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Dynamic Filtering Method for GPS Based on Multi-Scale

Wen An[1], Youming Liu*[2], Guowei Wang[2], Zhiyong Liu[2] ,Tingjun Li[2],
[1]School of Information and Electronics, Shandong Institute of Business and Technology, Yantai, China, 264005
[2]Naval Aeronautical and Astronautical University, Yantai, China, 264001
*Corresponding author, e-mail: liuym@bjut.edu.cn

Abstract

Aiming at GPS dynamic filtering method, this paper uses the method of multi-scale analysis, combines the "current" statistical model of automotive carrier with multi-scale signal transformation which is based on statistical characteristic, establishes the new algorithm of multi-scale data fusion for GPS dynamic filter combining with normal Kalman filter algorithm, at last achieves the optimal fusion estimated value of the target states based on global information at the finest scale. When the above algorithm is used to GPS dynamic filter, the simulated results show that the proposed algorithm can effectively increase estimated precision of target states compared with the conventional KF.

Keywords: *GPS dynamic position; multi-scale estimation; dynamic filtering method; wavelet transformation*

1. Introduction

In this paper, based on the detail research on multi-scale theory and traditional multi-model estimation method, combining the GPS dynamic filtering method and multi-scale signal transformation method which based on statistical properties, proposed a distributed dynamic model of GPS multi-scale information fusion estimation algorithm.

2. DWT

The basic ideal of multi-scale analysis is: the pending signals using wavelet transform methods in different scales of decomposition, and then get the corresponding smoothed signal and detail signal; wavelet transform is to connect the signal on the bridge at different scales.

If wrote the vector sequence $x(i,k)\in R^{n\times 1}\ (k\in Z)$ on scale i to block:

$$\boldsymbol{X}_m(i)=\left[\boldsymbol{x}^T(i,mM_i+1),\boldsymbol{x}^T(i,mM_i+2),\cdots,\boldsymbol{x}^T(i,mM_i+M_i)\right]^T \tag{1}$$

In this Mi = 2i-1, so Wavelet analysis and synthesis transform operator form are:

$$\boldsymbol{X}_{Vm}(i-1)=\boldsymbol{L}_{i-1}^T diag\{\boldsymbol{H}_{i-1},\cdots,\boldsymbol{H}_{i-1}\}\boldsymbol{L}_i\boldsymbol{X}_m(i) \tag{2}$$

$$\boldsymbol{X}_{Dm}(i-1)=\boldsymbol{L}_{i-1}^T diag\{\boldsymbol{G}_{i-1},\cdots,\boldsymbol{G}_{i-1}\}\boldsymbol{L}_i\boldsymbol{X}_m(i) \tag{3}$$

$$\boldsymbol{X}_m(i)=\boldsymbol{L}_i^T diag\{\boldsymbol{H}_{i-1}^T,\cdots,\boldsymbol{H}_{i-1}^T\}\boldsymbol{L}_{i-1}\boldsymbol{X}_{Vm}(i-1)+\boldsymbol{L}_i^T diag\{\boldsymbol{G}_{i-1}^T,\cdots,\boldsymbol{G}_{i-1}^T\}\boldsymbol{L}_{i-1}\boldsymbol{X}_{Dm}(i-1) \tag{4}$$

In formula (1) to (3), subscript V and D stand for the projection on smooth and detail signal space of Xm(i); Subscript m stands for the number m data block; $\boldsymbol{L}_i\in R^{nM_i\times nM_i}$ is the linear operator in needed form to transform block vector Xm(i) into wavelet, the number of diagonal matrix is n; Hi-1 and Gi-1$\in R^{M_{i-1}\times M_i}$ is the scale operator and wavelet operator from scale i to scale i-1.

