Improved newton-raphson with schur complement methods for load flow analysis

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Article Info	ABSTRACT
	The determination of power and voltage in the power load flow for the purpose of design and operation of the power system is very crucial in the assessment of actual or predicted generation and load conditions. The load flow studies are of the utmost importance and the analysis has been carried out by computer programming to obtain accurate results within a very short period through a simple and convenient way. In this paper, Newton-Raphson method which is the most common, widely-used and reliable algorithm of load flow analysis is further revised and modified to improve the speed and the simplicity of the algorithm. There are 4 Newton-Raphson algorithms carried out, namely Newton-Raphson, Newton-Raphson constant Jacobian, Newton-Raphson Schur Complement and Newton-Raphson Schur Complement constant Jacobian. All the methods are implemented on IEEE 14-, 30-, 57- and 118-bus system for comparative analysis using MATLAB programming. The simulation results are then compared for assessment using measurement parameter of computation time and convergence rate. Newton-Raphson Schur Complement constant Jacobian requires the shortest computational time.

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1. INTRODUCTION

An electric power system must be able to provide powers demanded by loads ranging from household electrical appliances to industrial heavy machines sufficiently, efficiently and economically, at all time while maintaining a stable voltage level and frequency [1]. Power system becomes more important as the load demand increases worldwide [2]. Power flow analysis plays an important role in planning environments for different network configurations design to sufficiently serve an expected future increasing load. Besides, it is an operational tool for monitoring the real-time status of the network in terms of voltage magnitudes and circuit flows. Power flow analysis has been used in a very large extent in system planning, design and operation to review the requirements for steady-state conditions [3-7]. Therefore, research works on advanced power flow analysis have been carried out actively.

Early developments of load flow analysis techniques focusing on Newton-Raphson method started with the development of the first efficient sparsity-oriented implementation of Newton-Raphson power flow algorithm by Tinney and Hart. There are two important features to be referred and emphasized for further evolvements, i.e. robust convergence and computational efficiency [8]. Therefore, Newton-Raphson load flow analysis method has obviously become the center of load flow algorithms as many enhancements have been formed based on it [9-15]. The main disadvantage of the Newton-Raphson method is the computational complication which is the necessity for factorizing and updating the Jacobian matrix during the iterative solution process [16]. One of the issues in Newton-Raphson method is the formation and update of Jacobian

matrix and its inversion repeatedly for every iteration. One of the enhancements on Newton-Raphson method is constant Jacobian power flow methods which were developed to improve and enhance the computational efficiency. One of the examples is Stott and Alsac's Fast-decoupled power flow algorithm which has emerged as widely-used general purpose algorithm [17, 18]. This paper presents the improved Newton-Rahpson with Schur Complement methods for load flow analysis. Modification and improvement are implemented by combining Newton-Raphson method with Schur Complement methods. Moreover, Newton-Raphson method is modified by having a constant Jacobian matrix throughout the whole iterative computing process which is simpler and has faster increasing computational efficiency while preserving the accuracy precisely.

2.   RESEARCH METHOD

Newton-Raphson approach in load flow analysis is improved to reduce computing time and memory required. In the enhanced method, Newton-Raphson using Schur complement method, dividing the Jacobian matrix into two separated matrices with reasonable computation time is believed to help in avoiding divergence of the solution. The Schur complement technique is a procedure to eliminate the interior variables in each subdomain and derive a global, reduced in size and linear system involving only the interface variables. Schur method can be used to separate the unknown variables in interface unknown variables and sub-domain internal unknown variables meaning that the off-diagonal effects can be reduced or eliminated [19]. This algorithm has been applied successfully to reduce computation time of load flow iterations, memory required and convergence strategy. The mathematical detail of Newton-Raphson with Schur Complement Method is presented in this section.

