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Hybrid conjugate gradient parameter for solving symmetric systems of nonlinear equations

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ABSTRACT

Mathematical models from recent research are mostly nonlinear equations in nature. Numerical solutions to such systems are widely needed and applied in those areas of mathematics. Although, in recent years, this field received serious attentions and new approach were discovered, but yet the efficiency of the previous versions suffers setback. This article gives a new hybrid conjugate gradient parameter, the method is derivative-free and analyzed with an effective inexact line search in a given conditions. Theoretical proofs show that the proposed method retains the sufficient descent and global convergence properties of the original CG methods. The proposed method is tested on a set of test functions, then compared to the two previous classical CG-parameter that resulted the given method, and its performance is given based on number of iterations and CPU time. The numerical results show that the new proposed method is efficient and effective amongst all the methods tested. The graphical representation of the result justify our findings. The computational result indicates that the new hybrid conjugate gradient parameter is suitable and capable for solving symmetric systems of nonlinear equations.

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1. INTRODUCTION

In recent years, many researchers [1-5] focused on the techniques which resulted to emergence of several CG formulas coefficient (β_k) in solving unconstrained optimization problems, but hybridizing the CG coefficient (β_k) gives so many advantages. Hybrid CG parameters are basically designed based on an adaptive switch from a CG parameter, these includes the hybridizations of PRP and FR methods in proposed by Touati-Ahmed and Storey [6], Hu and Storey [7] and Gilbert and Nocedal [1], the hybridizations of HS and DY methods proposed by Dai and Yuan [8] and the hybridization of LS and CD methods proposed by Touati-Ahmed and Storey [6]. In the hybrid CG methods of [6, 8], the CG parameter is computed based on discrete combinations of the CG parameters of the two categories. Recently, Andrei [9-13] proposed several efficient hybrid CG methods based on convex combinations of the CG parameters of the two categories which are continuous combinations. More exactly, in [9] a hybridization of HS and DY methods has been proposed in which the hybridization parameter is computed based on the standard secant (quasi-Newton) equation [14, 15]. In [10], using an acceleration scheme, Andrei proposed another hybridization of HS and DY methods in which the hybridization parameter is computed based on a modified secant equation proposed by Li et al. [3]. The hybridization parameter in our methods is computed from a modified secant equation obtained based on the search direction of the Hager-Zhang nonlinear CG method. The survey by Hager and Zhang (2006) discussed

extensively on some methods with special emphasis on their global convergence. The main aim of this proposed hybrid method is to find out new formula for conjugate methods such that they are not only globally convergent for general functions but also have good numerical performance.

2. DERIVATION OF THE HYBRID CONJUGATE GRADIENT PARAMETER FOR SOLVING SYMMETRIC SYSTEMS OF NONLINEAR EQUATIONS

In this section, the new formula was derived from [16] and [17], the resulting hybridizations formula is given below. The attractive features of these methods are to attain good computational performance and to maintain the attractive feature of strong global convergence. In order to take advantage of these, a hybridizations of these methods is carried out using the Andrei's approach together with the nonnegative restriction of the CG parameters suggested by Powell [5].

From [16], the CG parameter is given by

$$\beta_{k_1} = \frac{(\theta_k y_k - s_k)^T g_{k+1}}{y_k^T \theta_k}, \theta_k = \frac{s_k^T s_k}{s_k^T y_k} \; k = 1,2,3,\cdots \tag{1}$$

Also, from [17], the CG parameter is given by

$$\beta_{k_2} = \frac{(y_k - s_k)^T F(x_{k+1})}{y_k^T d_k}, \; k = 1,2,3,\cdots \tag{2}$$

Considering the concept of [18,18,20], the new CG parameter β_k is given by:

$\beta_k = (1-\alpha) + \alpha\beta_{k_2}$, where α is the hybridization parameter and is a scalar satisfying $0 < \alpha < 1$. If $\alpha = 1$, then $\beta_k = \beta_{k_2}$ while $\beta_k = \beta_{k_1}$ if $\alpha = 0$.

Thus,

$$\beta_k = (1-\alpha)\frac{(\theta_k y_k - s_k)^T g_{k+1}}{y_k^T \theta_k} + \alpha \frac{(y_k - s_k)^T F(x_{k+1})}{y_k^T d_k} \text{ for } k = 1,2,3,\cdots, \theta_k = \frac{s_k^T s_k}{s_k^T y_k} \tag{3}$$

The direction d_k is assumed to be a decent one and is obtained using

$$d_k = \begin{cases} -F(x_k) \;\; if \; k = 0 \\ -F(x_{k-1}) + \beta_k d_{k-1} \quad if \; k \geq 1 \end{cases} \tag{4}$$

where β_k is term as conjugate gradient parameter. The CG methods for solving nonlinear systems of equations generates an iterative points $\{x_k\}$ from initial given point x_0 via

$$x_{k+1} = x_k + \gamma_k d_k \tag{5}$$

where $\gamma_k > 0$ is attained via line search. Using the above procedure, the following is the algorithm for the new update.

