

A Survey on Weighted Network Measurement and Modeling

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Abstract

Real networks have complex topological structure characteristics such as pairs of nodes have different strength and capacity connection. There are many instances in our real life. Every person has strong or weak relationship with others in social network and different reaction paths have non-uniform flow in metabolic network. In the food chain predator and prey have diverse relation and in the neural network electrical signal transmission paths have different capacity. All these systems could be described well by the concept of weighted network and weight of each edge in weighted network stand for the connection strength among individuals. This paper aims to provide an overview of recent advances in such study area. We first introduce a series of weighted network statistical characteristic parameter and concepts. Then in particular, we also discuss some practical network application examples. Empirical results show that pure topological model is not sufficient to explain the abundant and complex characteristics observed in real systems and it is necessary to improve such model. So we finally focus on some latest optimized weighted network models and give comparisons.

Keywords: Weighted network; Statistical characteristic; Model;

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1. Introduction

Many things in real world such as variety of entities and complex relations between entities could be expressed by the form of weighted network. From Internet [1] to WWW [1], from large power network [2] to global transportation network [3], from brain in the organism to various metabolic network [4], and from Scientific collaboration network [5] to all kinds of economic, political and social relation network [5,6], we can say we have already have lived in the world filled with various weighted networks. In such situation, weighted network topology structure study has begun to be the hotspot.

The rest of paper is organized as follows. Section 2 introduces some statistics and concepts of weighted network. Section 3 describes some weighted network modeling methods, including YJBT [7], ZTZH [8] and AK [9]. Section 4 points out the defaults of above three modeling methods and gives the improvements. In section 5 we conclude and discuss future work.

2. Weighted Network Measurement

A weighted network could be expressed as $G^W = (N, l, W)$ where node $N = \{n_1, n_2, \dots, n_N\}$ and edge $l = \{l_1, l_2, \dots, l_K\}$ and weight in different edges $W = \{w_1, w_2, \dots, w_K\}$. The weighted network could be shown as Fig.1 where thickness of edges expresses the weight. When in the form of matrix, G^W is usually expressed as weighted matrix \mathbf{W} with the size of $N \times N$ and the element value w_{ij} stands for the corresponding edge weight. $w_{ij} = 0$ when node i and j have no connection. In this paper we discuss only two situations. One is $w_{ij} = w_{ji} \geq 0$ and the other is $w_{ii} = 0 \forall i$. In certain circumstance the weight

of edge may be negative such as the strange degree among people in social network. One primary characteristic of weighted network is the weight distribution $Q(w)$, which stands for the probability that the edge weight is w .



Figure 1. Weighted network

2.1 Node Strength and Intensity Distribution and correlation

In weighted network, the degree k_i of node i is called node strength which often expressed as s_i . The definition of s_i is [10-12]:

$$s_i = \sum_j w_{ij} \quad (1)$$

We can draw a conclusion from formula (1) that node strength includes both node degree and edge weight connecting the node. When the weight and the topological structure of the network is not relevant and if the node degree is k , the node strength is $s(k) \approx \langle w \rangle k$ where $\langle w \rangle$ is the average weight. If we consider the correlation, $s(k) \approx Ak^\beta$.

Given a node i with degree k_i and strength s_i , w_{ij} has the same order of s_i / k_i . The discrepancy in different edge weight of node i could be described as [13-14]:

$$Y_i = \sum_{j \in N_i} \left[\frac{w_{ij}}{s_i} \right] \quad (2)$$

where N_i indicates the first-order neighbor nodes set of node i . Y_i is well correlated with node degree. If all edges have similar weight, $Y(k)$ is proportional to $1/k$. If one edge could influence formula (2) then $Y(k) \approx 1$. In another word, $Y(k)$ is irrelevant to k in such situation [15]. Intensity distribution $R(s)$ stands for the probability of selecting a node with intensity s . $R(s)$ and $P(k)$, which is often called degree distribution, describe the structural properties of weighted network well.