The modifications are in the Jacobian matrix.

$$J = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{1}$$

where A = J_1, B = J_2, C = J_3 and D = J_4

J_1 has the order of $(n-1) \times (n-1)$ while J_4 has the order of $(n-1-m) \times (n-1-m)$. Both J_1 and J_4 have the square dimension which is invertible.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \tag{2}$$

Solving (2),

$$Ax + By = a \tag{3}$$

$$Cx + Dy = b \tag{4}$$

Expressing x from (3),

$$x = A^{-1}(a - By) \tag{5}$$

Substituting (5) into (4),

$$C[A^{-1}(a - By)] + Dy = b \tag{6}$$

Rearranging (6),

$$(D - CA^{-1}B)y = b - CA^{-1}a \tag{7}$$

Expressing y from (7),
$$y = (D - CA^{-1}B)^{-1}(b - CA^{-1}a)$$
$$= -(D - CA^{-1}B)^{-1}CA^{-1}a + (D - CA^{-1}B)^{-1}b \tag{8}$$

Expressing y from (4),

$$y = D^{-1}(b - Cx) \tag{9}$$

Substituting (9) into (3),

$$Ax + B[D^{-1}(b - Cx)] = a \tag{10}$$

Rearranging (10),

$$(A - BD^{-1}C)x = a - BD^{-1}b \tag{11}$$

Expressing x from (11),

$$\begin{aligned}x &= (A - BD^{-1}C)^{-1}(a - BD^{-1}b) \\ &= (A - BD^{-1}C)^{-1}a - (A - BD^{-1}C)^{-1}BD^{-1}b\end{aligned} \tag{12}$$

From (2),

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} a \\ b \end{bmatrix} \tag{13}$$

By referring to (8) and (12),

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} (A - BD^{-1}C)^{-1} & -(A - BD^{-1}C)^{-1}BD^{-1} \\ -(D - CA^{-1}B)^{-1}CA^{-1} & (D - CA^{-1}B)^{-1} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \tag{14}$$

Therefore by comparing (13) and (14),

$$J^{-1} = \begin{bmatrix} (A - BD^{-1}C)^{-1} & -(A - BD^{-1}C)^{-1}BD^{-1} \\ -(D - CA^{-1}B)^{-1}CA^{-1} & (D - CA^{-1}B)^{-1} \end{bmatrix} \tag{15}$$

Expressing J^{-1} in terms of X_1, X_2, Y_1 and Y_2,

$$J^{-1} = \begin{bmatrix} X_1 & -X_1 Y_1 \\ -X_2 Y_2 & X_2 \end{bmatrix} \tag{16}$$

where

$$X_1 = (A - BD^{-1}C)^{-1} \tag{17}$$

$$X_2 = (D - CA^{-1}B)^{-1} \tag{18}$$

$$Y_1 = BD^{-1} \tag{19}$$

$$Y_2 = CA^{-1} \tag{20}$$

By applying to Equation of Newton-Raphson,

$$\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} X_1 & -X_1 Y_1 \\ -X_2 Y_2 & X_2 \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{21}$$

