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# Study on The Error Transfer of Articulated Arm Coordinate Measuring Machines

**Guanbin Gao*[1], Wen Wang[2,3], Jianjun Zhou[3]**
[1]Faculty of Mechanical and Electrical Engineering, Kunming University of Science and Technology, Kunming 650500, China
[2]Department of Mechanical Engineering, Zhejiang University, Hangzhou 310027, China
[3]School of Mechanical Engineering, Hangzhou Dianzi University, Hangzhou 310018, China
*Corresponding author, e-mail: gbgao@163.com

***Abstract***

*The articulated arm coordinate measuring machine (AACMM) is a new type coordinate measuring machine (CMM) base on the linkage structure. The kinematic model of a 6-DOF AACMM with DH method was established, and from the kinematic model the coordinate systems and joint structural parameters of the AACMM are obtained. The Jacobian matrix was deduced by differential transformation from the kinematic model of the AACMM, and according to the Jacobian matrix, the error transfer coefficients of the joint structural parameters were calculated. Then with the calculation results the influence of the joint structural parameters on the measuring accuracy of the AACMM was analyzed, which provides a theoretical basis for calibration, tolerance distribution of the joint parts and components' selection of the AACMM.*

***Keywords****: coordinate measuring machine, error transfer, kinematic model, Jacobian matrix*

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## 1. Introduction

The AACMM is a multi-DOF (typically 6-DOF) and non-Cartesian coordinate measuring machine (CMM), which is modeled according to the structure of human joints: a series of linkages connected by rotating joints. Comparing with traditional CMMs the AACMM has the features of small size, light weight, large measurement range, flexible and can be applied in industrial site [1]. With these unique advantages the AACMM has been applied in the field of mold design, product quality online testing, equipment maintenance and assembly [2].

The AACMM belongs to open chain structure mechanisms which are seemingly simple, but in fact it can result in complex kinematic modeling [3]. The transformation from the joint space to the measuring space of the AACMM is rather complex and nonlinear, which results in that it is difficult to identify the influence of the joint structural parameters on the measurement accuracy. Studied the influence of the joint structural parameters on the measurement accuracy with numerical simulation based on DH method [4]. Studied the problem with the same method, but the kinematic modeling is based on the local product of exponentials , and reached substantially the same conclusions [5]. However, only some specific parameter errors were used to calculate the measurement error in [4] [5], so it is not an integral result. The physical meaning of the Jacobian matrix is the error transfer coefficient [6] [7], which can completely describe the error transfer from the joint space to the measuring space.

## 2. Kinematic Modeling of the AACMM

### 2.1. Kinematic modeling method for the AACMM

The kinematic model, also known as measurement model, is the mathematical basis of the AACMM, which can achieve the coordinate transformation from the joint space to the measurement space. Due to the structures of the AACMM and robot are similar, the kinematic modeling methods of the AACMM are also used in robot field. With the characteristics of simple, clear physical meaning and easy to program implementation DH method has been the mainstream method of robot kinematic modeling. The main problems of DH method is parameters in DH model are difficult to be measured or identified, and when two adjacent joint

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axes are parallel or nearly parallel the homogeneous transformation matrix will be singular and ill-conditioned. The adjacent joint axes of the AACMM are generally perpendicular. This feature makes the AACMM particularly suitable to model with DH method, so most AACMM kinematic model are established with DH method.

As there is some flexibility in modeling with DH method, the kinematic models of AACMM established by different people are not exactly the same. Contains a product result of 6 matrixes and a vector [8] , which contains 27 parameters. Contains a product result of 7 matrixes and a vector, which contains 25 parameters [9] and the kinematic model contains 23 parameters [1].

In recent years, in addition to DH method there has been a new modeling method for robots named product of exponentials (POE). Huang [5] established the kinematic model of a AACMM with POE method in which there are 88 parameters. Compared with the kinematic model based on DH method the kinematic model based on POE can describe the motion characteristics of the AACMM completely [10]. But too many parameters results in too complex description, and make it is even more complex for the further work [11] (calibration, motion control and error compensation) of AACMM. In this paper, we use DH method for kinematic modeling of the AACMM.