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3. GPS dynamic filter model

The state variables of three-dimensional motion vector can be described in:

$$\boldsymbol{X} = \begin{bmatrix} x & v_x & a_x & \varepsilon_x & y & v_y & a_y & \varepsilon_y & z & v_z & a_z & \varepsilon_z \end{bmatrix}^T \tag{5}$$

Mobile carrier a(t) random acceleration model can be expressed as:

$$\dot{a}_i(t) = -\frac{1}{\tau_{ai}} a(t) + \frac{1}{\tau_{ai}} \bar{a}_i(t) + w_{ai}$$

i=x, y, z (6)

The system state equation can be written as:

$$\dot{\boldsymbol{X}}(t) = \boldsymbol{A}\boldsymbol{X}(t) + \boldsymbol{\Gamma}(t)\boldsymbol{U}(t) + \boldsymbol{W}(t) \tag{7}$$

The positioning of the GPS receiver outputs the result take place for the appearance of measurement, which can create the measurement equation. From the analysis, the three groups there is no coupling between state variables. Take x axis as example:

$$\boldsymbol{X}(k+1) = \begin{bmatrix} 1 & T & (T/\tau_{a_x} - 1 + e^{-T/\tau_{a_x}})\tau_{a_x}^2 & 0 \\ 0 & 0 & (1-e^{-T/\tau_{a_x}})\tau_{a_x} & 0 \\ 0 & 0 & e^{-T/\tau_{a_x}} & 0 \\ 0 & 0 & 0 & e^{-T/\tau_x} \end{bmatrix} \boldsymbol{X}(k) + \begin{bmatrix} 0 & 0 & \tau_{a_x}[-T + \frac{T^2}{2\tau_{a_x}} + \tau_{a_x}(1-e^{-T/\tau_{a_x}})] & 0 \\ 0 & 0 & T-(1-e^{-T/\tau_{a_x}})\tau_{a_x} & 0 \\ 0 & 0 & 1-e^{-T/\tau_{a_x}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \bar{a}_x(k) \\ 0 \end{bmatrix}$$
$$+ \boldsymbol{W}(k) \equiv \boldsymbol{A}(k)\boldsymbol{X}(k) + \boldsymbol{\Gamma}(k)\boldsymbol{U}(k) + \boldsymbol{W}(k) \tag{8}$$

In formula (6) to (8), the definition of each parameter can be found in reference (6).

4. Multi-scale distributed estimation

A System Description

Suppose a class of single sensor single model dynamic system set up in a scale is :

$$\boldsymbol{x}(N,k+1) = \boldsymbol{A}(N,k)\boldsymbol{x}(N,k) + \boldsymbol{\Gamma}(N,k)\boldsymbol{u}(N,k) + \boldsymbol{w}(N,k), \quad k \ge 0 \tag{9}$$

$$\boldsymbol{z}(N,k) = \boldsymbol{C}(N,k)\boldsymbol{x}(N,k) + \boldsymbol{v}(N,k), \quad k \ge 0 \tag{10}$$

$$E\{\boldsymbol{w}(N,k)\} = \boldsymbol{0}, \quad E\{\boldsymbol{w}(N,k)\boldsymbol{w}^T(N,j)\} = \boldsymbol{Q}(N,k)\delta_{kj}, \quad k, j \ge 0 \tag{11}$$

$$E\{\boldsymbol{v}(N,k)\} = \boldsymbol{0}, \quad E\{\boldsymbol{v}(N,k)\boldsymbol{v}^T(N,j)\} = \boldsymbol{R}(N,k)\delta_{kj}, \quad k, j \ge 0 \tag{12}$$

The initial value of system is $\boldsymbol{x}(N,0)$ and it's a random vector, and:

$$E\{\boldsymbol{x}(N,0)\} = \boldsymbol{x}_0, \quad E\{[\boldsymbol{x}(N,0) - \boldsymbol{x}_0][\boldsymbol{x}(N,0) - \boldsymbol{x}_0]^T\} = \boldsymbol{P}_0 \tag{13}$$

Assume that $\boldsymbol{x}(N,0)$ $\boldsymbol{w}(N,k)$ $\boldsymbol{v}(N,k)$ are statistically independent.

B Multi-scale distributed estimation

Divide the state vectors and measurement vectors into data blocks that its block length is $M = 2^{N-1}$

$$\boldsymbol{X}_m(N)=\left[\boldsymbol{x}^T(N,mM+1),\boldsymbol{x}^T(N,mM+2),\cdots,\boldsymbol{x}^T(N,mM+M)\right]^T \tag{14}$$

$$\boldsymbol{Z}_m(N)=\left[\boldsymbol{z}^T(N,mM+1),\boldsymbol{z}^T(N,mM+2),\cdots,\boldsymbol{z}^T(N,mM+M)\right]^T \tag{15}$$

$$\overline{\boldsymbol{w}}(N,(m+1)M+1)=\sum_{i=1}^{M-1}\left[\frac{i}{M}\prod_{r=M}^{i+1}\boldsymbol{A}(N,mM+r)\boldsymbol{w}(N,mM+i)\right]+\boldsymbol{w}(N,mM+M) \tag{16}$$

