

# Application of Fractal Dimensions and Fuzzy Clustering to Tool Wear Monitoring

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## Abstract

*Monitoring of metal cutting tool wear states is a key technology for automatic, unmanned and adaptive machining. As tool wear increases, the vibration signals of cutting tool become more and more irregular in the turning processes. The degree of tool wear can be indirectly monitored according to these changes of vibration signals. In order to quantitatively describe these changes, fractal theory and fuzzy clustering method were introduced into the cutting tool wear monitoring area. Firstly, wavelet de-noising method was used to reduce the noise of original signals, and eliminate the effect of noise on fractal dimensions. Secondly, the fractal dimensions based on fractal theory were got from the de-noised signals, including box dimension, information dimension, and correlation dimension. Finally, the relationship between the fractal dimensions and tool wear states was studied; the affinities between the known and unknown states can be obtained through fuzzy c-mean clustering algorithm; tool wear states can be recognized by those affinities based on fractal dimensions. The experiment results demonstrate that wavelet de-noising method can efficiently eliminate the effect of noise on fractal dimensions, and tool wear states can be real-timely and accurately recognized through the fuzzy clustering analysis on fractal dimensions.*

**Keywords:** Wavelet De-noising; Fractal Dimension; Tool Wear Monitoring; Fuzzy Clustering;

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## 1. Introduction

As market competition intensifies and the improving of product quality, the automatic and intelligence level of modern machining equipments is greatly improved. In order to ensure the reliability and safety of machining processes, it is necessary to monitor the machining systems real-timely and accurately, especially to cutting tools [1]. The wear of cutting tool directly affects the quality, efficiency and safety of production. Many scholars, at home and abroad, have been doing many researches into tool wear states monitoring technology. R. Teti [2], Roth, J. T. [3] and Abellan-Nebot [4] analyze the development status, trends and existing issues of tool wear condition monitoring technology from the angles of sensing signals, signal processing and pattern recognition.

During actual machining, the tool wear was monitored by indirect monitor instead of by stopping the machine. Some common indirect monitoring signals include cutting force, acoustic emission, cutting vibration signal, and so on. Among them, vibration signal is very sensitive to tool wear, and free from the influence of chips and coolant [1, 2]. So the cutting vibration signal was widely used to monitor tool wear indirectly. Because the process of tool wear is quite complex and affected by many factors, there are high randomness and nonlinearity in the vibration signals of tool wear [5]. However, further research indicates that the seemingly chaotic phenomenon reflects the dynamical behavior of metal cutting operation system. In order to reveal the potential information of chaotic vibration signal and monitor the tool wear, the wavelet and fractal theory were introduced into the cutting tool wear monitoring area. Fractal dimension based on fractal theory is an important parameter to quantitatively describe singularity degree of the chaotic attractors. It is widely used to describe the numerical characteristic in nonlinear system. It can qualitatively and quantitatively analyze the system running state [6], [7]. Recently, fractal dimension is applied in mechanical device fault diagnosis area, some achievements reported [8], [9]. In this paper, fractal dimensions are used as tool wear monitoring, including box dimension, information dimension, and correlation dimension. Considering the excessive noise in the original vibration signals and the effect of noise on fractal dimensions which would

be used to recognize tool wear states, it is necessary to remove noise from the original signals. Wavelet theory has been a topic of research in application math and engineering science. Wavelet de-noising has become an important tool to suppress the noise due to its effectiveness and producing better results [10], [11]. In this paper, the wavelet de-noising method is used to eliminate the effect of noise on fractal dimension in original signals; the fractal dimensions based on fractal theory were got from the de-noised signals.

Cutting tool gradually wear out until it is failure, the fractal dimensions of tool wear states have fuzziness in many cases. Fuzzy mathematics just provides a new method to solve the problems of fuzziness. Fuzzy clustering analysis based on fuzzy mathematics can categorize and identify fuzziness samples by calculating the affinities between those samples [12], [13]. Nowadays, neural network technology is always used to recognize tool condition, which has a very good fault tolerance and strong adaptive ability to the environment. But it needs a large number of features and samples as the training input of the network. So the application of the neural networks is limited in industry. Fuzzy clustering does not rely on expert experience and other subjective evaluation, and also can resolve the ambiguity problem. Therefore, it can improve the efficiency of pattern recognition. In this paper, the FCM (Fuzzy C-means) clustering algorithm is used to recognize tool wear states. The experiment results show that the tool wear states monitoring system based on fractal dimensions and FCM clustering can accurately recognize tool wear states, and has comparatively higher reliability.

