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Random Weighting Estimation of Poisson Distributions

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Abstract

This paper presents a new random weighting method for estimation of Poisson distributions. A theory is established for random weighting estimation of the population parameters of two Poisson distributions with partially missing data. The strong convergence of the random weighting estimation is rigorously proved under the condition of $E\left|X_1\right|^{1+\delta} < \infty \quad (0 < \delta < 1)$. *The random weighting estimation of one-sided confidence intervals in Poisson distribution is also constructed, and its coverage probability is rigorously proved by using the Edgeworth expansion under certain conditions.*

Keywords: *random weighting estimation, population parameters, Poisson distribution, one-sided confidence intervals, and coverage probability.*

1. Introduction

Poisson distribution is an approximation of Binomial distribution. For *n*-times independent and repeated random experiments with two results only, Poisson distribution is a precise approximation solution when n is sufficiently large, the probability p of a given result is sufficiently small, and np remains fixed. Further, Poisson distribution is also an effective method to describe the probability of sparse events such as the rates or ratios for diseases of the nervous system, the number of traffic accidents in a city and the number of a specific kind of particles emitted from radiant materials. In general, Poisson distribution can be used to describe the probability for a number of events occurring within a fixed time interval when there is no memory on the stochastic process.

Maximization algorithm is one of the most basic the logical method of mathematical statistics, and has been applied in various fields [1-8, 10]. Maximum likelihood estimation is a commonly used method to estimate the parameters of Poisson distribution. David and Johnson studied the estimation of Poisson distribution based on sample moments, and derived the maximum likelihood estimate of asymptotic variance [1]. Irwin improved this work and constructed the maximum likelihood estimation of the population mean of a Poisson distribution with the zero class missing [2]. Wu and Wan discussed maximum likelihood estimation and likelihood ratio test for the identical populations of two Poisson distributions with partially missing data, in which the strong coincidence and asymptotic normality were proved [3]. Barreto-Souza and Cribari-Neto constructed the maximum likelihood estimation for a generalized exponential Poisson distribution [4]. An EM algorithm was developed for maximum likelihood estimation of the parameters in Poisson distribution [5, 6]. This algorithm overcomes the limitation of the Newton-Raphson algorithm used in maximum likelihood estimation, i.e., second derivatives of the log-likelihood are required for all iteration.

Random phenomenon can be seen everywhere in life, such as gambling, transportation etc [9]. Similarly, random theory can also be used to as estimate a parameter. The random weighting method is an emergent computational method in statistics [10-12]. It is simple in computation and suitable for large samples, and does not require the knowledge on the distribution function. It can also be used to calculate a statistic with a probability density function, since the resultant statistical distribution actually provides a probability density function. Therefore, the random weighting method has been widely applied to solve different

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problems [10-19]. However, there has been very limited research to use the random weighting method for estimation of Poisson distributions.

This paper adopts the random weighting method to estimation of Poisson distributions. The strong convergence for random weighting estimation of the Poisson population parameters with partially missing data is rigorously proved under the condition of $E\left|X_1\right|^{1+\delta} < \infty \quad (0 < \delta < 1)$. Random weighting estimation is also constructed for one-sided confidence intervals in Poisson distribution, and its coverage probability is achieved under certain conditions.

2. Random Weighting Estimation of Population Parameters of Two Poisson Distributions

Suppose that there are two Poisson populations, and their distributions are

$$p_i(k,\lambda_i) = \frac{\lambda_i^k}{k!} e^{-\lambda_i} \tag{1}$$

where $k = 0,\ 1,\ 2,\ \cdots,\quad i = 1,\ 2$, and $\lambda_i > 0$ is a unknown parameter.

