# SVD-based MEMS Dynamic Testing Technology

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### Abstract

To obtain the MEMS micro structure's high resolution measurement dynamic parameters, this paper, by obtaining stroboscopic imaging technology of micro structure of moving image sequence, proposed one kind based on the phase correlation technique and singular value decomposition technique combining sub-pixel measurement method. By the method of phase correlation, image space coordinate can transform into the frequency domain to the coordinates of the parameter space. And then through the singular value decomposition technique to obtain the correlation matrix, using the least-squares fitting, get the sub-pixel level displacement measurement results. The experimental results show that, with this method of measuring MEMS micro structure plane motion amplitude can reach sub-pixel accuracy, and can be effectively reduced by uneven illumination effect on the measuring result, thereby increasing the measurement results of stability and reduces the measuring error.

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#### 1. Introduction

MEMS (micro electro mechanical system, MEMS) is a emerging discipline.Its development has opened up a whole new area of technology and industry,the use of MEMS technology production come into contact with almost all areas in aviation, aerospace, automotive, biomedical, environmental monitoring, military, and very broad application prospects. And The basic properties of MEMS (micro electro mechanical system, MEMS) devices,which have the moving parts,such as resonator, vibration gyro, accelerometer and light switch, are the dynamic characteristics, especially in different frequency or under the condition of different voltage the movement amplitude. Therefore, whether the precise measurement of the movement amplitude is very critical. Applying motion estimation method to detect the target translational motion between two similar images has become a hot topic in many image processing.

At present, many detection methods have been developed, such as block matching pixel recursive method and the phase correlation method. Which block matching is a relatively simple one, it is the image is divided into fixed-size blocks, and assuming that these blocks for independent translational motion; Pixel recursive method is based on the differential operation of the pixel differential method, the disadvantage is more sensitive to noise, and the large amount of movement when the error; Phase correlation was first proposed by Kuglin and Hines [1], is based on the Fourier transform frequency domain, because it is the use of phase information in the cross-power spectrum to match the image, the image not sensitive to changes in the brightness and anti-jamming capability, and therefore have a higher matching accuracy, However, due to the computational complexity of the algorithm is unable to meet the requirements of real-time image processing system, and typically only the pixel-level precision, can not meet the precise measurement of the amplitude of plane motion of MEMS has been can not be applied to the actualreal-time detection [2]. Found from the simulation, image processing method in the frequency domain to solve the airspace under difficult or even unsolvable problem, and can improve the measurement of the level of the image processing system, but most of the real-time image processing system uses also are airspace-based algorithms, there is little based on frequency domain algorithm. In this kind of situation, this article through to no noise cases phase correlation matrix analysis of characteristics, this paper puts forward a singular value decomposition based phase correlation algorithm. The algorithm is mainly the

case of noise-free matrix rank and least squares linear fitting method can be very effective to overcome the impact of noise, and can achieve sub-pixel accuracy, so as to meet the precise measurement of the MEMS plane motion amplitude .

## 2. The MEMS dynamic test technology

MEMS testing techniques can be divided into two major MEMS common characteristics testing technology and MEMS device characterization testing technology.MEMS common characteristics of the testing techniques, including the tiny geometry and threedimensional topography measurement, measurement and reconstruction of the micron scale three-dimensional movement, material properties and mechanical parameters of the measurement and reliability testing measurement; MEMS device characterization testing technology based on inertial MEMS, optical MEMS, RF MEMS and bio-MEMS and other specific requirements of the different classes of devices using different test methods and equipment.

Stroboscopic imaging, computer vision, and interferometry widely used in MEMS dynamic testing technology.Faster movement of the MEMS devices, the technology uses a stroboscopic imaging using ordinary CCD camera to capture every movement of the MEMS device instantly quasi-static position, then using computer vision, image sequence-based motion estimation algorithm to measure the plane micromovement, the main limitation of this technique is the only cycle micro-motion or repeated transient micro-motion measurements.

