

## Clonal Selection Algorithm Based Iterative Learning Control with Random Disturbance

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### Abstract

*Clonal selection algorithm is improved and proposed as a method to solve optimization problems in iterative learning control. And a clonal selection algorithm based optimal iterative learning control algorithm with random disturbance is proposed. In the algorithm, at the same time, the size of the search space is decreased and the convergence speed of the algorithm is increased. In addition a model modifying device is used in the algorithm to cope with the uncertainty in the plant model. In addition a model is used in the algorithm cope with the uncertainty in the plant model. Simulations show that the convergence speed is satisfactory regardless of whether or not the plant model is precise nonlinear plants. The simulation test verify the controlled system with random disturbance can reached to stability by using improved iterative learning control law but not the traditional control law.*

**Key words:** *Iterative learning control, Clonal selection algorithm, Martingale, Stable in the conditional expectation*

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### 1. Introduction

Iterative learning control (ILC) is a well-known technique to increase the tracking accuracy of a system repeating a given operation, or to suppress a repetitive disturbance acting on system. Significant progress has been made for both technologies and applications of ILC in the past decade for batch processes, e.g., chemical batch process, (Wang, Shi, and Zhou 2006) industrial robot manipulators, (Krzysztof Tchon 2010) etc.

Recently, a number of references have presented incorporate control methods with ILC. Jian-Xin Xu and Zhi Hua propose a novel nonlinear control scheme for propose systems with robust system. Gunnarsson and Norrlof consider ILC problem with input constrains and use Lagrange multiplier methods to solve this problem. Kira L. Barton, Douglas A. Bristow and Andrew G. Alleyne has presented an alternative method for calculating the ILC norm, which is used as an analysis tool for comparing the convergence rate of different learning algorithms that have been designed elsewhere. And more recently, Dabid H. Owens and Bing Chu has modelled observed behaviour of norm optimal iterative learning control (NOILC) when the plant has one or more non-minimum phase (NMP) zeros. Li Heng-jie, Hao Xiao-hong, and Zeng Xian-qiang realize effective tracking of output of nonlinear systems with constraint and model uncertainty in specified time domain, improved clonal selection algorithms are employed to solve optimization problems in iterative learning control. A clonal selection algorithm based nonlinear optimal iterative learning control is proposed. Fang Guo-hua, Zhong Lin-juan, Wu Xue-wen, Tan Wei-xiong Based on the optimal control theory and relevant technology the optimal input-output control model for water resources utilization and water pollution control is established. The method for determining the main parameters of the model were analyzed and investigated, and the model was solved. Then output level of each sector of national economy, speed of development and the corresponding total amount of water resources use and water pollution control from 2006 to 2010 which meet the sustainable utilization of water resources and the requirement of water pollution control of the Jiangsu Province were obtained.

In this article, Suppose the controlled system is iterative learning control system with random disturbance. Traditional stability definition is generalized for stable in the conditional expectation, clonal selection algorithm is improved and proposed as a method to solve optimization problems in iterative learning control with random disturbance. And the improved

control law is applied into iterative learning control system with random disturbance which its stability theory is constructed. These can meet the requirements of economic development, water resources for the sustainable utilization in the Gansu Province. Some simulation results show the rationability of stable in the conditional expectation and the efficiency of improved iterative learning control law.

## 2. Problem Formulation

Consider the following discrete time, linear time invariant system:

$$\begin{cases} x(t+T_s) = f(x(t), u(t), t) \\ y(t) = f(x(t), u(t), t) \end{cases},$$

where  $t \in [0, T_s, 2T_s, \dots, T_f]$ ,  $T_f = NT_f$ .

The initial condition  $x_k(0) = x_0$ , denotes  $y_d(t)$  as the desired output trajectory.

Ideally, the ILC controller will iteratively generate a command signal, from trial to trial, such that the error  $e_k(t)$  converges to zero.

