Fractals on IPv6 Network Topology

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Abstract
The coarse-grained renormalization and the fractal analysis of the Internet macroscopic topology can help people better understand the relationship between the part and whole of the Internet, and it is significant for people to understand the essence of the research object through a small amount of information. Aiming at the complexity of Internet IPv6 IP-level topology, we put forward a method of core-threshold coarse-grained to renormalize its topology. By analyzing the degree distribution and degree correlation characteristics in each k-core network topology, the scale invariance of the networks of coarse-grained renormalization was illustrated. The fractal dimension of Internet IPv6 IP-level topology was further computed which shows that the Internet IPv6 IP-level topology has got fractals.

Keywords*: complex network, coarse-grained renormalization, fractal, self-similarity.*

1. Introduction

Being a prototypical instance of complex network, the analysis on the Internet microscopic topology has become a hot topic at present and has attracted more and more attention of academia. In recent years, researchers in this field have made considerable progress [1-8]. But most of them were concentrated in the whole of the topology, while rare researches was put on the internal features of the network, as well as the relationship between parts and whole, part and part of the network. For understanding the architecture of complex networks in the real world, easy to find links between them and the inherent laws, dividing or crunching the whole topology into a small group in accordance with certain rules is a meaningful typical research project in the fields of complex network. Song [9] creatively described the self-similarity of complex networks with the fractal dimension of the fractal theory which provides a new perspective for people to study the complex network. The coarse-grained renormalization can more detailed describe the complexity of real world network topology from small Groups.

For deeply understanding the essence of the Internet, it is necessary to choose an appropriate coarse-grained method to renormalize the network and further explore the relationship of topology features between the renormalized networks with the original network. In the paper, the core-threshold coarse-grained method was introduced to renormalize the IPv6 IP-level network topology based on the massive actual data of CAIDA ARK from January 2010 to December 2010. Then the self-similarity between the whole and part, part and part of the network was analyzed by analyzing some topological characteristics, and thus the fractals of the Internet IPv6 IP-level topology was studied.

2. Research Method

Core-threshold coarse-grained renormalization chooses the coreness of the network as the basis for renormalizing the network. The related definitions are defined as follows:

Let us consider a graph $G = (V, E)$ of $|V| = n$ vertices and $|E| = e$ edges, the k-core is defined as follows:

Definition 1 *k*-core [10]: A sub-graph $H = (C, E|C)$ induced by the set $C \subseteq V$ is a *k*-core or a core of order k iff $\forall v \in C : \deg ree_H(v) \geq k$, and H is the maximum sub-graph with this property. The number of the nodes in the *k*-core is called the size of it.

Definition 2 coreness [11]: A vertex *i* has coreness k if it belongs to the *k*-core but not to (*k*+1)-core. The highest coreness of the vertex in the graph is called the coreness of the graph, also known as the topology coreness of the network.

Definition 3 coarse-grained renormalization: Coarse-grained renormalization picks out the nodes with some common features from the original network to constitute a new network with fewer nodes. This process can repeat until the network is empty.

The core-threshold coarse-grained renormalization recursively removes the nodes of degree less than k and edges attached to them in the network, until all nodes in the remained network have at least degree k. So we can use the core-threshold coarse-grained renormalization to peel the original network layer by layer until the inner core of the network, that is the max k which the network is not empty, we denote it as k_{max}, and thus reveal the hierarchical features of the network.

Using core-threshold coarse-grained method to renormalize the IPv6 IP-level topology from January 2010 to December 2010, the results are illustrated in table 1.

Table 1. The Results of the Coarse-grained Renormalization of the IPv6 IP-level Topology from January 2010 to December 2010

Time / k-core	2010.01	2010.02	2010.03	2010.04	2010.05	2010.06	2010.07	2010.08	2010.09	2010.10	2010.11	2010.12
1	8508	8641	9065	9154	9099	9502	9406	12451	13291	13859	14513	16165
2	6433	6558	6915	6955	6871	7339	7210	9432	10324	10869	11507	12696
3	3214	3292	3445	3538	3286	3582	3467	4832	5555	6075	6533	7328
4	1739	1785	1898	1861	1629	1752	1657	2533	2983	3571	3761	4384
5	957	1047	1061	1127	922	1008	996	1459	1677	2118	2198	2646
6	507	549	644	682	549	636	646	903	1050	1398	1457	1805
7	279	377	413	404	302	462	488	627	779	1033	1150	1284
8	218	210	259	294	120	269	329	418	465	729	894	977
9	104	117	181	165	80	76	231	289	338	552	605	694
10	71	55	84	35		62	173	152	244	404	467	560
11		24	32	24			46	66	130	274	321	471
12								26	87	165	181	266
13									78	152	160	172
14										129	140	143
15										55	46	128

