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Design Based the δ Operator of the Minimum Beat System

Zhou Shiqun[1], Wang Ping[2]
School of electrical engineering and automation, Tianjin Polytechnic University
NO.399, Extention Line of West Binshui Road of the Xiqing Qu, Tianjin, China
zhoushiqun0@163.com
*corresponding author, e-mail: wangping@tjpu.edu.cn

Abstract

Digital controller simulation using MATLAB the minimum beat deviation system z domain, and transformed it in δ domain. Using the δ transformation's nature of approximating the continuous domain in small sampling period, we simulated the controller.

Keyword: *minimum beat error, z transform, δ transform*

1. Introduction

In Digital servo system, require the system output values as soon as possible to track a given value change. The minimum beat error is designed for the requirement. But in the z-transform traditional discretization method, if using a high sampling frequency, discrete-time model of minimum phase system becomes unstable zero non-minimum phase systems, and also generates computational accuracy deterioration and other problems. This article will take advantage of the δ domain nearly continuous domain in a high sampling frequency to design the controller, and to overcome the lack of the z-domain transform [1].

This article simulates the digital controller's minimum beat error system in the z domain, then transformed it in δ domain, compared the controller's two situations, and analyzed the experimental results in both cases.

2. Research Method

2.1.Minimum beat design error system in z-domain

The so-called Minimum beat system state is the closed-loop system for a particular input to achieve no static error steady state with the minimum sampling period. The closed-loop pulse transfer function has the following form [2]:

$$\Phi(z) = \Phi_1 z^{-1} + \Phi_2 z^{-2} + \cdots + \Phi_N z^{-N} \tag{1}$$

N is the smallest integer possible. Control system structure is shown in Figure 1.

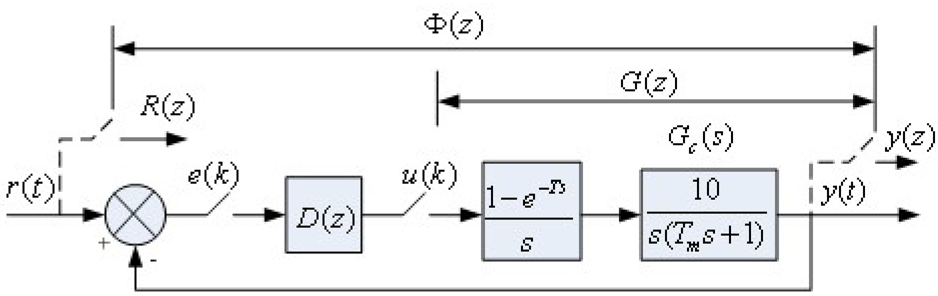

Figure 1. Schematic diagram of the whole control system

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$D(z)$ is the pulse transfer function of the digital controller. $G_c(s)$ is the transfer function of the controlled object. $W(s)$ is the correction element in series of the continuous control system. Its role is to change the pole-zero configurations to achieve the desired continuous control law. The more common zero-order hold was added to the figure to make $u^*(t)$ change into a continuous signal applying to the controlled object.

The pulse transfer function of $E(z)$ is [3]:

$$\Phi_e(z) = \frac{E(z)}{R(z)} = \frac{R(z) - Y(z)}{R(z)} \quad (2)$$

$E(z)$ is the z-transformation of the digital controller input signal. Error function $E(z)$ is [4]

$$E(z) = \Phi_e(z)R(z) = [1 - \Phi(z)]R(z) \quad (3)$$

According to the final value of the z-transform theorem, the steady-state error of the system is

$$e(\infty) = \lim_{x \to 1}(1 - z^{-1})E(z) = \lim_{z \to 1}(1 - z^{-1})\Phi_e(z)R(z) \quad (4)$$

Typical power function of the input function is

$$r(t) = A_0 + A_1 t + \cdots + \frac{A_{q-1}}{(q-1)!}t^{q-1} \quad (5)$$

Its z-transform is

$$R(z) = \frac{B(z)}{(1 - z^{-1})^q} \quad (6)$$

$B(z)$ is does not contain factor $(1 - z^{-1})$ polynomial for z^{-1}.