**2.2. Kinematic modeling for an AACMM**

The structure of the AACMM studied in this paper is shown in Figure 1. It mainly composed of a base, six rotating joints, two linkages and a probe.

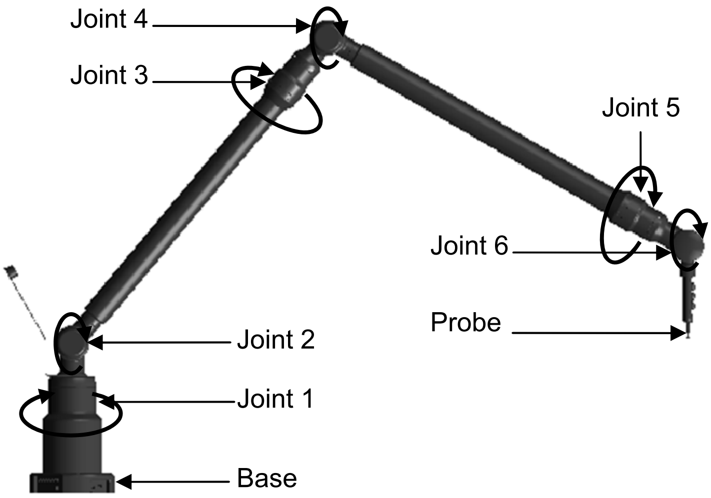


Figure 1. The structure of the AACMM

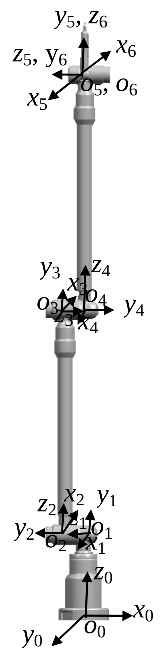


Figure 2. The coordinate systems of the AACMM

According to DH method the first step of kinematic is to establish corresponding coordinate system on each linkage of the AACMM, and then the kinematic equation is the product of the coordinate transformation. The coordinate systems of the AACMM established with DH method are shown in Figure 2.

There are five groups of parameters in kinematic model of the AACMM: linkage length $d_i$, joint length $a_i$, torsion angle $\alpha_i$, joint angle $\theta_i$ and offset of probe $l$, where $i$=1~6. The values of these parameters are shown in Table 1.

The coordinate transformation of adjacent joint coordinate system $\{x_i y_i z_i\}$ and $\{x_{i-1} y_{i-1} z_{i-1}\}$ can be achieved through two rotations and two translations [12]:

$$T_{i-1,i} = Rot(z_{i-1}, \theta_i) Trans(0,0,d_i) Trans(a_i,0,0) Rot(x_i, \alpha_i). \tag{1}$$

Table 1. Joint Structural Parameters of the AACMM

| *i* | $d_i$ [mm] | $a_i$ [mm] | $\alpha_i$ [°] | $\theta_i$ [°] | *l* [mm] |
|---|---|---|---|---|---|
| 1 | 300 | 0 | 90 | $\theta_1$ | 183.5 |
| 2 | 65.5 | 0 | 90 | $\theta_2$ | - |
| 3 | 792.7 | 0 | 90 | $\theta_3$ | - |
| 4 | 65.5 | 0 | 90 | $\theta_4$ | - |
| 5 | 883.2 | 0 | 90 | $\theta_5$ | - |
| 6 | 0 | 0 | 90 | $\theta_6$ | - |

According to the homogeneous transformation principle Eq. 1 can be written as Eq. 2.

$$T_{i-1,i}=\begin{bmatrix}\cos\theta_i & -\sin\theta_i & 0 & 0\\ \sin\theta_i & \cos\theta_i & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & d_i\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & a_i\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\alpha_i & -\sin\alpha_i & 0\\ 0 & \sin\alpha_i & \cos\alpha_i & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}\cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i\\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i\\ 0 & \sin\alpha_i & \cos\alpha_i & d_i\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{2}$$