So, the relationship between m-th data block $X_m(N)$ and $m+1$-th data block $X_{m+1}(N)$ is:

$$\boldsymbol{X}_{m+1}(N)=\boldsymbol{A}_m(N)\boldsymbol{X}_m(N)+\boldsymbol{\Gamma}_m(N)\boldsymbol{U}_m(N)+\boldsymbol{B}_m(N)\boldsymbol{W}_m(N) \tag{17}$$

$$\boldsymbol{A}_m(r,c)=\frac{1}{M-r+1}\prod_{l=M+r-1}^{c}\boldsymbol{A}(N),\quad c=r,r+1,\cdots,M,\quad r=1,2,\cdots,M \tag{18}$$

None-zero entries of $\Gamma_m(N)$ are:

$$\boldsymbol{\Gamma}_m(r,c)=\begin{cases}\dfrac{c-r+1}{M-r+1}\boldsymbol{A}^{M-(c-r+1)}\boldsymbol{\Gamma} & c=r,r+1,\cdots,M\\ \boldsymbol{A}^{M-(c-r+1)}\boldsymbol{\Gamma} & c=M+1,M+2,\cdots,M+r-1\end{cases},\quad r=1,2,\cdots,M \tag{19}$$

None-zero entries of $B_m(N)$ are:

$$\boldsymbol{B}_m(r,c)=\begin{cases}\dfrac{c-r+1}{M-r+1}\boldsymbol{A}^{M-(c-r+1)} & c=r,r+1,\cdots,M\\ \boldsymbol{A}^{M-(c-r+1)} & c=M+1,M+2,\cdots,M+r-1\end{cases},\quad r=1,2,\cdots,M \tag{20}$$

For $l=1,2,\cdots,M$, the measurement equation is:

$\boldsymbol{z}(N,mM+l)=\boldsymbol{C}(N,mM+l)\boldsymbol{x}(N,mM+l)+\boldsymbol{v}(N,mM+l)$, so:

$$\boldsymbol{Z}_m(N)=\boldsymbol{C}_m(N)\boldsymbol{X}_m(N)+\boldsymbol{V}_m(N) \tag{21}$$

$\boldsymbol{R}_m(N)$, $\boldsymbol{C}_m(N)$ are diagonal matrixes.

Signal $X_m(N)$ on scale N produce smooth signals and detail signals to each coarse scale i, there are:

$$\overline{\boldsymbol{X}}_m(N)=\begin{bmatrix}\boldsymbol{X}_{V,m}(i)\\ \boldsymbol{X}_{D,m}(i)\\ \boldsymbol{X}_{D,m}(i+1)\\ \vdots\\ \boldsymbol{X}_{D,m}(N-1)\end{bmatrix}=\begin{bmatrix}\prod_{r=i}^{N-1}\overline{\boldsymbol{H}}_r\\ \overline{\boldsymbol{G}}_i\prod_{r=i+1}^{N-1}\overline{\boldsymbol{H}}_l\\ \vdots\\ \overline{\boldsymbol{G}}_{N-1}\end{bmatrix}\boldsymbol{X}_m(N)=\overline{\boldsymbol{T}}(i)\boldsymbol{X}_m(N) \tag{22}$$

And $\overline{\boldsymbol{T}}^T(i)\overline{\boldsymbol{T}}(i)=\boldsymbol{I}$. For (18), we can get:

$$\overline{\boldsymbol{X}}_{m+1}(N)=\overline{\boldsymbol{A}}_m(N)\overline{\boldsymbol{X}}_m(N)+\overline{\boldsymbol{\Gamma}}_m(N)\overline{\boldsymbol{U}}_m(N)+\overline{\boldsymbol{W}}_m(N) \tag{23}$$

In this formula, $\overline{\boldsymbol{A}}_m(N)=\overline{\boldsymbol{T}}(i)\boldsymbol{A}_m(N)\overline{\boldsymbol{T}}^T(i)$, $\overline{\boldsymbol{W}}_m(N)=\overline{\boldsymbol{T}}(i)\boldsymbol{B}_m(N)\boldsymbol{W}_m(N)$.