## 2. Research Method

The tool wear monitoring system is composed of an accelerometer, data-acquisition devices and a micro-computer. The flank wear value of the cutting tool is the monitoring object. Multi-channel vibration signals are collected and converted to digital signals to feed into the computer which will accomplish data processing. Figure 1 is the block diagram of the cutting tool wear states monitoring system.

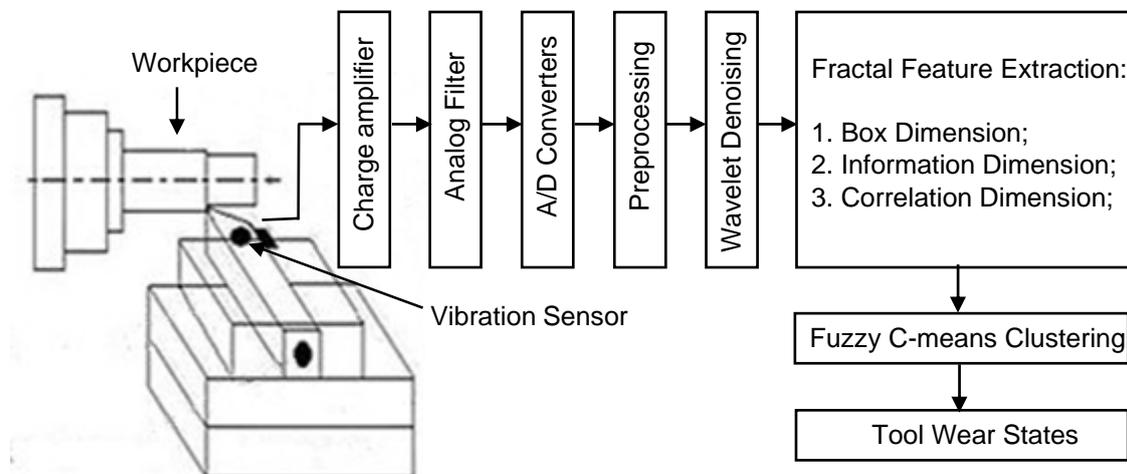


Figure 1. The block diagram of tool wear monitoring system

### 2.1. Wavelet De-noising

Wavelet de-noising can be viewed as an estimation problem trying to recover a true signal component  $f(n)$  from an observation  $X(n)$  where the signal component has been degraded by a noise component  $W(n)$ :  $X(n)=f(n)+W(n)$ . The estimation is computed with a thresholding estimator in an orthonormal basis  $B = \left\{ \left[ \phi_{J,m}(n) \right]_{m \in Z}, \left[ \psi_{j,m}(n) \right]_{L < j \leq J, m \in Z} \right\}$  as [10]:

$$\tilde{F} = \sum_{j=L+1}^J \left\{ \sum_m \rho_T \left( \langle X, \psi_{j,m} \rangle \right) \psi_{j,m} + \sum_m \rho_T \left( \langle X, \phi_{J,m} \rangle \right) \phi_{J,m} \right\} \quad (1)$$

Where,  $\rho_T$  is a threshold function that aims at eliminating noise components (via attenuating of decreasing some coefficient sets) in the transform domain while preserving the true signal coefficients. If the function is modified to rather preserve or increase coefficient values in the transform domain, it is possible to enhance some features of interest in the true signal component.

## 2.2. Fractal Dimensions

The research object of fractal theory is the complex phenomenon which has irregularity and self-similarity structure, such as coastline, sierra, changefulness clouds, and so on. According to these properties, fractal theory has become a powerful tool in the areas of monitoring and diagnostic for mechanism equipment. A lot of related researches have been done at home and abroad. In this paper, it was used in the study of tool wear states monitoring.