Algorithm:

Step 1: Given $x_0, \gamma > 0, \alpha = 0.6, \; \sigma\epsilon(0,1)$ and $\epsilon > 0$ compute $d_0 = -F(x_0)$, set $k = 0$.

Step 2: Compute $F(x_k)$ and test the stopping criterion, i.e. $||F(x_k)|| \leq \epsilon$, if yes, then stop, otherwise continue with next step.

Step 3: Compute γ_k by using the line search in [16,17,21,22]

Step 4: Compute $\theta_k = \frac{y_k^T y_k}{y_k^T s_k}$, and new β_k.

Step 5: Compute $x_{k+1} = x_k + \alpha_k d_k$

Step 6: Compute search direction using (4)

Step 7: Set $k = k + 1$ and go to step 2.

3. NUMERICAL RESULT

In this section, the numerical results of the proposed method is given. The proposed hybrid method is denoted as M1, and its performance is compared with the classical conjugate gradient parameter methods denoted as M2 and M3 respectively. In Tables 1 and 2, each n-dimensional problem with initial starting point is considered as one problem, therefore a total of 40 benchmark problems were solved. The computational experiment is based on the number of iterations and CPU time. To ascertain the global convergence of the proposed method, the benchmarks problems in [21, 22] was used with two different ISP, and the output of the performance of the methods was based on the performance profile presented by Dolan and More [24].

Table 1. Performance profile of M1, M2 and M3 based on iteration number & CPU time

Prob	ISP	Dim	M1		M2		M3	
			Iter	CPU	Iter	CPU	Iter	CPU
1a	$(0.5,0.5,\cdots,0.5)^T$	10	1	0.006806	13	0.015034	17	0.014147
		100	1	0.000632	14	0.002887	18	0.002249
		500	1	0.000366	14	0.003004	18	0.008648
		1000	1	0.000433	14	0.003024	19	0.009221
		5000	1	0.001403	14	0.123704	21	0.057777
1b	$(0.2,0.2,\cdots,0.2)^T$	10	5	0.000567	10	0.001078	15	0.001418
		100	6	0.001142	10	0.001165	18	0.003607
		500	6	0.001000	11	0.001951	18	0.004433
		1000	6	0.001395	11	0.004255	18	0.007740
		5000	7	0.006488	11	0.013589	21	0.055354
2a	$(0.5,0.5,\cdots,0.5)^T$	10	6	0.000580	4	0.000809	18	0.001765
		100	6	0.000565	4	0.000893	19	0.002758
		500	6	0.000904	5	0.001237	20	0.004726
		1000	6	0.001257	5	0.001637	21	0.008809
		5000	6	0.006039	6	0.006157	22	0.056553
2b	$(0.2,0.2,\cdots,0.2)^T$	10	4	0.000594	5	0.002852	15	0.002374
		100	5	0.000736	6	0.000828	15	0.002038
		500	5	0.000884	6	0.001150	16	0.004103
		1000	5	0.001982	6	0.001678	16	0.007631
		5000	5	0.005914	7	0.005991	18	0.045659

Table 2. Performance profile of M1, M2 and M3 based on iteration number & CPU time (cont)

Prob	ISP	Dim	M1		M2		M3	
			Iter	CPU	Iter	CPU	Iter	CPU
3a	$(0.5,0.5,\cdots,0.5)^T$	10	6	0.000583	4	0.000748	18	0.001662
		100	6	0.000502	4	0.000791	19	0.002683
		500	6	0.000705	5	0.001285	20	0.004508
		1000	6	0.000978	5	0.001404	21	0.007613
		5000	6	0.004791	6	0.005932	22	0.049457
3b	$(0.2,0.2,\cdots,0.2)^T$	10	4	0.000597	5	0.000625	15	0.001215
		100	5	0.000629	6	0.000764	15	0.001608
		500	5	0.000863	6	0.001065	16	0.003806
		1000	5	0.001085	6	0.002203	16	0.006194
		5000	5	0.005834	7	0.005344	18	0.048274
4a	$(0.5,0.5,\cdots,0.5)^T$	10	3	0.000345	1	0.009970	51	0.005631
		100	3	0.000391	1	0.000614	63	0.008490
		500	3	0.000423	1	0.000783	69	0.019573
		1000	4	0.000483	1	0.001370	73	0.032623
		5000	4	0.001014	1	0.004664	79	0.149735
4b	$(0.2,0.2,\cdots,0.2)^T$	10	2	0.105179	F	0.000493	60	0.006403
		100	4	0.126227	F	0.000784	70	0.013660
		500	4	0.212208	F	0.000963	78	0.022243
		1000	4	0.334854	F	0.005148	80	0.033186
		5000	4	1.369668	F	0.004322	88	0.159905