The content in the table described the size of the network (k-core), which was obtained by renormalizing the network using core-threshold coarse-grained method. It is showed that with the depth of the coarse-grained renormalization of the network, the scale of the network continues to shrink, after a dozen steps, the process of renormalization is over.

3. Results and Discussion
3.1. Scale Invariance
Definition 4 scale invariance: When enlarge or shrink any local area within the effective range of fractal system, its characteristics such as shape, complexity or irregularity do not change. All self-similarity system must meet this feature.

Degree distribution is the most common characteristic considered in the complex network. The degree distribution of k-cores obtained by the core-threshold coarse-grained renormalization of the IPv6 IP-level topology in June 2010 is shown in Figure 1.

Here, Figure (a) describes the topologies of core 1 to 5, and Figure (b) describes the topologies of core 6 to 10. It can be seen from the Figure that the number of the nodes with the lowest degree in each core is maximum. With the increase of the degree, the number of the nodes with the degree decreases rapidly. The fitness results are shown as Figure 2 which we can see that the distribution of the degree meets power-low in lower cores (Figure (a)), while the power-low of the degree distribution in higher cores (Figure (b)) is not obvious.

The key factor to measure a network or set has got fractals or not is the change of the degree distribution exponent under different observation scales. If the network is fractals, the degree distribution exponent under different observation scales has little change, that is, it has got scale invariance.
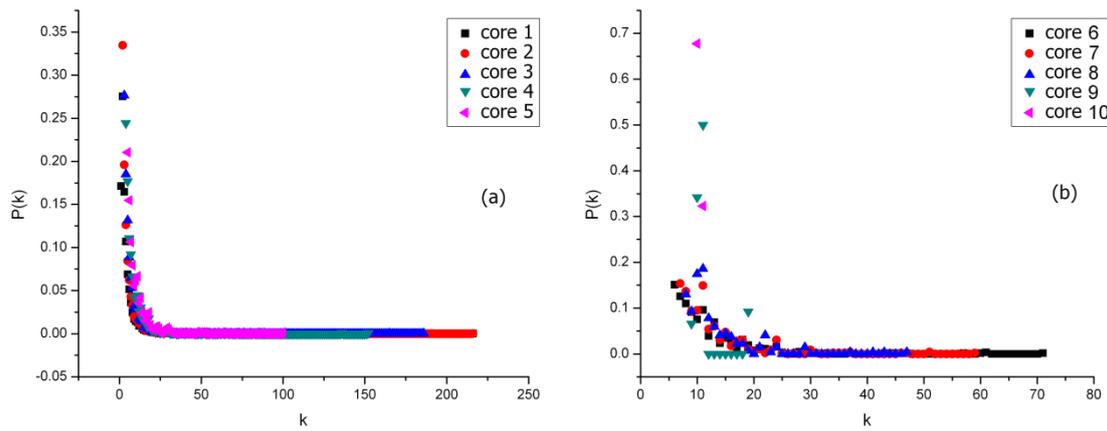
Figure 1. The Degree Distribution of *K*-Cores of the IPv6 IP-Level Topology In June, 2010
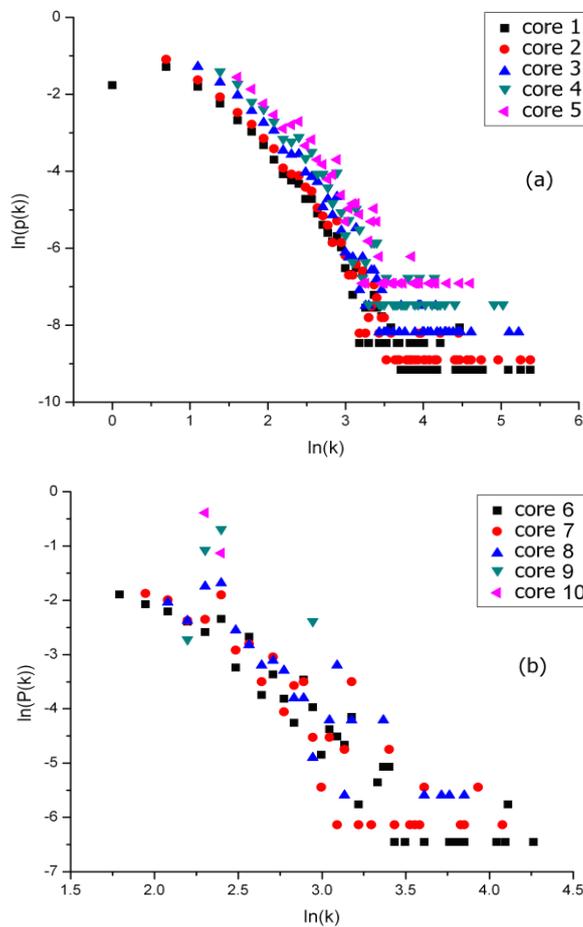
Figure 2. The Fitness Results of The Degree Distribution of *K*-Cores of the IPv6 IP-level Topology in June, 2010