n-times independent experiments are conducted on these two populations, respectively. Denote the samples for *n*-times independent experiments as $(Z_1, Z_2, \cdots, Z_n)$ and $(Y_1, Y_2, \cdots, Y_n)$, respectively. For the observation of the first population, Z_i may be lost by the probability of $1-p$ (p is an unknown constant within $0 < p < 1$), i.e., the actual observation values for the first population are $(Z_i, \delta_i) \quad (i = 1, 2, \cdots, n)$, where $(\delta_1, \delta_2, \cdots, \delta_n)$ and $(Z_1, Z_2, \cdots, Z_n)$ are independent of one another, and $p(\delta_i = 1) = 1 - p(\delta_i = 0) = p$. Obviously, $n_1 = \sum_{j=1}^{n} \delta_j$ is a random variable, which obeys the Binomial distribution with probability parameter p. It represents the number of observation values for the first population. We shall denote the n_1 observation values for the first population as vector $(X_1, X_2, \cdots, X_{n_1})$. Then

$$\sum_{j=1}^{n_1} X_j = \sum_{j=1}^{n} Z_j \delta_j \tag{2}$$

2.1. Maximum Likelihood Estimation of Poisson Population Parameters

In order to establish random weighting estimation, the mathematical representations of the Poisson population parameters have to be given. In this paper, the maximum likelihood estimation [2, 3], which is a popular statistical method to fitting a mathematical model to data, is used to establish the mathematical representations for the Poisson population parameters.

If only the estimation of λ_1 is considered, the measured likelihood function is

$$L_1(\lambda_1) = \prod_{j=1}^{n_1} \lambda_1^{X_j} e^{-\lambda_1} (X_j!)^{-1} \tag{3}$$

The maximum likelihood estimation of λ_1 may be written as

$$\hat{\lambda}_1 = \frac{1}{n_1} \sum_{j=1}^{n_1} X_j \tag{4}$$

Similarly, if only the estimation of λ_2 is considered, the maximum likelihood estimation of λ_2 may be given as

$$\hat{\lambda}_2 = \frac{1}{n_2} \sum_{j=1}^{n_2} Y_j \tag{5}$$

When examining if two populations are equal, the original assumption $H_0 : \lambda_1 = \lambda_2 = \lambda$ (unknown) has to be considered. Then, the likelihood function of λ may be written as

$$l(\lambda) = \prod_{j=1}^{n_1} \lambda^{X_j} e^{-\lambda} (X_j !)^{-1} \prod_{j=1}^{n_2} \lambda^{Y_j} e^{-\lambda} (Y_j !)^{-1} = e^{-(n_1+n_2)\lambda} \lambda^{(\sum_{j=1}^{n_1} X_j + \sum_{j}^{n_2} Y_j)} \left(\prod_{j=1}^{n_1} X_j ! \prod_{j=1}^{n_2} Y_j ! \right) \tag{6}$$

The maximum likelihood estimation of λ can be easily obtained as

$$\hat{\lambda} = \frac{1}{n_1 + n_2} (\sum_{j=1}^{n_1} X_j + \sum_{j=1}^{n_2} Y_j) \tag{7}$$

2.2. Random Weighting Estimation of Poisson Population Parameters and Convergence Analysis

Assuming that $X_1, X_2, \cdots, X_n$ is a sample of independent random variables with common distribution function $F(x)$. Let $x_1, \cdots, x_n$ be the corresponding observed realizations. Further, we shall denote $\tilde{X}_n = (X_1, X_2, \cdots, X_n)$ and $\tilde{x}_n = (x_1, x_2, \cdots, x_n)$. Then, the corresponding empirical distribution function of X may be written as

$$F_n(x) = \frac{1}{n} \sum_{i=1}^{n} I_{(X_i \le x)} \tag{8}$$

where $I_{(X_i \le x)}$ is the indicator function.

The random weighting estimation of $F_n(x)$ is

$$H_n(x) = \sum_{i=1}^{n} V_i I_{(X_i \le x)} \tag{9}$$

where random vector $(V_1, V_2, \cdots, V_n)$ obeys Dirichlet distribution $D(1, \cdots, 1)$, that is, $\sum_{i=1}^{n} V_i = 1$ and the joint density function of $(V_1, V_2 \cdots, V_n)$ is $f(V_1, V_2 \cdots, V_n) = \Gamma(n)$, where $(V_1, V_2 \cdots, V_n) \in D_n$, and $D_n = \left\{ (V_1, V_2, \cdots, V_n) : V_k \ge 0, k = 1, \cdots, n, \sum_{k=1}^{n} V_k \le 1 \right\}$.