The performance profile P: $R \rightarrow [0,1]$ is defined as follows: Let P and S be the set of problems and set of solvers respectively. For n_s solvers and n_p problems, and for each problem $p \in P$ and for each solver $s \in S$, $t_{p,s}$:=(number of iterations required to solve problem p by solver s) is defined. The performance ratio is given by:

$$r_{p,s} := t_{p,s}/min\{t_{p,s}\}.$$

Then the performance profile is defined by:

$$P(\tau) := \frac{1}{n_p} size\{p \in P: r_{p,s} \leq \tau\},$$

for all $\tau \in R$ where $P(\tau)$ is the probability for solver $s \in S$ that a performance ratio $r_{p,s}$ is within a factor $\tau \in R$ of the best possible ratio. The code for the proposed method was done using MATLAB 7.1, R2009b programming environment and run on a personal computer 2.4GHz, Intel (R) Core (TM) i7-5500U CPU processor, 4GB RAM memory and on windows XP operator. Both the methods was implemented with the same parameters as $\gamma_1 = 0.01, r = 0.2, \sigma_1 = \sigma_2 = 10^{-4}$, and $\eta_k = \frac{1}{\{k+1\}^2}$. The search is stopped if: (i) $\| F(x_k) \| < \epsilon$ with $\epsilon < 10^{-4}$ or (ii) The total number of iteration exceeds 1000.

The meaning of each column in the tables are stated as follows, "P": stands for Benchmark problem, "ISP": stands for initial starting points, "n": stands for dimension of the test problems, "Iter": the total number of iterations and "CPU": the processing time in seconds. A particular problem i, performs better if the number of iteration (iter) and/or the CPU time in seconds (Time) is less than the number of iteration or the CPU time corresponding to the other methods respectively.

From the above tables, the proposed method outperformed the two other methods in terms of CPU times and number of iterations, as shown in Figures 1 and 2.

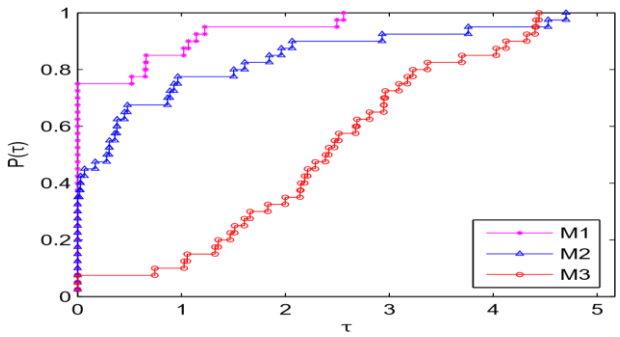

Figure 1. Performance profile of M1, M2 and M3 with respect to the number of iterations

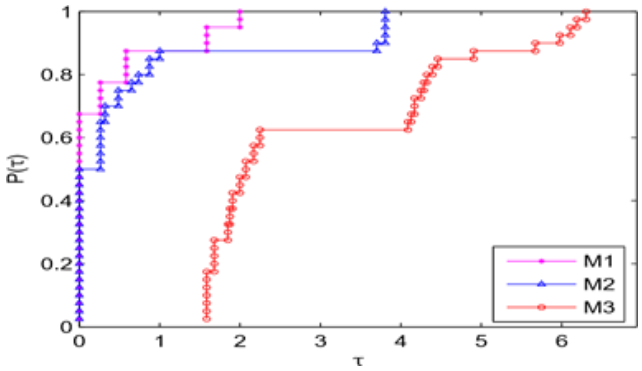

Figure 2. Performance profile of M1, M2 and M3 with respect to the CPU time

4. CONCLUSION AND FUTURE WORK

In this article, a derivative free method for solving symmetric nonlinear systems of equations is given. The hybrid method achieved its objectives fully, i.e. it is faster in terms of processing time and effective in terms of number of iterations. Thus, the hybrid conjugate gradient parameter is a very good alternative for solving symmetric nonlinear systems of equations. For further research work, the method can be modified to solve nonsmooth nonlinear equations.

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