The coordinates of the probe in the base coordinate system $\{o_0x_0y_0z_0\}$ are:

$$\begin{bmatrix}x\\ y\\ z\\ 1\end{bmatrix}=T_{0,1}\cdot T_{1,2}\cdot T_{2,3}\cdot T_{3,4}\cdot T_{4,5}\cdot T_{5,6}\cdot\begin{bmatrix}0\\ 0\\ l\\ 1\end{bmatrix}$$
$$=\prod_{i=1}^{6}\begin{bmatrix}\cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i\\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i\\ 0 & \sin\alpha_i & \cos\alpha_i & d_i\\ 0 & 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}0\\ 0\\ l\\ 1\end{bmatrix} \tag{3}$$

## 3. Jacobian Matrix of the AACMM

The coordinates of the probe can be wrriten in the functional form of the joint structural parameters:

$$x=f_x(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6,\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5,\alpha_6,d_1,d_2,d_3,d_4,d_5,d_6,a_1,a_2,a_3,a_4,a_5,a_6,l) \tag{4}$$

$$y=f_y(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6,\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5,\alpha_6,d_1,d_2,d_3,d_4,d_5,d_6,a_1,a_2,a_3,a_4,a_5,a_6,l) \tag{5}$$

$$z=f_z(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6,\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5,\alpha_6,d_1,d_2,d_3,d_4,d_5,d_6,a_1,a_2,a_3,a_4,a_5,a_6,l) \tag{6}$$

After fully differential of Eq.4 ~ Eq.6 the follow equations can be obtained.

$$dx=\sum_{i=1}^{6}\frac{\partial f_x}{\partial\theta_i}d\theta_i+\sum_{i=1}^{6}\frac{\partial f_x}{\partial\alpha_i}d\alpha_i+\sum_{i=1}^{6}\frac{\partial f_x}{\partial d_i}dd_i+\sum_{i=1}^{6}\frac{\partial f_x}{\partial a_i}da_i+\frac{\partial f_x}{\partial l}dl \tag{7}$$

$$dy=\sum_{i=1}^{6}\frac{\partial f_y}{\partial\theta_i}d\theta_i+\sum_{i=1}^{6}\frac{\partial f_y}{\partial\alpha_i}d\alpha_i+\sum_{i=1}^{6}\frac{\partial f_y}{\partial d_i}dd_i+\sum_{i=1}^{6}\frac{\partial f_y}{\partial a_i}da_i+\frac{\partial f_y}{\partial l}dl \tag{8}$$

$$dz=\sum_{i=1}^{6}\frac{\partial f_z}{\partial\theta_i}d\theta_i+\sum_{i=1}^{6}\frac{\partial f_z}{\partial\alpha_i}d\alpha_i+\sum_{i=1}^{6}\frac{\partial f_z}{\partial d_i}dd_i+\sum_{i=1}^{6}\frac{\partial f_z}{\partial a_i}da_i+\frac{\partial f_z}{\partial l}dl \tag{9}$$

Eq.7 ~ Eq.9 can be written in matrix format:

$$\begin{bmatrix} \mathrm{d}x \\ \mathrm{d}y \\ \mathrm{d}z \end{bmatrix} = \begin{bmatrix} \frac{\partial f_x}{\partial \theta_1} & \frac{\partial f_x}{\partial \theta_2} & \frac{\partial f_x}{\partial \theta_3} & \frac{\partial f_x}{\partial \theta_4} & \frac{\partial f_x}{\partial \theta_5} & \frac{\partial f_x}{\partial \theta_6} & \frac{\partial f_x}{\partial \alpha_1} & \frac{\partial f_x}{\partial \alpha_2} & \frac{\partial f_x}{\partial \alpha_3} & \frac{\partial f_x}{\partial \alpha_4} & \frac{\partial f_x}{\partial \alpha_5} & \frac{\partial f_x}{\partial \alpha_6} & \frac{\partial f_x}{\partial d_1} & \frac{\partial f_x}{\partial d_2} & \frac{\partial f_x}{\partial d_3} & \frac{\partial f_x}{\partial d_4} & \frac{\partial f_x}{\partial d_5} & \frac{\partial f_x}{\partial d_6} & \frac{\partial f_x}{\partial a_1} & \frac{\partial f_x}{\partial a_2} & \frac{\partial f_x}{\partial a_3} & \frac{\partial f_x}{\partial a_4} & \frac{\partial f_x}{\partial a_5} & \frac{\partial f_x}{\partial a_6} & \frac{\partial f_x}{\partial l} \\ \frac{\partial f_y}{\partial \theta_1} & \frac{\partial f_y}{\partial \theta_2} & \frac{\partial f_y}{\partial \theta_3} & \frac{\partial f_y}{\partial \theta_4} & \frac{\partial f_y}{\partial \theta_5} & \frac{\partial f_y}{\partial \theta_6} & \frac{\partial f_y}{\partial \alpha_1} & \frac{\partial f_y}{\partial \alpha_2} & \frac{\partial f_y}{\partial \alpha_3} & \frac{\partial f_y}{\partial \alpha_4} & \frac{\partial f_y}{\partial \alpha_5} & \frac{\partial f_y}{\partial \alpha_6} & \frac{\partial f_y}{\partial d_1} & \frac{\partial f_y}{\partial d_2} & \frac{\partial f_y}{\partial d_3} & \frac{\partial f_y}{\partial d_4} & \frac{\partial f_y}{\partial d_5} & \frac{\partial f_y}{\partial d_6} & \frac{\partial f_y}{\partial a_1} & \frac{\partial f_y}{\partial a_2} & \frac{\partial f_y}{\partial a_3} & \frac{\partial f_y}{\partial a_4} & \frac{\partial f_y}{\partial a_5} & \frac{\partial f_y}{\partial a_6} & \frac{\partial f_y}{\partial l} \\ \frac{\partial f_z}{\partial \theta_1} & \frac{\partial f_z}{\partial \theta_2} & \frac{\partial f_z}{\partial \theta_3} & \frac{\partial f_z}{\partial \theta_4} & \frac{\partial f_z}{\partial \theta_5} & \frac{\partial f_z}{\partial \theta_6} & \frac{\partial f_z}{\partial \alpha_1} & \frac{\partial f_z}{\partial \alpha_2} & \frac{\partial f_z}{\partial \alpha_3} & \frac{\partial f_z}{\partial \alpha_4} & \frac{\partial f_z}{\partial \alpha_5} & \frac{\partial f_z}{\partial \alpha_6} & \frac{\partial f_z}{\partial d_1} & \frac{\partial f_z}{\partial d_2} & \frac{\partial f_z}{\partial d_3} & \frac{\partial f_z}{\partial d_4} & \frac{\partial f_z}{\partial d_5} & \frac{\partial f_z}{\partial d_6} & \frac{\partial f_z}{\partial a_1} & \frac{\partial f_z}{\partial a_2} & \frac{\partial f_z}{\partial a_3} & \frac{\partial f_z}{\partial a_4} & \frac{\partial f_z}{\partial a_5} & \frac{\partial f_z}{\partial a_6} & \frac{\partial f_z}{\partial l} \end{bmatrix} \tag{10}$$

$$\times \left[ d\theta_1, d\theta_2, d\theta_3, d\theta_4, d\theta_5, d\theta_6, d\alpha_1, d\alpha_2, d\alpha_3, d\alpha_4, d\alpha_5, d\alpha_6, dd_1, dd_2, dd_3, dd_4, dd_5, dd_6, da_1, da_2, da_3, da_4, da_5, da_6, dl \right]^T$$

Assigning the 3×25 matrix to **J**, then **J** is the Jacobian matrix of the AACMM.

## 4. Analysis of Error Transfer Coefficients

In this section, the error transfer coefficients of the joint structral parameters ($d_i$, $a_i$, $l$, $\alpha_i$ and $\theta_i$) was calculated base on the Jacobian matrix.