Defined by the solution of optimization problem

$$u_{k+1} = f(u_k, u_{k-1}, \dots, u_{k-r}, e_{k+1}, e_k, \dots, e_{k-s}) \quad (2)$$

$$\lim_{k \rightarrow \infty} \|e_k\| = 0 \text{ and } \lim_{k \rightarrow \infty} \|u_k - u^*\| = 0$$

where

$$y_k = [y_k(0), y_k(T_s), y_k(2T_s), \dots, y_k(T_f)]^T,$$

$$u_k = [u_k(0), u_k(T_s), u_k(2T_s), \dots, u_k(T_f)]^T,$$

$$e_k = [(y_d(0) - y_k(0) | u_0(0), \dots, u_{k-1}(0)), (y_d(T_s) - y_k(T_s) | u_0(T_s), \dots, u_{k-1}(T_s)), \\ (y_d(2T_s) - y_k(2T_s) | u_0(2T_s), \dots, u_{k-1}(2T_s)), \\ \dots, (y_d(T_f) - y_k(T_f) | u_0(T_f), \dots, u_{k-1}(T_f))]^T$$

## 3. Analysis of Convergence

Consider the following discrete time, linear time invariant system:

$$x_{k+1}(t) = Ax_k(t) + Bu_k(t)$$

$$y_k(t) = Cx_k(t) \quad (1)$$

$$e_k(t) = y_d(t) - y_k(t)$$

$$u_{k+1}(t) = u_k(t) + E[e_{k+1}(t) | u_0(t), \dots, u_k(t)] \quad (2)$$

where  $t$  is the time index (i.e. sample number),  $k$  is the iteration number and  $u_k(t)$ ,  $x_k(t)$  and  $y_k(t)$  are input, state and output of the system on iteration  $k$ . The initial condition  $x_k(0) = x_0$ , ( $k = 1, 2, \dots$ ) is the same for all iteration.  $E[e_{k+1}(t) | u_0(t), \dots, u_k(t)]$

is conditional mean. Note that the matrices  $A$ ,  $B$ ,  $C$  are invertible matrix.

Theorem 1: Consider System (1), control laws (2), if match all of the following

- (1) The matrix  $A - BKC$  is stability matrix,  
 (2)  $\|I - (j\omega I - A + BKC)^{-1}\|_{\infty} < 1$ ,  
 (3)  $x_k(0) = x_0$ , ( $k = 1, 2, \dots$ ), where  $\omega \in (0, \omega_0)$ ,  $\omega_0$  is Frequency of desired trajectory.

As a consequence

$\{u_k(t), k = 1, 2, \dots\}$  for stable in the conditional expectation.

Proof Consider  $\forall t \in [0, T]$ , we have

$$\begin{aligned} x_{k+1}(t) &= e^{At} x_{k+1}(0) + \int_0^t e^{A(t-\tau)} B u_{k+1}(\tau) d\tau \\ &= e^{At} x_{k+1}(0) + \int_0^t e^{A(t-\tau)} B u_k(\tau) d\tau + \int_0^t e^{A(t-\tau)} BKE[e_{k+1}(\tau)|u_0(\tau), \dots, u_k(\tau)] d\tau. \end{aligned}$$

$$\text{So } x_{k+1}(t) = x_k(t) + \int_0^t e^{A(t-\tau)} BKE[e_{k+1}(\tau)|u_0(\tau), \dots, u_k(\tau)] d\tau \quad (3)$$

$$\text{For } x_{k+1}(t) = x_k(t) + \int_0^t e^{A(t-\tau)} BKE[e_{k+1}(\tau)|u_0(\tau), \dots, u_k(\tau)] d\tau,$$

we have

$$\begin{aligned} E[\Delta x_{k+1}(t)] &= E[\Delta x_k(t)] - \int_0^t e^{A(t-\tau)} BKE[e_{k+1}(\tau)|u_0(\tau), \dots, u_k(\tau)] d\tau \\ &= E[\Delta x_k(t)] - \int_0^t e^{A(t-\tau)} BKE[Cx_d(\tau) - x_{k+1}(\tau)] d\tau \\ &= E[\Delta x_k(t)] - \int_0^t e^{A(t-\tau)} BKCE[\Delta x_{k+1}(\tau)] d\tau \end{aligned}$$

Take the Laplace transform, we have

$$\begin{aligned} E[\Delta x_{k+1}(s)] &= E[\Delta x_k(s) - (sI - A)^{-1} BKCx_{k+1}(s)] \\ &= [I + (sI - A)^{-1} BKC]^{-1} E[\Delta x_k(s)] \\ &= [I + (sI - A + BKC)^{-1} BKCE]^{-1} E[\Delta x_k(s)]. \end{aligned}$$

We obtain System (1) is stability.