### 3. Phase Related Technical Principle

The traditional phase correlation motion estimation algorithm is based on the Fourier power spectrum analysis of frequency-domain nonlinear, depend mainly on the power spectrum analysis, phase correlation method [3]. The idea of this method:

Let  $R(\mu, v)$  and  $S(\mu, v)$ , respectively, for the reference image and test images of the Fourier transform, the relationship between  $R(\mu, v)$  and  $S(\mu, v)$  is:

$$S(\mu, \nu) = R(\mu, \nu) \exp\{-j(\mu x_0 + \nu y_0)\}$$
(1)

Among them,  $(\mu, \nu)$  in the frequency domain coordinates,  $x_0$  and  $y_0$  denote the horizontal and vertical displacement between the two plans.

The first step to calculate the phase correlation matrix:

$$Q(\mu,\nu) = \frac{R(\mu,\nu)^* S(\mu,\nu)}{|R(\mu,\nu)^* S(\mu,\nu)|} = \exp\{-j(\mu x_0 + \nu y_0)\}$$
(2)

Among them,  $R^*(\mu, \nu)$  is  $R(\mu, \nu)$  's the conjugate.

In the second step of calculating the  $Q(\mu, \nu)$ 's inverse Fourier transform, the peak coordinates that is, ask the displacement value:

$$Q(x, y) = \delta(x - x_0, y - y_0)$$
<sup>(3)</sup>

However, in image processing if x and y are integers, then Q will be expressed as a function of F; if x and Y are non-integers, then Q peak energy will be around to adjacent pixel dispersion, performance, in the peak appears around the many small burr. Therefore, reducing the accuracy of motion estimation. In addition, when sampling, the image is band-limited, there will be aliasing, and (1) will not be valid for all  $(\mu, \nu)$ . The image edge effects can also affect the degree of tapering and clarity of the peak.

Ideal noise-free conditions, the correlation matrix A of rank 1 A of each element can be decomposed into:

 $Q(\mu,\upsilon) = \exp\{-j\mu x_0\}\exp\{-j\upsilon y_0\}$ (4)

Therefore, we can define two vectors:  $q_x(\mu) = \exp\{-j\mu x_0\}$  and  $q_y(\nu) = \exp\{-j\nu y_0\}$ . Then the phase the the correlation matrix to can be expressed as:

$$Q(\mu,\nu) = q_x q_y^H \tag{5}$$

So (1) can be rewritten as:

$$S = (q_x q_y^H) \circ R \tag{6}$$

Where,  $\{\square\}^H$  represents the complex conjugate transpose,  $\circ$ , said between the elements and the elements multiplied by.

## 4. Based on the SVD Phase Correlation Technique 4.1 Singular Value Decomposition

In practical applications, because the noise and sampling limitations, making the phase correlation matrix rank does not equal to 1. This paper uses the SVD decomposition, correlation matrix for processing [4], can obtain the phase correlation matrix of rank 1 estimation [5]. Image A is the size of the  $M \times N$  matrix, and its rank R, then A can use the singular value decomposition method is decomposed into a series of  $M \times N$ -size rank 1 matrix weighted. Singular value decomposition is an orthogonal transformation between a matrix A and its singular value c and singular vectors u and v, if all the row vectors u and arranged in matrix U-, and arranged all the row vector vmatrix V, then:

$$\begin{cases} AV = U\Sigma \\ A^{T}U = V\Sigma \end{cases}$$
(7)

Among them, the main diagonal elements of the matrix  $\Sigma$  is the corresponding singular value  $\sigma$ , the other elements are zero. Matrix U and V are unitary matrix, that is, two orthogonal matrix within the column to the volume, and the column vector of length 1, therefore, the matrix A can be written as  $A = R \sum_{i=1}^{R} \sigma U_i V_i^T$ , that is the singular value decomposition of A, for the expansion

of the image and, although the formula is only suitable for R a base image, but a base image with the corresponding singular value weighted.