A uniformly distributed density function of $(V_1, \cdots, V_n)$ can be defined as

$$f(V_1, \cdots, V_n) = (n-1)! \tag{10}$$

If only the maximum likelihood estimation $\hat{\lambda}_1$ of λ is considered, the random weighting estimation of $\hat{\lambda}_1$ may be written as

$$H_1(x) = \sum_{i=1}^{n_1} V_i X_i \tag{11}$$

Similarly, if only the estimation of $\hat{\lambda}_2 = \frac{1}{n_2} \sum_{j=1}^{n_2} Y_j$ is considered, the random weighting estimation of $\hat{\lambda}_2$ may be written as

$$H_2(y) = \sum_{i=1}^{n_2} V_i Y_i \tag{12}$$

Theorem 1 *Suppose* $E\left|X_1\right|^{1+\delta}<\infty$. *For any* $0<\delta<1$ *and* $\varepsilon>0$, *when* $N\to\infty$,

$$p\{\sup_{n\geq N}\left|H_1(x)-\bar{X}_n(x)\right|>\varepsilon\}\to 0 \qquad (a.s) \tag{13}$$

The following Lemma is used to prove the theorem.
Lemma 1 [15] *When* $n\geq 2$ *and* $r\geq 1$, *there exists a limited number* $C>0$ *such that*

$$E^*\left[\left(\sum_{i=1}^{n}V_iX_i-\bar{X}_n\right)^{2r}\left|X_n\right.\right]\leq Cn^{-r}S_n^{2r} \tag{14}$$

where E^* *represents the conditional expectation under the condition that* $X_1,X_2,\cdots X_n$ *are given, and* $S_n^2=\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X}_n)^2$.

We now start to prove Theorem 1. Let

$$W_{n,\delta}=\frac{1}{n}\sum_{i=1}^{n}\left|X_i-\mu\right|^{1+\delta} \tag{15}$$

where $\mu=EX_i<\infty$.
Thus,

$$\begin{aligned}
S_n^2&=\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X}_n)^2\leq\frac{1}{n}\sum_{i=1}^{n}(X_i-\mu)^2\\
&\leq\left\{\max_{1\leq i\leq n}\left|X_i-\mu\right|^{1-\delta}\right\}\cdot\frac{1}{n}\sum_{i=1}^{n}\left|X_i-\mu\right|^{1+\delta}\\
&=\left\{\max_{1\leq i\leq n}\left|X_i-\mu\right|^{1-\delta}\right\}\cdot W_{n,\delta}\\
&\leq\left\{\sum_{i=1}^{n}\left|X_i-\mu\right|^{1+\delta}\right\}^{(1-\delta)/(1+\delta)}\cdot W_{n,\delta}\\
&=\left\{nW_{n,\delta}\right\}^{(1-\delta)/(1+\delta)}\cdot W_{n,\delta}\\
&=n^{(1-\delta)/(1+\delta)}\cdot W_{n,\delta}^{2/(1+\delta)}
\end{aligned} \tag{16}$$

If $n\geq 2$, by (14) and (16), there exists *C* such that

$$\begin{aligned}
&E^*\left[\left(\sum_{i=1}^{n}V_iX_i-\bar{X}_n\left|X_n\right.\right)^{2r}\right]\leq Cn^{-r}\left\{\left[nW_{n,\delta}\right]^{(1-\delta)/(1+\delta)}\cdot W_{n,\delta}\right\}^r\\
&=Cn^{-2\delta r/(1+\delta)}\cdot W_{n,\delta}^{2r/(1+\delta)}
\end{aligned} \tag{17}$$