And $\overline{\boldsymbol{\Gamma}}_m(N)=\overline{\boldsymbol{T}}(i)\boldsymbol{\Gamma}_m(N)\overline{\boldsymbol{T}}^T(i)$, $\overline{\boldsymbol{U}}_m(N)=\overline{\boldsymbol{T}}(i)\boldsymbol{U}_m(N)$ $E\{\overline{\boldsymbol{W}}_m(N)\}=\boldsymbol{0}$, $E\{\overline{\boldsymbol{W}}_m(N)\overline{\boldsymbol{W}}_m^T(N)\}=\overline{\boldsymbol{Q}}_m(N)$, $\overline{\boldsymbol{Q}}_m(N)=\overline{\boldsymbol{T}}(i)\boldsymbol{B}_m(N)\boldsymbol{Q}_m(N)\boldsymbol{B}_m^T(N)\overline{\boldsymbol{T}}^T(i)$.

Use nature $\overline{T}^T(i)\overline{T}(i)=I$ to change formula (22) into:

$$Z_m(N)=C_m(N)\overline{T}^T(i)\overline{T}(i)X_m(N)+V_m(N)=\overline{C}_m(N)\overline{X}_m(N)+V_m(N) \quad (24)$$

In case to dynamic filtering, we can use the formula below to get the original conditions of state block $X_m(N)$:

$$\hat{X}_{0|0}(N)=\begin{bmatrix}\hat{x}(N,1)\\ \hat{x}(N,2)\\ \vdots\\ \hat{x}(N,M)\end{bmatrix}=\begin{bmatrix}A(N,0)\\ A(N,1)A(N,0)\\ \vdots\\ \prod_{j=0}^{M-1}A(N,j)\end{bmatrix}x_0 \quad (25)$$

$$\mathbf{P}_{0|0}(N)=\begin{bmatrix}\mathbf{A}(N,0)\\ \mathbf{A}(N,1)\mathbf{A}(N,0)\\ \vdots\\ \prod_{j=0}^{M-1}\mathbf{A}(N,j)\end{bmatrix}\mathbf{P}_0\begin{bmatrix}\mathbf{A}(N,0)\\ \mathbf{A}(N,1)\mathbf{A}(N,0)\\ \vdots\\ \prod_{j=0}^{M-1}\mathbf{A}(N,j)\end{bmatrix}^T+\mathbf{B}_m(N)\mathbf{Q}_m(N)\mathbf{B}_m^T(N) \quad (26)$$

And $\overline{X}_{0|0}(N)=\overline{T}(i)\hat{X}_{0|0}(N)$, $\overline{P}_{0|0}(N)=\overline{T}(i)P_{0|0}(N)\overline{T}^T(i)$.

Use the newly created system model (23) and observation model (24) for Kalman filtering:

$$\hat{\overline{X}}_{m+1|m+1}(N)=\hat{\overline{X}}_{m+1|m}(N)+\overline{K}_{m+1}(N)\cdot\left[Z_{m+1}(N)-\overline{C}_{m+1}(N)\hat{\overline{X}}_{m+1|m}(N)\right] \quad (27)$$

$$\hat{\overline{X}}_{m+1|m}(N)=\overline{A}_m(N)\hat{\overline{X}}_{m|m}(N)+\overline{\Gamma}_m(N)U_m(N) \quad (28)$$

$$\overline{P}_{m+1|m}(N)=\overline{A}_m(N)\overline{P}_{m|m}(N)\overline{A}_m^T(N)+\overline{Q}_m(N) \quad (29)$$

$$\overline{K}_{m+1}(N)=\overline{P}_{m+1|m}(N)\overline{C}_{m+1}^T(N)\left[\overline{C}_{m+1}(N)\overline{P}_{m+1|m}(N)\overline{C}_{m+1}^T(N)+\overline{Q}_{m+1}(N)\right]^{-1} \quad (30)$$

$$\overline{P}_{m+1|m+1}(N)=\left[I-\overline{K}_{m+1}(N)\overline{C}_{m+1}(N)\right]\overline{P}_{m+1|m}(N) \quad (31)$$

In final, we can get $\hat{X}_{m|m}(N)=\overline{T}^T(i)\hat{\overline{X}}_{m|m}(N)$, and able to multi-scale estimate for data on minimal scale N.