### (1) The Calculation of Box Dimension

The Box Dimension  $D_B$  is the simplest and most obvious fractal dimension. For the unit hypervolume attractor,  $D_B$  can be got by:

$$D_B = \lim_{\Delta \rightarrow 0} \left[ \frac{\ln N_\Delta}{\ln(1/\Delta)} \right] \quad (2)$$

Where,  $N$  is the number of hypercube which is used to cover the attractor whose side length is  $\Delta$ .

Use the Box whose side length is  $\Delta$  to cover attractor in the calculation of Box Dimension. If the number of full Boxes is  $N$ , the curve of  $\{\ln(N\Delta)-\ln(1/\Delta)\}$  can be drawn in the bilogarithmic diagram. The Box Dimension is determined according to the slope of the curve.

### (2) The Calculation of Information Dimension

If assign mentioned box numbers and the probability is  $P_i$  when the attractor fills into the  $i$ -box, the box can be expressed by Shannon equation:

$$I(\delta) = -\sum_{i=1}^N P_i \ln P_i \quad (3)$$

If using  $I(\delta)$  instead of  $N(\delta)$  in Box Dimension, Information Dimension  $D_I$  can be got by:

$$D_I = \lim_{\delta \rightarrow 0} \frac{I(\delta)}{\ln \delta} = \lim_{\delta \rightarrow 0} \frac{-\sum_{i=1}^N P_i \ln P_i}{\ln \delta} \quad (4)$$

### (3) The Calculation of Correlation Dimension

There is a time-series  $x_i$  in the experiment. The first  $n$  points was adopted to reconstruct  $m$ -dimensional phase space, the distance between these nodes can be got. The correlation function is as follows [13]:

$$C(r) = \lim_{N \rightarrow \infty} \frac{1}{N^2} \sum_i^N \sum_j^N H(u); i \neq j. \text{Where, } u = r - |x_i - x_j|; \begin{cases} H(u)=1 & u > 0 \\ H(u)=0 & u < 0 \end{cases} \quad (5)$$

$C(r)$  represents the ratio of the nodes whose distance is less than  $r$  in the reconstructed phase space. When choosing a suitable value for  $r$ , the following relation can be got:

$$\lim_{r \rightarrow 0} C(r) = r^{D_C} \text{ So } D_C = \lim_{r \rightarrow 0} \frac{\ln C(r)}{\ln r} \quad (6)$$

$D_C$  is the correlation dimension.

### 2.3. Fuzzy C-means Clustering Algorithm

The basic idea of cluster analysis is using the similarity metrics to judge the relationship of objects which is close or distant. According to this idea, the classification can be achieved. This paper focuses on the fuzzy c-means algorithm which uses cluster centers and Euclidean distance function [9], [10]. First of all, in this method, a number of cluster centers are selected randomly and the fuzzy membership to certain cluster center is assigned for all the dates. And then the cluster center is revised constantly by iterative methods. In the process of iterative, the weighted sums of minimizing distance between all the points to each cluster center and the membership values is used as the optimization objective. Iterative process is end when reaching the maximum iteration number or the decrease degree of the objective function value in two iterations is less than the given minimum increment.

On the mathematical level, fuzzy C-means clustering is to find the fuzzy dividing matrix  $U=[u_{ik}]_{c \times n}$  that makes clustering objective function  $J$  minimum and the clustering center  $P$ . Objective function  $J$  is calculated as [15]:

$$J_m(U, P) = \sum_{k=1}^n \sum_{i=1}^c (u_{ik})^m (d_{ik})^2 \quad (7)$$

Where,  $(d_{ik})^2 = \|x_k - p_i\|^2$  is the distance between the two vectors  $x_k$  and  $p_i$ ,  $x_k$  is the  $k$ -th samples of data,  $p_i$  is the  $i$ -th clustering prototype,  $i = 1, 2, \dots, c$ ;  $k = 1, 2, \dots, n$ ,  $m \in (1, \infty)$  is weighting exponent, the objective function  $J$  is the square sum of the weighted distance between a variety of data and the corresponding cluster center.