where

$$\Delta\delta = X_1(\Delta P - Y_1 \Delta Q) \tag{22}$$

$$\Delta V = X_2(\Delta Q - Y_2 \Delta P) \tag{23}$$

The steps taken in Newton-Raphson with Schur Complement load flow analysis method are simplified in Figure 1.
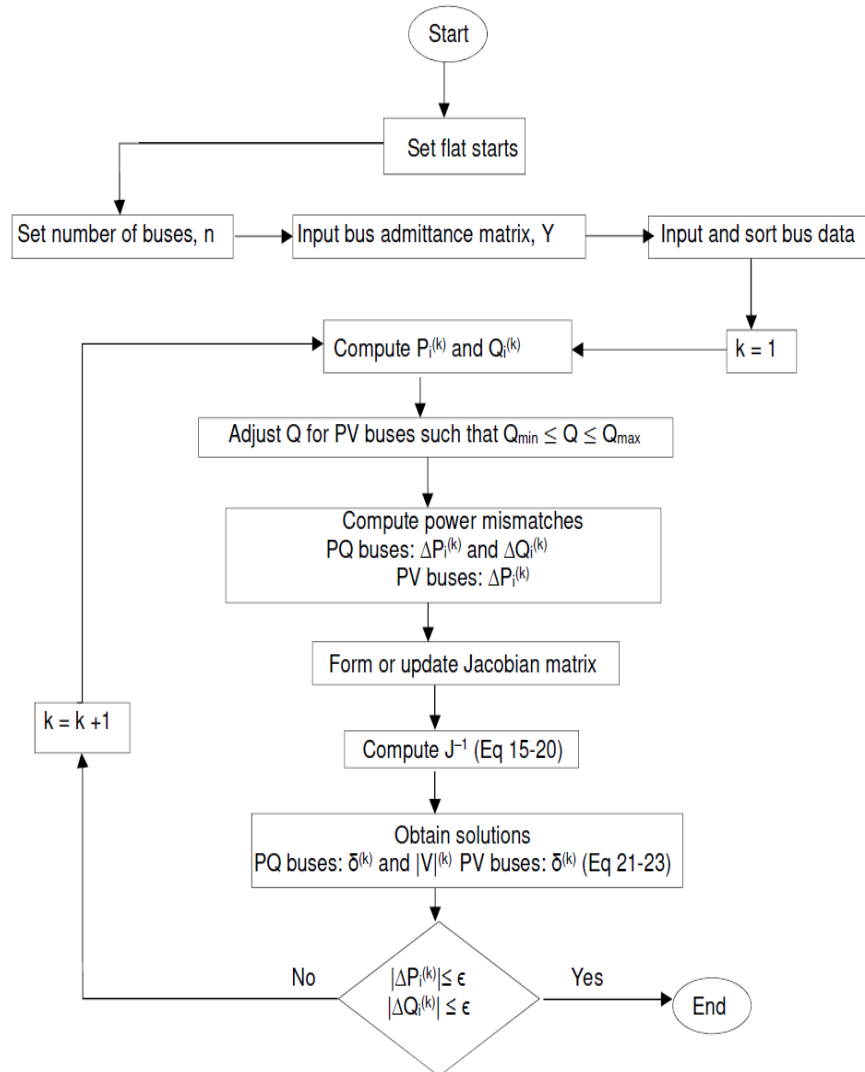
Figure 1. Flowchart for Newton-Raphson with Schur Complement method

Equations formed in power system analysis are generally nonlinear, thus linearized Jacobian matrix is employed to determine the final solutions by iterative method. Generally, Jacobian matrix is a matrix of first-order partial derivatives of each vector function comprising at least 2 or more variables.

In conventional Newton-Raphson method, Jacobian matrix is constructed by taking first-order partial differentiations of real powers and reaction powers with respect to voltage magnitudes and voltage angles [20]. Jacobian matrix is updated for every iteration using the values obtained from last iteration. Using this appropriate approach, the solutions with mismatches which are equal to or less than the range of the pre-defined error can obtain in 4 to 5 iterations regardless of the number of buses in the handled system implying excellent converging characteristics. Nonetheless, in an n-bus system with a slack node and m PV nodes, Jacobian matrix is with an order of $(2n - 2 - m) \times (2n - 2 - m)$ meaning that the dimension of Jacobian matrix increases with the number of system buses.

On the other hand, Newton-Raphson with Schur Complement method has a simpler way to evaluate the inverse of Jacobian matrix as the matrix is separated into 4 main matrixes with smaller dimensions namely $(n - 1) \times (n - 1)$ A, $(n - 1) \times (n - 1 - m)$ B, $(n - 1 - m) \times (n - 1)$ C and $(n - 1 - m) \times (n - 1 - m)$ D. The inverse of Jacobian matrix involves only the computation of the inverse of matrix A and matrix D which theoretically needs less time compared to Newton-Raphson method, but this method takes long time required for re-calculating the inverse of Jacobian matrix for every iteration.

To solve this issue, the methods are improved using constant Jacobian matrix throughout the whole computing process which means using Jacobian matrix calculated in the first iteration for the following

iterations until the results converge [21-25]. It is not surprising that the number of iterations increases a lot compared to using iteratively updated Jacobian matrix as the converging rate decreases. However, by eliminating the time-consuming process of repeatedly calculating Jacobian matrix, the computational speed increases significantly.