It can be seen from the above analysis that the degree distribution of the nodes in lower cores has not been affected by the observation scales. It shows clear power-law features, and the exponent has little change, it is fluctuate at 2.07. While the feature of power-low of degree distribution disappears in higher cores, which is because the number of the nodes in higher

cores is little. The above analysis shows that the whole IPv6 IP-level topology and the coarse-grained networks have all got the power-law of degree distribution which indicates that the IPv6 IP-level topology is self-similarity.

3.2. Degree Correlation

Degree correlation is another important measure in the research fields of complex network. Definition 5 average degree of nearest neighbours [12]: Average degree of nearest neighbours of the node with degree k, $k_{nn}(k)$, is defined as follows：

$$k_{nn}(k) = \frac{1}{n_k} \sum_{j/k_j=k} \frac{1}{n_j} \sum_{i \in V(j)} k_i \qquad (1)$$

In which, $V(j)$ is the set of the neighbour nodes of node j, n_j is the number of the neighbour nodes of node j, n_k is the number of the nodes with degree k.

Definition 6 accumulation proportion of the average degree of nearest neighbours: Accumulation proportion of the average degree of nearest neighbours is defined as follows:

$$K(k \geq d) = \frac{\sum_{i=d}^{\max(\deg ree)} k_{nn}(i)}{\sum_{i=1}^{\max(\deg ree)} k_{nn}(i)} \qquad (2)$$

Figure 3 describes the accumulation proportion of the average degree of nearest neighbours of each k-core obtained by the core-threshold coarse-grained renormalization of IPv6 IP-level topology of April, 2010. In which, Figure (a) and (b) describe the changes of the accumulation proportion of the average degree of nearest neighbours of core 1 to 6 and 7 to 11 separately.
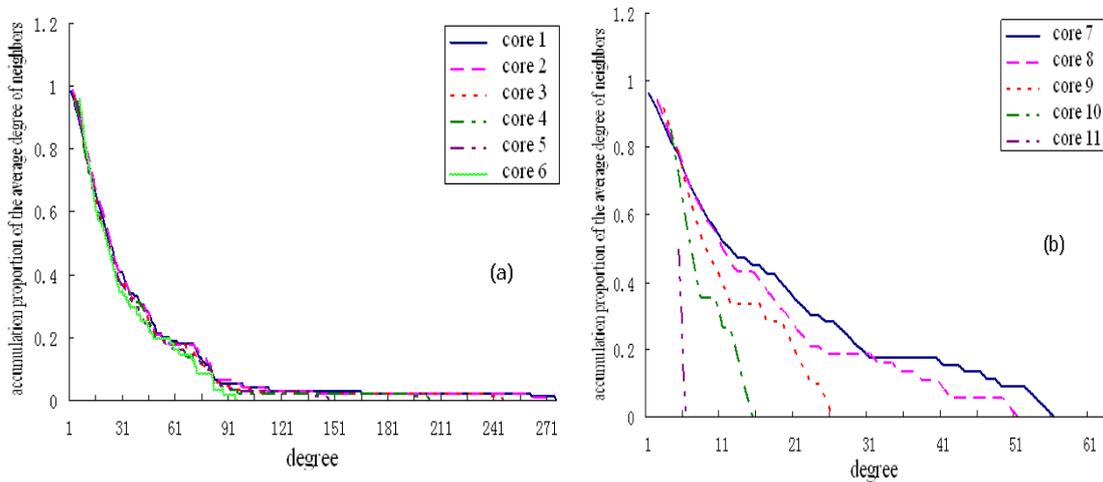
Figure 3. The Changes of the Accumulation Proportion of the Average Degree of Nearest Neighbours of the Coarse-Grained Networks of IPv6 IP-level Topology