2.2 Lowest-weight path

Networks in n -dimensional Euclid Space are often two or three dimensional and so the distance of node i and j could be represented by Euclidean distance yardstick. Edge length in weighted network can be defined as a function of weight, for instance $l_{ij} = 1/w_{ij}$. In weighted network, an optimal path is usually not the lowest-hop path for considering the edge weight. Choosing of the lowest-weight path is completely dependent on the definition of edge weight.

2.3 Weight cluster coefficient

In the literature [16], Barrat et al. defined clustering coefficient on node i in weighted network as:

$$c_i^w = \frac{1}{s_i(k_i - 1)} \sum_{j,m} \frac{(w_{ij} + w_{im})}{2} a_{ij} a_{jm} a_{mi} \quad (3)$$

Weight clustering coefficient considers not only the triangle numbers adjacent to node i but also the relative weight of relevant nodes intensity or strength. $s_i(k_i - 1)$ is a normalization factor to ensure $0 \leq C_i^w \leq 1$. C^w is the average value of all nodes' weight clustering coefficient and $C^w(k)$ is the average value of nodes' weight clustering coefficient with the degree being k . In the case of large-scale random network(not considering degree relevant) $C^w = C$ and $C^w(k) = C(k)$ where C is the average value of clustering coefficient of un-weighted network with the same topology and $C(k)$ is the average value of such network nodes' clustering coefficient with degree being k .

In real weighted network we may meet two reciprocal situations. One is $C^w > C$ and then edges forming triangle usually have larger weight. The other is $C^w < C$ and these edges have smaller weight.

3. Weighted network modeling

3.1 Yook–Jeong–Barabási–Tu (YJBT)

The simplest way to construct a weighted network is to build a random network with the degree distribution is $P(k)$ and then try to make it submit to mutual and independent distribution, supposing the edges' weight are independent of each other. One interesting situation is that the node degree and weight are often coupled. S.H. Yook et al. proposed the Yook–Jeong–Barabási–Tu (YJBT) free scale weighted network model. The topology of the model and the edge weight distribution in the model are generated based on preferential attachment mechanism during the period of network growth [7]. The YJBT Model construction process is as follows:

Starting from m_0 isolated nodes, every time when introduced a new node j we make it connectable to m existing nodes where $m \leq m_0$. The probability that node j is connected to the existing node i obeys BA Network preferential connection characteristic. We can assign each edge weight of node j $w_{ji} (= w_{ij})$:

$$w_{ji} = \frac{k_i}{\sum_{\{i\}} k_i} \quad (4)$$

YJBT could generate a free scale network whose degree distribution is $P(k) \sim k^{-\gamma}$ and $\gamma = 3$. The network's power law intensity distribution is $R(s) \sim s^{-\gamma_s}$ where the value γ_s of is closely related to the choice of m .

3.2 Zheng–Trimper–Zheng–Hui (ZTZH)

Zheng–Trimper–Zheng–Hui (ZTZH) is the generalized form of YJBT. It combines random weight allocation mechanism based on the node degree distribution [8]. We add new edge l_{ji} on the probability of p and on the probability of $1 - p$ add edge l_{ji} weight:

$$w_{ji} = \frac{\eta_i}{\sum_i \eta_i} \quad (5)$$

where η_i is the optimum parameter and is randomly selected on the probability of $\rho(\eta)$ between 0 and 1 [17]. When $p=1$, it is YJBT model. ZTZH also leads to power law intensity distribution's being $R(s) \sim s^{-\gamma_s}$. γ_s is sensitive to p and if p increases from 0 then γ_s falls down from 3 continuously.