To make the steady-state deviation $e(\infty)$ is zero, so $\Phi_e(z)$ must contain at least a factor of $(1 - z^{-1})^q$ that is [5]:

$$\Phi_e(z) = 1 - \Phi(z) = (1 - z^{-1})^n F(z) \quad (7)$$

To make the deviation as soon as possible to zero, q should meet:

$$q = n \quad (8)$$

From the requirement of accuracy, $\Phi_e(z)$ should meet:

$$\Phi_e(z) = 1 - \Phi(z) = (1 - z^{-1})^q F(z) \quad (9)$$

To make the design of the digital controller is the most simple form of the lowest order, must make $F(z) = 1$.

2.2. δdomain Minimum beat error system design

δ transformation is a discrete domain description method developed in recent years. δ conversion when the sampling period is small, discrete model of pole-zero densely located near $z=1$, very prone to modeling errors [6]. Finite word-length computer will bring greater quantization error and nonlinear effects. δ conversion characteristics is discrete model close to the original continuous model in small sampling period, overcoming the deficiencies of the z-conversion. Better value characteristics in the realization of the digital computer, especially a lot of well-known continuous domain design concepts and design methods can be directly approximated for δ transformation, does not have to be converted to the appropriate concepts and methods of the z domain, which gives the designer convenience.

Define the δ transformation of the discrete function $f(k)$ [7]:

$$F(\delta)=\Delta[f(k)]=T\sum_{i=0}^{\infty} f(i)(T\delta+1)^{-i} \tag{10}$$

It is shown that δ conversion and the z conversion of the sampling signal have the same form. amplitude difference between the T-fold. A conversion can be defined as a direct.

$$\delta=\frac{z-1}{T} \text{ or } z=T\delta+1 \tag{11}$$

Make $s=\Sigma+j\Omega$, $\delta=\sigma+j\omega$, $\delta+\frac{1}{T}=\frac{e^{sT}}{T}$, we can get:

$$(\sigma+\frac{1}{T})+j\omega=\frac{e^{\Sigma T}e^{j\Omega T}}{T} \tag{12}$$

Calculate the value of both sides, we can get:

$$(\sigma+\frac{1}{T})^2+\omega^2=(\frac{e^{\Sigma T}}{T})^2 \tag{13}$$

δ domain's stability domain is $(-\frac{1}{T}, 0)$ as the center, and the radius of a circle of $\frac{1}{T}$.

The δ transform of the function of the Z-transform can be directly obtained [8]:

$$G_c(\delta)=G_c(z)|_{z=1+T\delta} \quad \Phi(\delta)=\Phi(z)|_{z=1+T\delta} \quad \Phi_e(\delta)=\Phi_e(z)|_{z=1+T\delta} \tag{14}$$

3. Results and Analysis

Transfer function of be controlled is $G_c(s)=\frac{5}{s(T_S+1)}$, $T=T_s=0.05s$. Fast system controller designed for the speed of the input function:

$$G(s)=\frac{1-e^{-Ts}}{s}\cdot\frac{5}{s(T_S+1)} \tag{15}$$

$G(s)$ Conversion into z-domain:

$$G(z)=5(1-z^{-1})[\frac{Tz^{-1}}{(1-z^{-1})^2}-\frac{T_S}{1-z^{-1}}+\frac{T_S}{1-e^{-\frac{T}{T_S}}z^{-1}}]=\frac{0.184z^{-1}(1+0.718z^{-1})}{(1-z^{-1})(1-0.368z^{-1})} \tag{16}$$

Designed for speed input, so

$$\Phi_e(z) = (1 - z^{-1})^2 \tag{17}$$

Closed-loop transfer function of speed input system to achieve the requirements of the system

$$\Phi(z) = 2z^{-1} - z^{-2} \tag{18}$$

$$D(z) = \frac{1}{G(z)} \cdot \frac{\Phi(z)}{1\text{-}\Phi(z)} = \frac{10.87(1-0.5z^{-1})(1-0.368z^{-1})}{(1\text{-}z^{-1})(1+0.718z^{-1})} \tag{19}$$

$D(z)$ is the pulse transfer function of the digital controller of a computer to achieve.