Let

$$\begin{aligned}
A_\nu&=\left\{\sup_{n\geq N}\left|H_1(x)-\bar{X}_n\right|>\varepsilon\right\}=\left\{\sup_{n\geq N}\left|\sum_{i=1}^{n}V_iX_i-\bar{X}_n\right|>\varepsilon\right\}\\
B_\nu&=\left\{\sup_{n\geq N}W_{n,\delta}\leq g\right\}
\end{aligned} \tag{18}$$

where $g>E\left|X_1-\mu\right|^{1+\delta}$ is a bounded constant, and $W_{n,\delta}$ is a backward martingale.

By the Kolmogorov's maximum inequality [10], we have

$$p(B_v) = P\left\{\sup_{n\geq N} W_{n,\delta} - E\left|X_1-\mu\right|^{1+\delta} > g - E\left|X_1-\mu\right|^{1+\delta}\right\}$$
$$\leq \left\{g - E\left|X_1-\mu\right|^{1+\delta}\right\}^{-1}\cdot E\left|W_{n,\delta} - E\left|X_1-\mu\right|^{1+\delta}\right| \to 0, \text{when } N\to\infty \quad (19)$$

By the Markov's inequality and (17),

$$p(A_v B_v) = \sum_{n\geq N} p\left\{\sup\left|\sum_{i=1}^{n} V_i X_i - \bar{X}_n\right| > \varepsilon, W_{n,\delta} \leq g\right\}$$
$$\leq \sum_{n\geq N} E\left[I_{(W_{n,\delta}\leq g)}\cdot Cn^{-2\delta r/(1+\delta)}\cdot W_{n,\delta}^{2r/(1+\delta)}\right]/\varepsilon^{2r}$$
$$\leq C_0(g,\varepsilon,r)\sum_{n\geq N} n^{-2\delta r/(1+\delta)}$$
$$\leq C_1(g,\varepsilon,r)N^{1-2\delta r/(1+\delta)} \quad (20)$$

where C_0 and C_1 are two bounded positive constants. For a constant $0 < h < 1$, choosing $r > 1$ such that

$$2\delta r/(1+\delta) = 1+h$$
$$N^{1-2\delta r/(1+\delta)} = N^{-h} \to 0 \quad (N\to\infty) \quad (21)$$

By (19) and (21), we have $P(B_v)\to 0$ and $P(A_v B_v)\to 0$ $(N\to\infty)$. Thus,

$$p(A_v) \leq P(A_v B_v) + P(B_v) \quad (22)$$

By appropriately selecting parameter *r*, (13) follows from (18)-(22). The proof of Theorem 1 is completed.

3. Random Weighting Estimations of One-Sided Confidence Intervals in Poisson Distribution

One-sided confidence intervals such as the standard Wald interval play an important role in many applications. However, the standard Wald interval suffers a pronounced systematic bias in the coverage [20, 21]. In this section, we first construct the random weighting estimation of the Wald interval for the mean μ of Poisson distribution, and then analyze the coverage probability of the random weighting estimation. For the sake of concise description, we shall focus on the random weighting estimation for the upper limit of the Wald interval. The random weighting estimation for the lower limit of the Wald interval can be constructed in the similar way.

Suppose that $X_1, X_2, \cdots, X_n$ are the random variable serials of an independent and identical distribution with common distribution function *F(x)*, and Poi(λ) is a member of the discrete natural exponential family with a quadratic variance function. In the case of Poisson distribution, $\mu=\lambda$. Therefore, the variance σ^2 is a quadratic function of mean μ in Poi(λ)

$$\sigma^2 \equiv V(\mu) = \mu + B\mu^2 \quad (23)$$

In the following, for the sake of convenient description, we shall denote by k the $100(1-\alpha)$ *th* percentile of the standard normal distribution, and let $X = \sum_{i=1}^{n} X_i$,

$\hat{\mu} \equiv \overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, and $\hat{\mu}^* = \sum_{i=1}^{n} V_i X_i = H_1(x)$.