The load flow analysis using Newton-Raphson algorithm and other improved methods based on Newton-Raphson method are derived and they are:
1. The conventional Newton-Raphson method
2. Newton-Raphson with Schur Complement method
3. Newton-Raphson method using constant Jacobian
4. Newton-Raphson with Schur Complement method using constant Jacobian

These four algorithms are applied to different standard bus systems to test and analyses the reliability, accuracy, convergence rate and also the computing time comparatively. There are 4 bus systems used in this research, i.e. IEEE 14-bus system, IEEE 30-bus system, IEEE 57-bus system and IEEE 118-bus system. The performance and effectiveness of the algorithms based on criteria such as converging rate denoted by the number of iterations and computing time are measured and analyzed.

3.   RESULTS AND ANALYSIS

The four algorithms described in previous section are implemented on four different bus systems to compare the effectiveness of these approaches. The four 4 different bus systems are:
1. IEEE 14-bus system consisting of 5 generators and 20 lines and transformers
2. IEEE 30-bus system consisting of 6 generators and 41 lines and transformers
3. IEEE 57-bus system consisting of 7 generators and 80 lines and transformers
4. IEEE 118-bus system consisting of 54 generator and 186 lines and transformers

The load flow algorithms are represented by using abbreviations as follows:
1. NR - The conventional Newton-Raphson method
2. NRSC - Newton-Raphson with Schur Complement method
3. NRcj - The conventional Newton-Raphson method using constant Jacobian
4. NRSCcj - Newton-Raphson with Schur Complement method using constant Jacobian

These algorithms are implemented on 64-bit laptop, Operating System of Windows 8, processor of Intel(R) Core(TM) i5-3337U CPU @ 1.80GHz, Ram of 4.00 GB (3.89 GB usable), using Matlab version R2011a. Each algorithm is tested on each bus for at least 5 times to record the computing time. This is to ensure the resultant computing time is as accurate as possible by taking the average of these 5 samples, Tavg. The results are then graphically analysed in different perspectives to identify the extent of improvements. The recorded results for 14-, 30-, 57- and 118-bus systems are presentation in Figure 2 for the computational time and Figure 3 for the iteration number, respectively.
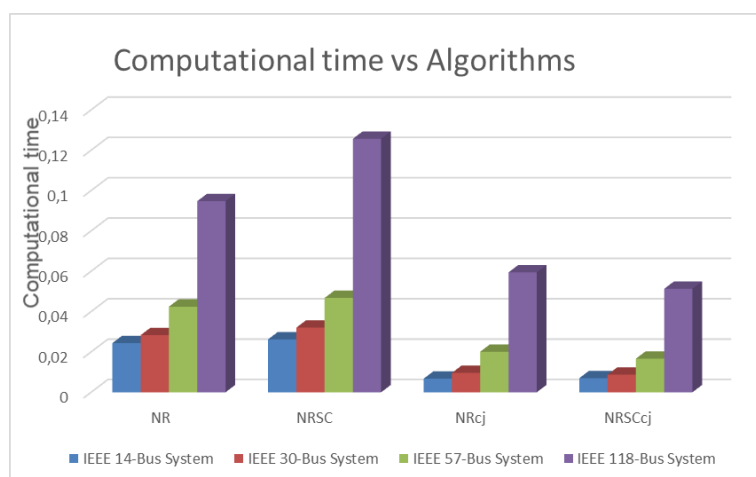

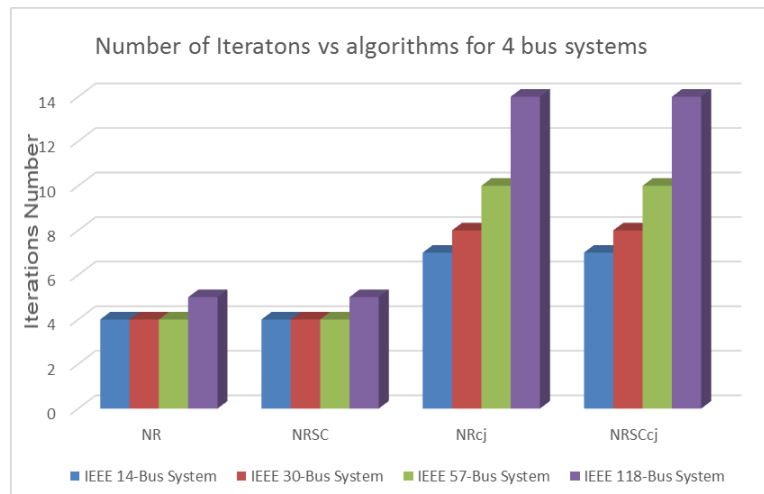Figure 2. Computational time for algorithms