3.3 Antal–Krapivsky (AK)

Antal–Krapivsky (AK) proposed a structural growth weighted network model based on edge and weight coupling [9]. The model generation process is as follows: Every step a new node connects to an existing node i and the connection probability is

$$\prod_{j \rightarrow i} = \frac{s_i}{\sum_l s_l} \quad (6)$$

This preferential connection rule is a strength-prior rule. That is the new node is always inclined to connect the node with higher strength. Many real networks naturally use this connection mechanism. Due to the consideration of the bandwidth and flow, in Internet, new routers should be connected to central routers [1]. In the scientific research network, authors are more easily to search cooperating with others. For each edge adding the weight which is positive and obedience to distribution $\rho(w)$, the final shape of the network is the tree form. When the connecting process lasts a long time and with $s \rightarrow \infty$, the intensity distribution $R(s)$ of the network approaches a stable distribution $R(s) \sim s^{-3}$ and is irrelevant to $\rho(w)$, the edge weight distribution.

4. Model comparison and improvement

Above YJBT, ZTZH and AK models are all based on growth mechanism. Weight is assigned when edge is first established and keeps unchanged. These models all ignore the dynamic evolution characteristics of the edge weight when new nodes and edges are added to the network. Not only that, the weight evolution and edge reconnection is a common natural characteristics of network. For example in the aviation network, when a new route was established it would affect other routes traffic volume. In such situation an improved weighted network model appeared. That is Barrat–Barthélemy–Vespignani (BBV) model which constructed based on weight dynamic evolution caused by the local network growth and edge reconnection [18-20]. It starts from m_0 nodes and the edge weight is w_0 . Then every step a new node connects existing m nodes on the probability as shown in formula (7), supposing the original weight of m edges is w_0 .

$$P(k) = \begin{cases} 0 & k < K \\ \binom{N}{k-K} \left(\frac{Kp}{N}\right)^{k-K} \left(1 - \frac{Kp}{N}\right)^{N-k+K} & k \geq K \end{cases} \quad (7)$$

Let l_{ji} be a new edge and its appearance makes weight of edges between node i and its neighbor node set change as follows:

$$w_{il} \rightarrow w_{il} + \Delta w_{il} \quad \forall l \in N_i \quad (8)$$

where

$$\Delta w_{il} = \delta \frac{w_{il}}{s_i} \quad (9)$$

When a new edge with the weight of w_0 is connected to node i it would make communication traffic on the other edges of i increase by δ . During the process each side's increased amount is proportional to the edge weight and produces the result of $s_i \rightarrow s_i + \delta + w_0$ in the end. Specific evolution process is shown in figure 2.

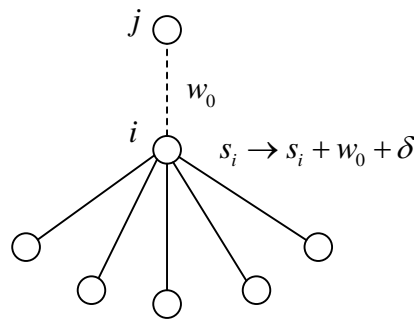


Figure 2. Edge weight evolution process of BBV

Intensity and weight generated by BBV model both obey exponential distribution. In order not to lose the general we can make $w_0 = 1$ and then the model is determined by parameter δ . After a few steps we could get the weight distribution as $Q(w) \sim w^{-\gamma_w}$, $\gamma_w = 2 + 1/\delta$ and degree distribution as $P(k) \sim k^{-\gamma}$ and intensity distribution as $R(s) \sim s^{-\gamma_s}$, where $\gamma = \gamma_s = (4\delta + 3)/(2\delta + 1)$. This result shows that BBV could generate a free scale network whose power law exponent $\gamma \in [2, 3]$. When $\delta = 0$ BBV is the same as BA.

5. Conclusion and Future Works

Weighted network has been an active and difficult research topic for recent years. Some researchers obtain fairly good results. But there still exist some unresolved and scarcely addressed problems such as community detection, consensus problem and node importance evaluation and so on. At the same time, some kinds of statistics of weighted networks have not a unified definition and their physical significances are not very clear. To solve such questions, we need to look for more reliable method to describe the weighted network. Moreover, we should try to find more effective methods to build weighted network end-to-end reliability in the condition of node failure.