5. Simulations and Conclusion

Consider the real-time requirements in filtering algorithm, so set the length of data block is M =4, and the maximum scale of signal decomposition is N =2, and here we take Harr wavelet as base wavelet because Harr Wavelet transform for a limited number of columns does not cause distortion effects of the boundary.

Take real-time filtering of the x axis positioning result that GPS outputs, original conditions are $x(0)=L_x(0)$, $v_x(0)=L_{vx}(0)$, $a_x(0)=0\text{m/s}^2$, $\varepsilon_x(0)=0\text{m}$, $P(0)=\mathbf{0}$, related parameters are $\sigma_{a_x}^2=3^2$, $\sigma_x^2=40^2$, $R_x=\text{diag}(12^2,0.2^2)$, $\tau_{a_x}=1$, $\tau_X=0.5$, and the simulation time is all 600s.

To illustrate the superiority of the algorithm, this paper takes into account the complex dynamics of change acceleration movement. Initial velocity is $v_0=10\text{m/s}$, initial acceleration is $a_0=0$. During $0\text{s}\le t<100\text{s}$, da/dt=0.2 m/s3, and during $100\text{s}\le t<200\text{s}$, da/dt=-0.2m/s3. Figure 1 shows the conventional KF algorithm and filter algorithm estimates the position error, velocity error and acceleration error comparison chart. Obviously, the algorithm in terms relative to the original algorithm can significantly improve the position, velocity and acceleration parameter estimation accuracy. Table 1 the roots mean square error of estimation was compared can be seen from

Table 1. Location accuracy improved by almost double the rate of accuracy is increased by nearly 2-fold acceleration accuracy is improved by nearly 1 time.

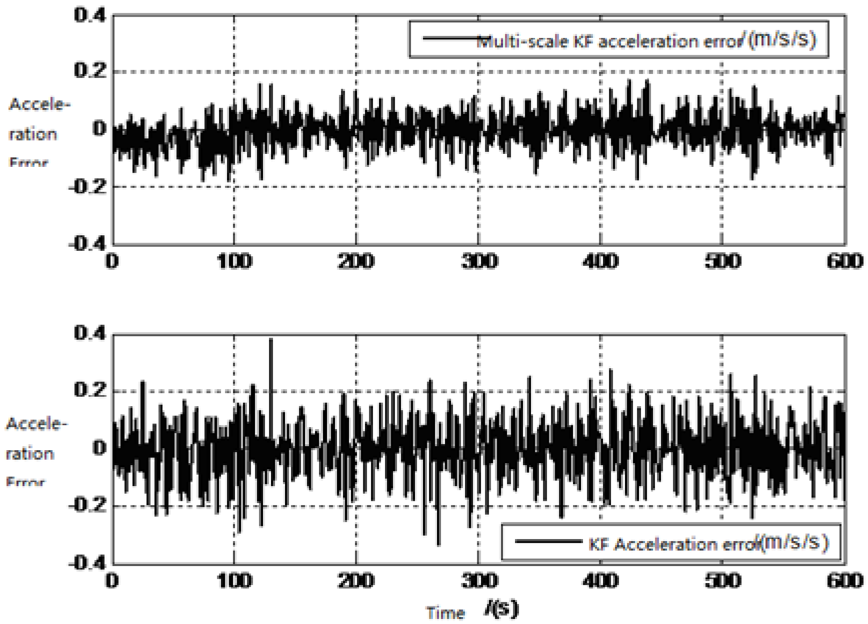

Figure 1. Comparison figure between this paper's algorithm and normal Kalman filter

Table 1. Comparison of RMS for every parameter estimation error

Location error/m		Velocity error/(m/s)		Acceleration error/(m/s2)	
Multi-scale KF algorithm	KF algorithm	Multi-scale KF algorithm	KF algorithm	Multi-scale KF algorithm	KF algorithm
0.820 1	1.409 5	0.010 0	0.036 6	0.005 7	0.012 8

This dynamic filtering problem for the GPS, the establishment of a multi-scale decomposition of random signals and optimal estimation of simultaneous multi-scale estimation algorithm, access to the original scale in the Kalman filter is difficult to obtain direct estimates of the effect, and the Haar wavelet simulation, and then verify the effectiveness of the algorithm. For other orthogonal wavelet, only properly handle border issues, this algorithm will also be applicable.

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