Now convert it to the δ domain:

$$G(\delta) = G(z) \,|\, z = T\delta + 1 = \frac{3.68(\delta + 34.36)}{\delta(\delta + 12.64)} \tag{20}$$

$$D(\delta) = \frac{1}{G(\delta)} \cdot \frac{\Phi(\delta)}{1\text{-}\Phi(\delta)} = \frac{10.87(\delta + 10)(\delta + 12.64)}{\delta(\delta + 34.36)} \tag{21}$$

In the system of the input signal for speed, figure 2 is the output of the system in the z-domain using MATLAB simulation, figure 3 is the output of the system in the δ-domain using MATLAB simulation. Compare the two figures, the output of system in the δ-domain can track the signal better, the output of system in the z-domain failed to produce such results. Figure 4 is the output of the controller in the z-domain. Figure 5 is the output of the controller in the δ-domain. The output of the controller in the z-domain has a lager overshoot, and spent a long time to reach steady state. The output of the controller in the δ-domain has a small overshoot, and reach the steady state fast. We can get better result in the δ-domain when using a high sampling frequency .In their formulation it must be considered that using appropriate control rules depending on the operating conditions can greatly improve the inverter performances in terms of dynamic response and robustness [9].

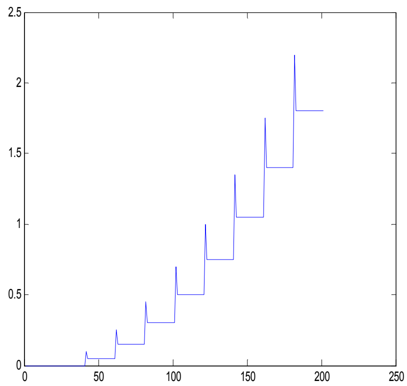

Figure 2. z-domain system output

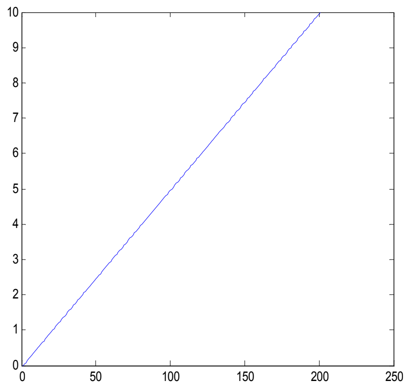

Figure 3. δ-domain system output

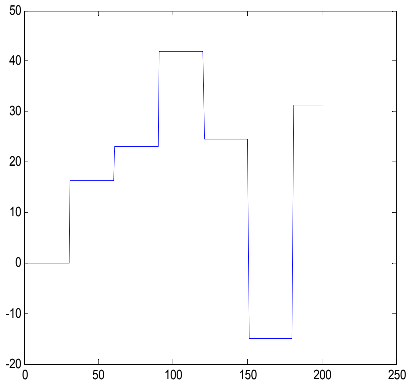

Figure 4. z-domain controller output

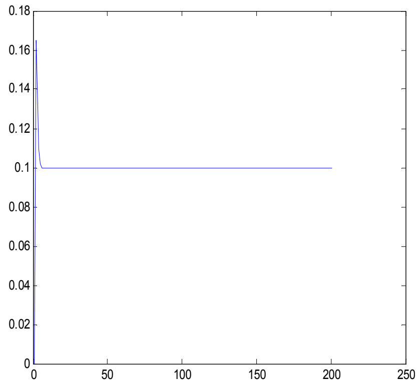

Figure 5. δ-domain controller output

4. Conclusion

Model-based design process is selected as design flow for acceleration and simplification of developing process [10]. Because of the computer “s finite word-length, the discretization of the system will become nonlinear and bring large error. We can take advantage of the δ domain nearly continuous domain in a high sampling frequency to overcome the lack of the z-domain transform.

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