3.1 Random Weighting Estimation of Wald Interval

By using the normal approximation [20], the Wald interval can be constructed as

$$W_n = \frac{\sqrt{n}(\hat{\mu}-\mu)}{V^{1/2}(\hat{\mu})} \to N(0,1) \quad (24)$$

Accordingly, the random weighting estimation of the Wald interval may be written as

$$W_n^* = \frac{\sqrt{n}(\hat{\mu}^*-\mu)}{V^{1/2}(\hat{\mu}^*)} \to N(0,1) \quad (25)$$

The $100(1-\alpha)$ % upper limit of the Wald interval is

$$CI_w^u = [0, \hat{u} + \kappa V^{1/2}(\hat{\mu}) n^{-1/2}] = [0, \hat{u} + \kappa(\hat{u} + b(\hat{u})^2)^{1/2} n^{-1/2}] \quad (26)$$

Accordingly, the $100(1-\alpha)$ % upper limit for random weighting estimation of the Wald interval may be written as

$$C^* I_w^u = [0,\ \hat{u}^* + \kappa V^{1/2}(\hat{\mu}^*) n^{-1/2}] = [0,\ \hat{u}^* + \kappa(\hat{u}^* + b(\hat{u}^*)^2)^{1/2} n^{-1/2}] \quad (27)$$

3.2 Coverage Probability of Wald Interval

Consider the coverage of a general upper limit interval

$$CI_* = [0, \frac{X+s_1}{n-bs_2} + k\left\{V(\hat{u}) + (V(\hat{u}) + r_2)n^{-1}\right\}^{1/2} n^{-1/2}] \quad (28)$$

where s_1, s_2 and r_2 are constants.

The random weighting estimation of CI_* may be written as

$$C^* I_* = [0, \frac{X+s_1}{n-bs_2} + k\left\{V(\hat{u}^*) + (V(\hat{u}^*) + r_2)n^{-1}\right\}^{1/2} n^{-1/2}] \quad (29)$$

It is not difficult to see that confidence interval $C^* I_w^u$ is a special case of $C^* I_*$.

By solving a quadratic equation, we have

$$P(\mu \in CI_*) = P(\frac{n^{1/2}(\hat{\mu}-\mu)}{\sigma} \geq z_W) \quad (30)$$

and

$$P(\mu \in C^* I_*) = P(\frac{n^{1/2}(\hat{\mu}^*-\mu)}{\sigma} \geq z_W) \quad (31)$$

where

$$z_W = (\frac{B-\sqrt{B^2-4AC}}{2A} - \mu)\sigma^{-1} n^{1/2} \quad (32)$$

where

$$A = n - bk^2(1 + r_1 n^{-1})(1 - bs_2 n^{-1})^2$$
$$B = 2n\mu - 2(s_1 + bs_2\mu) + k^2(1 + r_1 n^{-1})(1 - bs_2 n^{-1}) \quad (33)$$
$$C = n(\mu - (s_1 + bs_2\mu)n^{-1})^2 - r_2 k^2 n^{-1}(1 - bs_2 n^{-1})^2$$

where s_1, s_2, r_1 and r_2 are constants.

Theorem 2 *Assuming* $n\mu + n^{1/2}\sigma z_W$ *is not an integer, the coverage probability of the Wald interval* $C^* I_w^u$ *satisfies*

$$P_W = P(\mu \in C^* I_W^u) = (1-\alpha) - \frac{1}{6}(2k^2+1)(1+2b\mu)\sigma^{-1}\phi(k)n^{1/2} + \Theta_1(\mu, z_W)\sigma^{-1}\phi(k)n^{-1/2}$$
$$+\{-\frac{b}{36}(8k^5 - 11k^3 + 3k) - \frac{1}{36\sigma^2}(2k^5 - k^3 + 3k)\}\phi(k)n^{-1} \quad (24)$$
$$+\left\{\frac{1}{6}(2k^3+3)(1+2b\mu)\Theta_1(\mu, z_W) + \Theta_2(\mu, z_W)\right\} \times \sigma^{-2}k\phi(k)n^{-1} + O(n^{-3/2})$$

The following Lemma is used to prove Theorem 2.

