0,\varepsilon_{00}\rangle + \beta|1,\varepsilon_{01}\rangle. \quad (42)$$

After that, the state of the system reads

$$\rho' = |\alpha|^2|0,\varepsilon_{00}\rangle\langle 0,\varepsilon_{00}| + |\beta|^2|1,\varepsilon_{01}\rangle\langle 1,\varepsilon_{01}|\alpha\beta^*|0,\varepsilon_{00}\rangle\langle 0,\varepsilon_{01}| + \alpha^*\beta|1,\varepsilon_{01}\rangle\langle 0,\varepsilon_{00}|. \quad (43)$$

After encoding of four different unitary operations U_{00}, U_{01}, U_{10}, U_{11} which have been elaborate in section 3 with the probabilities p_0, p_1, p_2, p_3,here $p_0+ p_1+ p_2+ p_3= 1$, the state can be denoted as

$$\begin{aligned}\rho'' = &(p_0+p_3)(|\alpha|^2|0,\varepsilon_{00}\rangle\langle 0,\varepsilon_{00}| + |\beta|^2|1,\varepsilon_{01}\rangle\langle 1,\varepsilon_{01}|) + (p_0-p_3)(\alpha\beta^*|0,\varepsilon_{00}\rangle \\ &\langle 1,\varepsilon_{01}| + \alpha^*\beta|1,\varepsilon_{01}\rangle\langle 0,\varepsilon_{00}|) + (p_1+p_2)(|\alpha|^2|1,\varepsilon_{00}\rangle\langle 1,\varepsilon_{00}| + |\beta|^2|0,\varepsilon_{01}\rangle\langle 0,\varepsilon_{01}|) \\ &+(p_1-p_2)(\alpha\beta^*|1,\varepsilon_{00}\rangle\langle 0,\varepsilon_{01}| + \alpha^*\beta|0,\varepsilon_{01}\rangle\langle 1,\varepsilon_{00}|),\end{aligned} \quad (44)$$

which can be rewritten in the orthogonal basis $\{|0,\varepsilon_{00}\rangle, |1,\varepsilon_{01}\rangle, |1,\varepsilon_{00}\rangle, |0,\varepsilon_{01}\rangle\}$

$$\rho'' = \begin{pmatrix} (p_0+p_3)|\alpha|^2 & (p_0-p_3)\alpha\beta^* & 0 & 0 \\ (p_0-p_3)\alpha^*\beta & (p_0+p_3)|\beta|^2 & 0 & 0 \\ 0 & 0 & (p_1+p_2)|\alpha|^2 & (p_1-p_2)\alpha\beta^* \\ 0 & 0 & (p_1-p_2)\alpha^*\beta & (p_1+p_2)|\beta|^2 \end{pmatrix}. \quad (45)$$

The maximal information I_0 that can be extracted from this state is given by the *Von Neumann entropy*, namely,

$$I_0 = \sum_{i=0}^{3} -\lambda_i \log_2 \lambda_i. \quad (46)$$

where $\lambda_i (i = 0,1,2,3)$ are the eigen values of ρ'', which can be expressed as

$$\lambda_{0,1} = \frac{1}{2}(p_0+p_3) \pm \frac{1}{2}\sqrt{(p_0+p_3)^2 - 16p_0p_3(e-e^2)}, \quad (47)$$

$$\lambda_{2,3} = \frac{1}{2}(p_1+p_2) \pm \frac{1}{2}\sqrt{(p_1+p_2)^2 - 16p_1p_2(e-e^2)}. \quad (48)$$

Assuming that the probability of the four operations is equal to each other, namely, $p_i = 1/4$, the expression can be simplified to

$$\lambda_{0,1,2,3} = \frac{1}{4} \pm \frac{1}{2}.\left|\frac{1}{2} - e\right|, \tag{49}$$

namely,

$$\lambda_{0,2} = \frac{1}{2} - \frac{e}{2}, \lambda_{1,3} = \frac{e}{2}. \tag{50}$$

Then the entropy can be expressed as

$$I_0 = -(1-e)\log_2 \frac{1-e}{2} - e\log_2 \frac{e}{2}. \tag{51}$$

After the derivative operation, it can be concluded that the function $I_0(e)$ has a maximum at $e = 1/2$. This means that if the eavesdropper Eve exist, she has to face a detection probability $e(I_0) > 0$. If she wants to gain the full information ($I_0 = 1$), the detection probability is $e = 1/2$. This probability is big enough to detect any effective eavesdropping attack.

6. Discussion and summary

The protocol which has been discussed in section 3 is mainly based on the controlled quantum teleportation and the superdense coding. The integrator of those two technologies will make the transmission efficiency to be higher. Comparing with those trivial protocols, the present one has favorable and unfavorable aspects. In our scheme, there is no classical bit communicated at the bottleneck channel, namely, all the channel of the butterfly network is the quantum channel. The unity of the channel will make the deployment of the protocol much more convenient. Another improvement is that under our protocol, the probability of the particle being resumed by the receiver is one. While those quantum network coding protocols [14, 17] which adopted the classical bits as the complement of the encoding operation sending 1.5 times on average to guarantee that the receiver can obtain the sender’s particle accurately. Relative to the other protocols, the transmission efficiency of our protocol is greatly raised. Also, there is still having some shortcomings in our protocol, like many probabilistic teleportation protocols [13-14, 20-21], our protocol takes use of non-maximally entangled resources without first converting to a maximally entangled pair via local filtering or entanglement concentration. There is still existing some specificity in this channel.

In summary, it is provided that two senders sharing two non-maximally entangled GHZ pairs, a protocol is proposed to transmit two 2-level entangled states which have been shared by the two senders over the butterfly network crossly. If one particle transmission over the side links and two particles communication on the other channels are allowed, after the transmission, both of the receivers can reestablish the initial states with a certain probability 1. Furthermore, this protocol has also been generalized to transmit two multipartite entangled states. Finally, on purpose to ensure the security of the network transmission, several potential attack strategies were discussed and the appropriate detection mechanism was designed.

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