### 4.1. Error Transfer Coefficients of $d_i$, $a_i$ and $l$

From the Jacobian matrix of **J** the following equations can be deduced.

$$(\frac{\partial f_x}{\partial d_i})^2 + (\frac{\partial f_y}{\partial d_i})^2 + (\frac{\partial f_z}{\partial d_i})^2 = 1 \tag{11}$$

$$(\frac{\partial f_x}{\partial a_i})^2 + (\frac{\partial f_y}{\partial a_i})^2 + (\frac{\partial f_z}{\partial a_i})^2 = 1 \tag{12}$$

$$(\frac{\partial f_x}{\partial l})^2 + (\frac{\partial f_y}{\partial l})^2 + (\frac{\partial f_z}{\partial l})^2 = 1 \tag{13}$$

Eq.11 ~ Eq.13 show that the error transfer coefficients of the structual parameters of $d_i$, $a_i$ and $l$ are all equal to 1, which means that the errors of the $d_i$, $a_i$ and $l$ are directly transferred to the measurement error without being enlarged or reduced.

### 4.2. Error Transfer Coefficients of $\alpha_i$ and $\theta_i$

The error transfer coefficients of $\alpha_i$ and $\theta_i$ are $\frac{\partial f_x}{\partial \theta_i}$, $\frac{\partial f_y}{\partial \theta_i}$, $\frac{\partial f_z}{\partial \theta_i}$, $\frac{\partial f_x}{\partial \alpha_i}$, $\frac{\partial f_y}{\partial \alpha_i}$ and $\frac{\partial f_z}{\partial \alpha_i}$, After calculation it is known that $\frac{\partial f_z}{\partial \theta_1} = 0$, and the others coefficients contain a dozen or even dozens of expressions, so the other coefficients aren't listed in this paper for space limitation. The expressions of the coefficients consist of $d_i$, $a_i$, $l$ and the trigonometric functions of $\alpha_i$ and $\theta_i$, so the coefficients are not only related to the joint structural parameters but also the pose of the AACMM. The averages of the error transfer coefficients were calculated with 10000 poses generated through Mont Carlo method, as shown in Table 2.

Table 2. The Average Error Transfer Coefficients of $\alpha_i$ and $\theta_i$

| I | $\frac{\partial f_x}{\partial \alpha_i}$ | $\frac{\partial f_y}{\partial \alpha_i}$ | $\frac{\partial f_z}{\partial \alpha_i}$ | $\frac{\partial f_x}{\partial \theta_i}$ | $\frac{\partial f_y}{\partial \theta_i}$ | $\frac{\partial f_z}{\partial \theta_i}$ |
|---|---|---|---|---|---|---|
| 1 | 384.87 | 395.84 | 367.77 | 501.61 | 496.62 | 0 |
| 2 | 568.02 | 578.28 | 232.63 | 394.68 | 385.32 | 609.88 |
| 3 | 311.40 | 310.46 | 239.77 | 310.53 | 308.82 | 230.78 |
| 4 | 475.54 | 480.67 | 353.12 | 421.28 | 422.54 | 474.16 |
| 5 | 60.14 | 60.13 | 56.52 | 58.72 | 57.79 | 54.43 |
| 6 | 93.76 | 93.77 | 87.71 | 89.25 | 91.92 | 93.86 |

Table 2 shows that the error transfer coefficients of $\alpha_i$ and $\theta_i$ are very large, which means that $\alpha_i$ and $\theta_i$ have a significant influence on the measurement accuracy of the AACMM.

## 5. Conclusions

This paper studied the kinematic modeling methods for linkage structural robots and estalished the kinematic model for a 6-DOF AACMM with DH method. Jacobian matrix were used to calculate the error transfer coefficients, the results show that the joint structural parameters of $\alpha_i$ and $\theta_i$ have a significant influence on the measurement accuracy of the AACMM while $d_i$, $a_i$ and $l$ have relatively small influence. So the angle sensors, bearings and other components which influence the accuracy of $\alpha_i$ and $\theta_i$ greatly must be high precision.

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