## 3. Experimental results

### 3.1 Experimental design

Experiments are carried out on the CK6143 machining center. The experimental material is 45 steel. The cutter material is YT15. The cutting form is cutting and the cooling fluid wasn't used. Data was collected when the tool wear are 0.0mm, 0.1mm, 0.2mm, 0.3mm, 0.4mm, and 0.5mm. The length of sampled data is 10000. The experiments were performed at three working conditions. Their cutting velocity, cutting feed and depth respectively are as follows: (500r/min, 0.5mm/r, 0.5mm), (1000r/min, 0.5mm/r, 1mm), and (1500r/min, 0.8mm/r, 1.5mm). The vibration acceleration sensor is 8702B50M1 K-Shear produced by Kistler Switzerland, which can measure displacement, velocity and acceleration. The single-channel sampling frequency is 100 KHz. Figure 2 is the picture of sensors installation. Figure 3 is the picture of worn cutting tool.

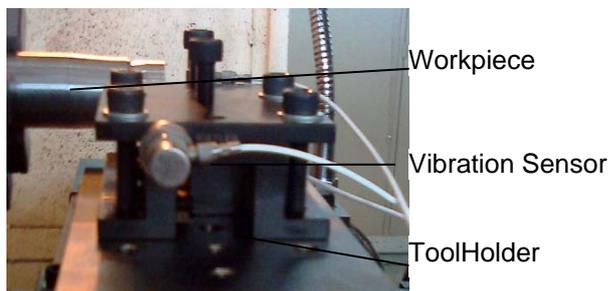


Figure 2. The picture of sensor installation

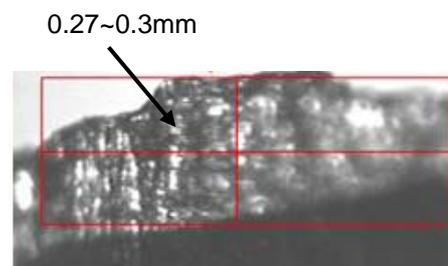


Figure 3. The picture of tool wear

### 3.2. The De-noising of Tool Wearing Vibration Signal

In order to ensure the accurate extraction of the fractal dimension, the wavelet theory was used to reduce the noise of the vibration signal. Take the box dimension for example, if there is a sinusoidal signal  $s = \sin(0.03 \cdot t)$ , then add noise to the signal, at last, use db4 wavelet to de-noise the signal by 4 level decomposition. The actual box dimension of the sinusoidal

signal is 0.99344, however the calculated dimension of the signal with noise is 1.41321, the box dimension is 1.11103 after the de-noising process. It indicates that noise has great influence on the calculation of the dimension. So does the correlation dimension. In the following, analyze the tool vibration signal of 0.5mm wear value under first working condition (cutting velocity 500r/min, cutting feed 0.5mm/r, cutting depth 0.5mm). Figure 4 is the time domain graph before and after de-noising.

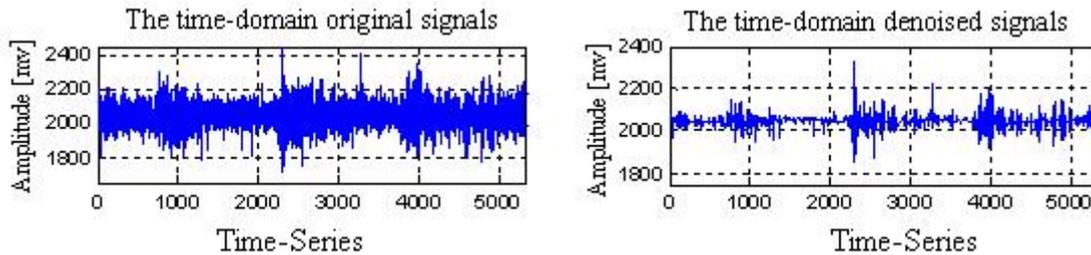


Figure 4. Time-domain graph before and after de-noising

### 3.3. Fractal Dimension Features of Tool Wear Signals

To get the quantitative changing information under different tool wear states, the fractal dimensions were calculated under all kinds of wear states, including box dimension, information dimension, and correlation dimension. An effective fractal dimensions feature has to include the following two points simultaneously: divisibility and repeatability.

#### (1) Calculation of Box Dimension

According to the box dimension calculation theory, the box dimension of tool wear states can be got based on the de-noised vibration signals. Considering the length of paper, Only the fractal dimension of different wear states under the first working condition are listed in Table1, including three samples in every wear condition.