Lemma 2 [20] *Assume that* $X_1, X_2, \cdots, X_n$ *is a sample of independent and identically distributed random variables with common distribution function* $F(x)$ *which is represented by* $Poi(\lambda)$. *Denote* $Z_n = n^{1/2}(\hat{\mu} - \mu)/\sigma$ *and* $F_n(z) = P(Z_n \le z)$. *Then, the two-term Edgeworth expansion of* $F_n(z)$ *is*

$$F_n(z) = \phi(z) + p_1(z)\phi(z)n^{-\frac{1}{2}} - \Theta_1(\mu, z)\sigma^{-1}\phi(z)n^{-\frac{1}{2}} + p_2(z)\phi(z)n^{-1}$$
$$+\left\{\Theta_1(\mu, z)\sigma p_3(z) + \Theta_2(\mu, z)\right\}\sigma^{-2}z\phi(z)n^{-1} + O(n^{-\frac{3}{2}}) \quad (35)$$

where

$$p_1(z) = \frac{1}{6}(1 - z^2)(1 + 2b\mu)\sigma^{-1}$$
$$p_2(z) = -\frac{1}{36}(2z^5 - 11z^3 + 3z)b - \frac{1}{72}(z^5 - 7z^3 + 6z)\sigma^{-2} \quad (36)$$
$$p_3(z) = -\frac{1}{6}(z^2 - 3)(1 + 2b\mu)\sigma^{-1}$$

and Θ_1 *and* Θ_2 *are represented as*

$$\Theta_1(\mu, z) = g(\mu, z) - \frac{1}{2}$$
$$\Theta_2(\mu, z) = -\frac{1}{2}g^2(\mu, z) + \frac{1}{2}g(\mu, z) - \frac{1}{12} \quad (37)$$

where

$$g(\mu, z) = g(\mu, z, n) = n\mu + n^{-\frac{1}{2}}\sigma z - (n\mu + n^{-\frac{1}{2}}\sigma z)_- \quad (38)$$

where (x)_ denotes the largest integer less than or equal to x.

If z depends on n, i.e.,

$$z = z_0 + c_1 n^{-\frac{1}{2}} + c_2 n^{-1} + O(n^{-\frac{3}{2}}) \tag{39}$$

where z_0, c_1 and c_2 are constants, then,

$$\begin{aligned} F_n(z) &= \phi(z_0) + \tilde{p}_1(z)\phi(z_0)n^{-\frac{1}{2}} - \Theta_1(\mu, z)\sigma^{-1}\phi(z)n^{-\frac{1}{2}} + \tilde{p}_2(z)\phi(z_0)n^{-1} \\ &+ \left\{\Theta_1(\mu, z)\sigma\tilde{p}_3(z_0) + \Theta_2(\mu, z)\right\}\sigma^{-2} z_0 \phi(z_0) n^{-1} + O(n^{-\frac{3}{2}}) \end{aligned} \tag{40}$$

where

$$\tilde{p}_1(z) = c_1 + \frac{1}{6}(1 - {z_0}^2)(1 + 2b\mu)\sigma^{-1} \tag{41}$$

$$\tilde{p}_2(z) = c_2 - \frac{1}{2} z_0 {c_1}^2 + \frac{1}{6}({z_0}^3 - 3z_0)(1 + 2b\mu)\sigma^{-1} + p_2(z_0) \tag{42}$$

$$\tilde{p}_3(z) = c_1 - \frac{1}{6}({z_0}^3 - 3)(1 + 2b\mu)\sigma^{-1} \tag{43}$$