References

- [1] Pastor-Satorras R, Vespignani A. *Evolution and Structure of the Internet: A Statistical Physics Approach*, Cambridge University Press, Cambridge, 2004.
- [2] Watts DJ. *Small Worlds: The Dynamics of Networks between Order and Randomness*, Princeton University Press, Princeton, NJ, 1999.
- [3] Guimerà R, Mossa S, Turtschi A. *The worldwide air transportation network: Anomalous centrality,*

- community structure, and cities' global roles*, Proc. Natl. Acad. Sci. USA, 2005; 102(22): 7794-7799.
- [4] Girvan M, Newman MEJ. *Community structure in social and biological networks*, PNAS, 2002; 99: 7821-7826.
- [5] Scott J. *Social Network Analysis: A Handbook*. London: Sage Publications, 2nd ed., 2002.
- [6] Latora V, Marchior M V, Marchior M, *Economic small-world behaviour in weighted networks*, Eur. Phys. J. B, 2003, 32: 249–263.
- [7] Song C, Havlin S, Makse HA. *Self-Similarity of Complex Networks*, Nature, 2005; 433: 392-395.
- [8] Broder A, Kumar R, Maghoul F, Stata R, Tomkins A, and Wiener J. *Graph structure in the web*, Computer Networks, 2000, 33: 309-314.
- [9] Klemm K, Eguíluz VM. *Highly clustered scale-free networks*, Phys. Rev. E, 2002; 65: 036123.
- [10] Barrat A, Barthélemy M, Pastor S R, *The architecture of complex weighted networks*, Proc. Natl. Acad. Sci. USA, 2004, 101: 3747-3800.
- [11] Yook SH, Jeong H, Barabási AL. *Weighted Evolving Networks*, Phys. Rev. Lett., 2001; 86: 5835.
- [12] Onnela J P, Chakraborti A, Kaski K. *Dynamics of market correlations: Taxonomy and portfolio analysis*, Phys. Rev. E, 2003, 68: 056110.
- [13] Barthélemy M, Gondran B, Guichard E. *Spatial structure of the Internet traffic*, Physica A, 2003; 31: 9633-643.
- [14] Derrida B, Flyvbjerg H. *Statistical properties of randomly broken objects and of multivalley structures in disordered systems*, J. Phys. A, 1987; 20: 5273-5288.
- [15] Barratand A, Weigt M. On the properties of small-world networks, Eur. Phys. J. B, 2000; 13: 547–560.
- [16] Molloy M, Reed B. *A critical point for random graphs with a given degree sequence*, Random Struct. Algorithm, 1995; 6: 161-179.
- [17] Barabási AL, Jeong H, Ravasz R. *Evolution of the social network of scientific collaborations*, Physica A, 2002; 311: 590-614.
- [18] Barrat A, Barthélemy M, Vespignani A. *Modeling the evolution of weighted networks*, Phys. Rev. E, 2004; 70: 066149.
- [19] Dorogovtsev SN, Mendes JFF, Samukhin AN. *Size-dependent degree distribution of a scale-free growing network*, Phys.Rev. E, 2001, 63: 062101.
- [20] Pandya RVR. A note on “*Weighted Evolving Networks: Coupling Topology and Weight Dynamics*”, 2004; 16: 644-657
- [21] Chen Wen, Zhao Yongjiu, Jun Zhouxiao. Compact and wide upper-stopband triple-mode broadband microstrip BPF, *Telkomnika*, 2012,10(2): 353-358
- [22] Ya nXijun, Meng Xiangwei, Yan Yan. A wireless sensor network in precision agriculture, *Telkomnika*, 2012; 10(4): 788-797