Table 1. The box dimension under the first working condition

Tool wear	0.0mm	0.1mm	0.2mm	0.3mm	0.4mm	0.5mm
Sample No.1	1.3594	1.3841	1.4307	1.5168	1.5260	1.5578
Sample No.2	1.3503	1.3939	1.4430	1.5237	1.5191	1.5590
Sample No.3	1.3572	1.3797	1.4502	1.5209	1.5321	1.5650

From the table above, box dimension under different wear states fluctuate in a wide range, having obvious divisibility. Through longitudinal comparison, the box dimensions of same wear status fluctuate in a small range under the first working condition. It shows that it has a good repeatability.

#### (2) Calculation of Information Dimension

According to the information dimension formula which was list above, the information dimension of tool wear states can be got based on the de-noised vibration signals. Table 2 are the information dimensions of all wear status and three samples under the first working condition.

Table 2. Information dimension under the first working condition

Tool wear	0.0mm	0.1mm	0.2mm	0.3mm	0.4mm	0.5mm
Sample No.1	1.5660	1.6385	1.6421	1.6816	1.6939	1.7521
Sample No.2	1.5669	1.6389	1.6419	1.6811	1.6929	1.7524
Sample No.3	1.5658	1.6392	1.6425	1.6823	1.6935	1.7538

From Table 2, the information dimension changes when tools wear. Although the change is very little, it can reflect the degree of deviation from normal working status of tools. At the same time, the calculation results of three samples are listed in Table 2, which reflect it has good repeatability.

### (3) Calculation of Correlation Dimension

The parameter which includes delay amount and embedding dimension must be determined before calculating tool wear correlation dimension. In this paper, the delay amount and embedding dimension were determined by mutual information theory and Cao method. Taking the first working condition as example, the delay amount is 2 and embedding dimension is 21 in every wear states. Their values of correlation dimension are listed in the Table 3.

Table 3. Correlation dimension under the first working condition

Tool wear	0.0mm	0.1mm	0.2mm	0.3mm	0.4mm	0.5mm
Sample No.1	1.4546	1.6801	2.3421	4.6552	6.9879	10.8707
Sample No.2	1.4593	1.6824	2.3515	4.6405	6.9746	10.7962
Sample No.3	1.4449	1.6726	2.3456	4.6623	6.9918	10.9168

From the Table 3, it shows that the value of correlation dimension gives large change with the change of tool wear and can be used to evaluate the tool condition. Similarly, the correlation dimension can be got in another working condition. They also have the divisibility and repeatability.

### 3.4. Tool Wear States Recognition Based on FCM Clustering

In the experiment, several groups of data under each working condition were collected to validate the effectiveness of the method. Taking first condition for example, the cluster identification process is as follows:

First of all, form a 18×3 matrix  $X$  for being clustered according to table 1, table 2, and table 3. The 1~3 columns of  $X$  are fractal dimensions when tool wear is 0.0mm. The 4~6 columns of  $X$  are fractal dimensions when tool wear is 0.1mm. The 7~9 columns of  $X$  are fractal dimensions when tool wear is 0.2mm. The 10~12 columns of  $X$  are fractal dimensions when tool wear is 0.3mm. The 13~15 columns of  $X$  are fractal dimensions when tool wear is 0.4mm. The 16~18 columns of  $X$  are fractal dimensions when tool wear is 0.5mm. The error of objective function is  $10^{-5}$ . The classification number is 6. The classification-matrix  $U$  can be obtained by FCM algorithm. The results of  $U$  are list in table 4.