We now start to prove Theorem 2. By (30) and (33),

$$\begin{aligned} z_W &= -k - (s_1 + bs_2\mu - \frac{1}{2}k^2(1 - 2b\mu))\sigma^{-1} n^{-\frac{1}{2}} - \{(\frac{1}{2}r_1 + bk^2 - bs_2)k \\ &- \frac{1}{2}k(s_1 + bs_2\mu)(1 + 2b\mu)\sigma^{-2} + (\frac{1}{2}r_2 + \frac{1}{8}k^3)\sigma^{-2}\} n^{-1} + O(n^{-\frac{3}{2}}) \end{aligned} \tag{44}$$

Denoting

$$\begin{aligned} c_1^* &= -(s_1 + bs_2\mu - \frac{1}{2}k^2(1 + 2b\mu))\sigma^{-1} \\ c_2^* &= -\{(\frac{1}{2}r_1 + bk^2 - bs_2)k - \frac{1}{2}k(s_1 + bs_2\mu)(1 + 2b\mu)\sigma^{-2} + (\frac{1}{2}r_2 + \frac{1}{8}k^3)\sigma^{-2}\} \end{aligned} \tag{45}$$

z_W may be rewritten as

$$z_W = -k + c_1^* n^{-\frac{1}{2}} + c_2^* n^{-1} + O(n^{-\frac{3}{2}}) \tag{46}$$

By (40), the coefficient of the non-oscillatory term $O(n^{-\frac{1}{2}})$ in the Edgeworth expansion of the coverage $P(\mu \in C^* I_*) = 1 - F_n(z_W)$ may be written as

$$\begin{aligned} p_1^*(z_W) &= -\tilde{p}_1(z_W) = -{c_1}^* + \frac{1}{6}(k^2 - 1)(1 + 2b\mu)\sigma^{-1} \\ &= \{(s_1 - (\frac{1}{3}k^2 + \frac{1}{6})) + (s_2 - 2(\frac{1}{3}k^2 + \frac{1}{6}))b\mu\}\sigma^{-1} \end{aligned} \tag{47}$$

Choose

$$s_1 = \frac{1}{6}(2k^2+1)$$
$$s_2 = \frac{1}{3}(2k^2+1) \quad (48)$$

such that $p_1^*(z_W)=0$ for all μ. Therefore, the $O(n^{-\frac{1}{2}})$ non-oscillatory term in (40) is disappeared.

Similarly, the coefficient of the non-oscillatory term $O(n^{-1})$ in the Edgeworth expansion of $P(\mu \in C^* I_*)$ may be obtained from (42) as

$$p_2^*(z_W) = -\tilde{P}_2(z_W) = \frac{1}{2}k\left\{r_1 - \frac{1}{18}(13k^2+17)b\right\} + \frac{1}{2}k\sigma^{-2}\left\{r_2 - \frac{1}{36}(2k^2+7)\right\} \quad (49)$$

Let

$$r_1 = \frac{1}{18}(13k^2+17)b$$
$$r_2 = \frac{1}{36}(2k^2+7) \quad (50)$$

such that $p_2^*(z_w)=0$. Consequently, the $O(n^{-1})$ non-oscillatory term in (40) is disappeared. Therefore,

$$s_1 = s_2 = r_1 = r_2 = 0 \quad (51)$$

Substituting (51) into (44),

$$z_W = -k + \frac{1}{2}k^2(1+2b\mu)\sigma^{-1}n^{-\frac{1}{2}} - (b+\frac{1}{8}\sigma^{-2})k^3n^{-1} + O(n^{-\frac{3}{2}}) \quad (52)$$

Since the sum of (34) and (40) is equal to one,

$$P(\mu \in C^* I_W^u) = 1 - F_n(z_W) \quad (53)$$

The proof of Theorem 2 is completed.

4. Conclusion

This paper presents a new random weighting method for estimation of Poisson distributions. The contribution of the paper is that theories are established for random weighting estimation of the population parameters of two Poisson distributions with partially missing data and for random weighting estimation of one-sided confidence intervals in Poisson distribution. Future research work will focus on applications of the established theories in engineering practices.

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