If  $\|I - (j\omega I - A + BKC)^{-1}\|_{\infty} < 1$  we have

$$\|E[\Delta x_{k+1}(t)]\|_{\infty} = \|[I + (sI - A + BKC)^{-1} BKC]\|_{\infty} \cdot \|E[\Delta x_k(s)]\|_{\infty}$$

It follows that

$$\lim_{k \rightarrow \infty} \|E[\Delta x_k(t)]\|_{\infty} = 0$$

We have

$$\|E[e_k(t)]\|_{\infty} \leq \|C\|_{\infty} \|E\Delta x_k(t)\|_{\infty}$$

$$\text{So } \lim_{k \rightarrow \infty} \|Ee_{k+1}(t)\|_{\infty} = 0$$

It follows that

$$\lim_{k \rightarrow \infty} E[e_{k+1}(t)|u_0(t), \dots, u_k(t)] = \lim_{k \rightarrow \infty} E[y_d(t) - y_{k+1}(t)|u_0(t), \dots, u_k(t)] = 0.$$

This completes the proof of the theorem.

#### 4. Simulation

A saturated nonlinear industrial controlled system by saturated nonlinear part and a linear portion and connected component. The saturated nonlinear part of the expression is as follows:

$$z(t) = \begin{cases} k\beta & u(t) \geq \beta \\ ku(t) & |u(t)| < \beta \\ -k\beta & u(t) \leq -\beta \end{cases}$$

The linear portion of the transfer function

$$G(s) = 1/(2s^2 + 2s + 1)$$

Consider the following system

$$\begin{cases} x(i+1) = Ax(i) + Bu(i) \\ y(i) = Cx(i) \end{cases}$$

$$A = \begin{bmatrix} 1.0646 & -0.7358 \\ 0.5 & 0 \end{bmatrix}, B = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix},$$

$$C = [0.3539 \quad 0.5055] \quad (4)$$

a sampling time of 0.1s.

The desired output trajectory  $y_d(t) = 0, t = 1,$

$$y_d(t) = 1.2(1 - 1/(1+t)^3) \quad t \in [0,25] \quad (5)$$

Where  $-1.5 = u_i^{\min} \leq u_i \leq u_i^{\max} = 1.5 \quad i = 1,2, \dots, 25.$

Figure 1 give output tracking curve, Figure 2 give error log form with the iteration convergence curve. We can see the implementation of the algorithm proposed in this paper, the system achieved good dynamic quality. Output curve is not completely tracking expected trajectory, it is because of the controlled object and the characteristics of the input constraints.

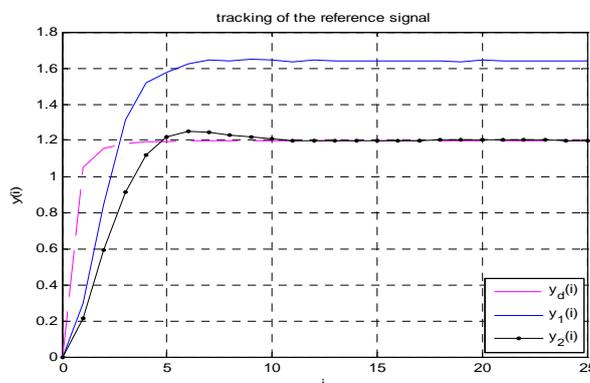


Figure 1. Linear system output to track the curve

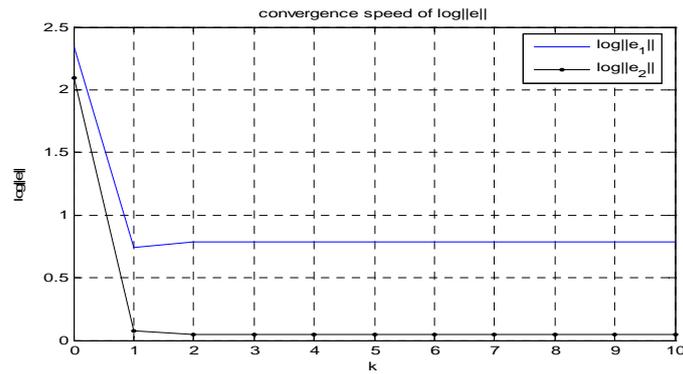


Figure 2.  $\log\|e_k(t)\|$  convergence curve

## 5. Conclusion

Suppose the controlled system is iterative learning control system with random disturbance. Traditional stability definition is generalized for stable in the conditional expectation, clonal selection algorithm is improved and proposed as a method to solve optimization problems in iterative learning control with random disturbance. And the improved control law is applied into iterative learning control system with random disturbance which its stability theory is constructed. Some simulation results show the rationality of stable in the conditional expectation and the efficiency of improved iterative learning control law. The simulation test verify the controlled system with random disturbance can reached to stability by using improved iterative learning control law but not the traditional control law.

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