Table 4. Classification-matrix  $U$

Classification-matrix $U$ (1-9 column)								
<b>9.997E-01</b>	<b>9.986E-01</b>	<b>9.988E-01</b>	1.105E-04	1.350E-03	1.324E-03	1.631E-04	3.527E-05	9.846E-05
3.030E-04	1.274E-03	1.106E-03	<b>9.999E-01</b>	<b>9.985E-01</b>	<b>9.985E-01</b>	2.956E-04	6.344E-05	1.778E-04
2.103E-05	8.659E-05	8.127E-05	1.423E-05	1.809E-04	1.566E-04	<b>9.995E-01</b>	<b>9.999E-01</b>	<b>9.997E-01</b>
1.919E-07	7.839E-07	7.561E-07	7.549E-08	9.511E-07	8.486E-07	1.802E-06	3.996E-07	1.102E-06
5.548E-07	2.268E-06	2.183E-06	2.261E-07	2.849E-06	2.538E-06	6.067E-06	1.348E-06	3.712E-06
1.655E-06	6.773E-06	6.495E-06	7.188E-07	9.068E-06	8.053E-06	2.446E-05	5.459E-06	1.500E-05
Classification-matrix $U$ (10-18 column)								
1.930E-06	1.557E-05	9.030E-06	3.293E-07	4.836E-06	2.929E-06	1.101E-06	4.842E-05	3.465E-05
2.237E-06	1.806E-05	1.046E-05	3.581E-07	5.260E-06	3.185E-06	1.156E-06	5.087E-05	3.637E-05
3.721E-06	3.013E-05	1.738E-05	4.688E-07	6.890E-06	4.168E-06	1.344E-06	5.924E-05	4.227E-05
5.156E-07	4.103E-06	2.430E-06	6.733E-07	9.771E-06	6.009E-06	<b>1.000E+0</b>	<b>9.994E-01</b>	<b>9.996E-01</b>
3.660E-06	2.890E-05	1.731E-05	<b>1.000E+0</b>	<b>9.999E-01</b>	<b>1.000E+0</b>	6.469E-06	2.912E-04	2.008E-04
<b>1.000E+0</b>	<b>9.999E-01</b>	<b>9.999E-01</b>	1.853E-06	2.739E-05	1.645E-05	2.527E-06	1.121E-04	7.914E-05

Through the definition of classification-matrix  $U$ , the row number of maximum of each column is classification number. The results of table 4 coincide with the actual results. The cluster center matrix  $V$  and the corresponding wears are list in Table 5.

Table 5. Cluster center matrix P and corresponding wears

Cluster center matrix P			Wears(mm)
1.3556	1.5662	1.4529	0.0
1.3859	1.6389	1.6784	0.1
1.4413	1.6422	2.3464	0.2
1.5606	1.7528	10.8612	0.5
1.5257	1.6934	6.9848	0.3
1.5205	1.6817	4.6527	0.4

The obtained cluster center can be a standard mode for tool wear states recognition. The tool wear states of a new sample can be determined according to the closeness degree of the new samples to the standard mode. Taking a new sample  $X_1$  as an example, its wear is 0.2mm in the first working condition:

$$X_1 = [1.4131 \ 1.6512 \ 2.5183]$$

After calculating the closeness degree, what we can get the distance  $D_1$  between  $X_1$  and cluster centers  $V$  are as follows:

$$D_1 = [1.0703 \ 0.8405 \ 0.1744 \ 8.3449 \ 4.4681 \ 2.1373]$$

So the closest row between  $X_1$  and cluster centers  $V$  is third. So the wear of the new sample 0.2mm, which coinciding with the actual condition. This tool wear states monitoring method is verified by the experiment in other processing conditions.

#### 4. Conclusion

Nowadays, fractal theory and fuzzy clustering are frontier research topics and applied to classification and pattern identification in many fields for the superiority of themselves. How to use the two theories to recognize tool wear states is a valuable research topic. Firstly, wavelet theory was used to de-noise the original vibration signals. Secondly, the fractal dimensions were extracted by fractal theory, including box dimension, correlation dimension, and information dimension. The three characteristic values were used as states indicator for tool wear monitoring. Finally, the fuzzy C-means clustering algorithm was used to recognize tool wear states.

The experiment results show that: Wavelet de-noising method can efficiently eliminate the effect of noise on fractal dimensions. The fractal dimensions of tool wear vibration signals are sensitive to tool wear states. The fractal dimensions based on fractal theory can reveal the underlying information in the vibration signals. The tool wear states can be accurately recognized by the fuzzy clustering analysis on fractal dimensions. Meanwhile, fuzzy clustering analysis, unlike neural networks which need a large number of samples to learn, can greatly reduce the diagnosis time and can be used for real-time monitoring; this method can also be used for other condition monitoring areas.

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