3.3. Fractal Dimension

The self-similarity of a system refers to that the structure characteristics of the system are similar from the different spatial scales or time scales, or the parts of the system similar to the whole. In addition, there also exists self-similarity between the whole and parts of the system or among parts. In general, self-similarity has a more complex form of expression, rather than simple coincident between the amplified local areas and the whole.

While, the quantitative nature of characterizing the self-similarity system, such as fractal dimension, does not change with the operation such as zoom in or out, the change is only its external manifestations [13].

The fractal dimension must be defined within some ranges, that is, there is an upper and lower limit. For objects existing in the reality, its fractal characteristics must be within a limited range of observation scales. The fractal dimension is meaningful in this range. The common methods for computing the fractal dimension are listed as follows [13]:

(1) Computing the fractal dimension according to the observation scales
(2) Computing the fractal dimension according to the relationship of measurement
(3) Computing the fractal dimension according to the correlation function
(4) Computing the fractal dimension according to the distribution function
(5) Computing the fractal dimension according to the spectrum.

Considering the hierarchy depth of the network, we adopt the forth method, according to the distribution function, to compute the fractal dimension.

The coreness of node i is denoted as k_i, the corresponding coreness distribution is denoted as $P(k_i)$, then the distribution function of k-core is denoted as follows:

$$P(k) = \sum_{k_i=k}^{k_{max}} p(k_i) \tag{3}$$

For any k larger than K, its distribution function $P(k)$ is:

$$P(k) = P(k > K) = \sum_{i=K+1}^{k_{max}} P(i) \tag{4}$$

For observation scale K, we transform it with $K = \lambda K$, then for any $\lambda > 0$, we have got:

$$P(K) \propto P(\lambda K) \tag{5}$$

Then the distribution function of IPv6 IP-level network topology, $P(K)$, satisfies the following power type:

$$P(K) \propto K^{-D} \tag{6}$$

In which, the exponent D is the fractal dimension of IPv6 IP-level network topology. For any k within the effective range of fractals, its fractal dimension D is constant. The fractal dimensions of IPv6 IP-level topology from January 2010 to December 2010 computed with above method are shown as table 2. It can be seen that the fractal dimensions are not integers, which shows that the Internet IPv6 IP-level topology has fractal characteristics.

Table 2. The Fractal Dimensions of IPv6 IP-level Topology From January 2010 to December 2010

Time	Fractal dimensions	Time	Fractal dimensions	Time	Fractal dimensions
2010.01	0.590	2010.05	0.652	2010.09	0.461
2010.02	0.596	2010.06	0.567	2010.10	0.387
2010.03	0.547	2010.07	0.465	2010.11	0.388
2010.04	0.554	2010.08	0.508	2010.12	0.360

By analyzing the fractal features of IPv6 IP-level topology on the time evolution shows that the fractal dimension is decrease in the time series which mean that the fractal feature is more and more obvious, it is because of the enhancement of mutually exclusive nature between the nodes with higher degree.

4. Conclusion

We proposed a core-threshold coarse-grained method to renormalize the IPv6 IP-level topology based on the concept of k-core in the paper. It is concluded that the degree distribution

exponent does not change with the coarse-grained measurement scale, as well as the curves of the accumulation proportion of the average degree of nearest neighbours in the renormalized networks are coincident. The above studies have shown that in the process of coarse-grained renormalization of the IPv6 IP-level topology, the obtained *k*-cores are self-similarity, they are similar with the whole network, but with the deepening of the process of renormalization, the self-similarity gradually disappeared. We further computed the fractal dimensions using distribution function method in fractal theory, and find that it is not integer which means that the IPv6 IP-level topology has fractal features.

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