Figure 3. Iteration numbers for algorithms

One of the issues in Newton-Raphson method is the formation and update of Jacobian matrix and its inversion repeatedly for every iteration. This issue causes long computational time required to obtain the final solution. Besides taking much computational time, this also increases the complexity and burdened the calculating process particularly for power systems with large number of buses as the memory requirements and computing time increase with the size of the power system. Moreover, almost immediate results from simulations are required for on-line applications to identify the exact problems during faults occurrence.

Newton-Raphson with Schur Complement involves the inversion of matrixes with fewer dimensions relatively and the inverse of Jacobian can be obtained directly using the derived formula. Besides, constant Jacobian matrix has been applied in NRcj and NRSCcj methods without modifying any of the computational procedures but only replacing the iteratively updated Jacobian matrix with the first iteration Jacobian matrix throughout the whole analysis. These techniques help in saving computing time which is a very crucial issue especially for on-line applications by eliminating the need of updating Jacobian matrix.

NRSC requires slightly longer time as compared to NR. Newton-Raphson and improved Newton-Raphson methods with constant Jacobian matrix, NRcj and NRSCcj show very satisfying results with great improvements in computational time which is less than half of the time of NR and NRSC respectively in most of the bus systems. NRSCcj has slightly shorter time than NRcj for bigger bus systems.

NR and NRSC take 4-5 numbers of iteration to converge although the number of buses has increased. NR and NRSC have lesser number of iterations meaning that the convergence rate is higher. By using constant Jacobian matrix modifications, it is expected to have higher number of iterations. Based on the results obtained, NRcj and NRSCcj need additional 3-9 iterations to converge for the four bus systems.

4.   CONCLUSION

Power flow analysis is carried out by implementing the improved Newton-Raphson algorithms using four types of bus systems via MATLAB. The accurate values of power system parameters such as bus voltages magnitudes and angles, active and reactive power, line flows and line losses have been successfully obtained. One of the improving approaches is carried out using constant Jacobian modifications in the conventional Newton-Raphson method (NR) as well as Newton-Raphson with Schur Complement method. By replacing of constant Jacobian matrix, the first iteration of Jacobian matrix formed using flat starts is used throughout the whole computing process instead of updating it for every iteration. This has saved up much computational time at the same time simplifying and reducing the computing steps. While for NRSC which is expected to have shorter computational time has a disappointing result of longer time taken than NR but it has the shortest computational time when being implemented with constant Jacobian. As a conclusion, the traditional Newton-Raphson load flow analysis method has been effectively improved and revised to be simpler and faster. These improvements have some positive influences on load flow analysis especially for on-line applications.

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REFERENCES

[1] T. Wildi, "Electrical Machines, Drives, and Power Systems Sixth Edition," Pearson Prentice Hall, pp. 706, 2006.
[2] Navid G., et al., "Improvement voltage Stability and Load Ability Enhancement by Continuation Power Flow and Bifurcation Theory," *Indonesian Journal of Electrical Engineering and Informatics (IJEEI),* vol. 1, 2013.
[3] J. Wood and B. F. Wollenberg, "Power Generation, Operation and Control," 2nd ed., New York, John Wiley & Sons, 1996.
[4] Diego I. and Jorge C., "Evaluation of the Forward-Backward Sweep Load Flow Method using the Contraction Mapping Principle," *International Journal of Electrical and Computer Engineering,* vol. 6, pp. 3229-3237, 2016
[5] W. Hubbi, "The mismatch theorem and second-order load-flow algorithms," *Generation, Transmission and Distribution, IEE Proceedings C*, vol. 132, pp. 189-194, 1985.
[6] J. D. Glover, et al., "Power System Analysis & Design," SI Version, Cengage Learning, 2011.
[7] L. Roy and T. A. Mohammed, "Comparative study of two iterative schemes applied to decoupled load flow methods," *Energy Management and Power Delivery, 1995. Proceedings of EMPD '95., 1995 International Conference on*, vol. 1, pp. 277-282, 1995.
[8] W. F. Tinney and C. E. Hart, "Power Flow Solution by Newton's Method," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-86, pp. 1449-1467, 1967.
[9] F. Milano, "Continuous Newton's Method for Power Flow Analysis," *Power Systems, IEEE Transactions on*, vol. 24, pp. 50-57, 2009.
[10] R. S. Salgado and L. V. Barboza, "Corrective Load Adjustments for Unsolvable Power Flows," *Power and Energy Engineering Conference (APPEEC), 2012 Asia-Pacific*, pp. 1-4, Mar 2012.
[11] P. J. Lagace, et al., "Improving power flow convergence by Newton Raphson with a Levenberg-Marquardt method," *Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century, 2008 IEEE*, pp. 1-6, Jul 2008.
[12] Y. Hui, et al., "Newton-Raphson on power flow algorithm and Broyden Method in the distribution system," *Power and Energy Conference, 2008. PECon 2008. IEEE 2nd International*, pp. 1613-1618, Dec 2008.
[13] Y. Yubin, et al., "The effect of small impedance branches on the convergence of the Newton Raphson power flow," *Electric Utility Deregulation and Restructuring and Power Technologies, 2008. DRPT 2008. Third International Conference on*, pp. 1141-1146, Apr 2008.
[14] D. J. Jiao, et al., "Convergence regions of Newton method in power flow studies: Numerical studies," *Circuits and Systems (ISCAS), 2013 IEEE International Symposium on*, pp. 1532-1535, May 2013.
[15] Y. Yubin, et al., "A improved Newton power flow in rectangular form for systems with small impedance branches," *Power Electronics and Intelligent Transportation System (PEITS), 2009 2nd International Conference on*, pp. 10-14, Dec 2009.
[16] S. Kamel, et al., "Improved NR Current Injection Load Flow Using Power Mismatch Representation of PV Bus," *Electrical Power and Energy Systems*, vol. 53, pp. 64-68, 2013.
[17] B. Stott and O. Alsac, "Fast Decoupled Load Flow. Power Apparatus and Systems," *IEEE Transactions on*, vol. PAS-93, pp. 859-869, 1974.
[18] P. R. Bijwe and M. S. Kelapure, "Nondivergent Fast Power Flow Methods," *IEEE Transactions on Power Systems*, vol. 18, pp. 633-638, 2003.
[19] A. A. Ahmed and N. Cristian, "Simulation Approach to Improve Power Analysis Network for Integration of Distributed Generation," *2014 International Conference and Exposition on Electrical and Power Engineering*, pp. 019-024, 2014.
[20] Pooja S. and Navdeep B., "Computational Analysis of IEEE 57 Bus System using N-R Method," *International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering*, pp. 8859-8869, 2015.
[21] L. T. Tay, et al., "Some New Findings on Gauss-Seidel Technique for Load Flow Analysis," *Lecture Notes in Electrical Engineering,* vol. 291, 2014.
[22] C. K. Seng, et al., "Load Flow Analysis Using Improved Newton-Raphson Method," *Applied Mechanics and Materials*, vol. 793, pp. 494-499, 2015.
[23] C. K. Seng, et al., "Load Flow Analysis Using Second-order Load Flow Methods and Its Variations," *Applied Mechanics and Materials,* vol. 785, pp. 73-77, 2015.
[24] L. T. Tay and K. S. Chieng, "Performance Study of Load Flow Algorithms in Well and Ill-Conditioned Systems," *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 8, pp. 414-419, Nov 2017.
[25] Tay L. T., et al., "Improved Newton-Raphson with Levenberg-Marquardt Methods for Load Flow Analysis," *Journal of Fundamental Applied Sciences*, vol. 10, pp. 460-471, 2018.