## 4.2 Phase correlation algorithm based on SVD

The proposed algorithm is based on this singular value decomposition, the phase correlation matrix after SVD decomposition can be broken down into two singular matrix [6]. According to the characteristics of an ideal noise-free case, the phase correlation matrix of rank is 1,and the phase correlation matrix after SVD will be able to get the singular vectors of the horizontal and vertical direction, this parper get the phase slop which is the translation vector.by least squareslinear estimates of the singular vectors of these two directions. Phase correlation algorithm based on SVD flow chart shown in Figure 1.



Figure 1. The flow chart SVD\_Phase\_Correlation algorithm

First, from the use of strobe technology in MEMS micro-structure of the moving image sequence, the image A and B, and they exist only between a simple flat shift the relationship between the Fourier transform of the two images.and then using Normalized cross-power spectrum to determine the phase correlation matrix.

Find normalized rank of a phase correlation matrix A for an estimated matrix to determine the translation between the two contain the noise of the image. The most direct way is to use the singular value decomposition, can obtain the singular vectors of the horizontal and vertical direction, so that the coefficients of the linear phase will be able to independently by these two singular vectors is established. Then the least squares method to the phase component obtained by the decomposition of the linear fit.

For the size of  $M \times N$  matrix Q, its airspace and frequency domain coordinates relations are  $\mu = 2\pi / M$ ,  $\upsilon = 2\pi / N$ , for the singular vectors of V are as follows:

$$W[\Delta \ c]^{T} = unwrap\left\{ \angle \ v \right\}$$
(8)

Where, *W* is a row vector,  $w = \{0, 1, 2, \dots, (s-1)\}$ , *S* represents the length of *V*,  $\Delta$  represents the slope. And ultimately by the least squares fit of the best value:

$$\begin{bmatrix} \Delta \\ c \end{bmatrix} = (W^T W)^{-1} W^T \text{ unwarp} \{ \angle v \}$$
(9)

The obtaining value of the slope of the fitted line,  $\Delta$  is translation. When  $v = q_x$ , v epresents the weight of the horizontal direction,  $x_0 = -\Delta(M/2\pi)$ ; When  $v = q_y$ , v represents the weight of the horizontal direction,  $y_0 = -\Delta(N/2\pi)$ .

Because of Noise and edge effects, the processed data will be mass. To improve the efficiency of the algorithm to consider, you can set the threshold to mask the datas less than the threshold in Q, and apply mask to mask the data the origin of the DC component with a radius of R other than the data. Generally set R = 0.6L/2, L, take the smaller value between the *M* and *N*. In the end, you can estimate the translation of non-integer values in a large range of horizontal and vertical direction.

## 5. Analysis

In practical applications, using stroboscopic imaging technique of micro-structure to get the moving image sequence. But the test results will have an impact , because of uneven illumination, gray and other factors. In testing, collecting two MEMS dynamic image with noise (subject to Gaussian noise) to interference test, and verify that the algorithm in this paper, noise immunity and effectiveness.

## 5.1 The graphical results

Computer simulation results are as follows:





Figure 2. Contains the noise matrix reconstruction



Figure 3. Phase correlation the original image correlation



Figure 4. The Phase of matrix of Rank1 estimates



Figure 5. The horizontal direction phase component after SVD decomposition

Figure 6. The vertical direction phase component after SVD decomposition

Figure 2 is the original image which size is 500pixels and with noise, Figure 3 there is a flat to shift the relationship between the two image phase correlation matrix, Figure 4 after the singular value decomposition rank 1 phase estimate matrix, obtained after treatment phase correlation matrix. Can be drawn from Figures 3 and 4 deal with the former phase stripes of noise pollution is serious, ambiguous, and post-processing phase stripes become very clear. Figures 5 and 6, respectively, are the phase component of the horizontal and vertical directions. In order to suppress the noise area, masking the phase correlation matrix of rank 1 from the central DC component in the high frequency components above the 76pixels ,shown as figure 7. Finally, after a least squares linear fit will be able to determine the translational value of the sub-pixel level.



Figure 7. The distance from the center to the DC component of 76 pixels within the scope of the correlation matrix

## 5.2 Analysis of experimental data

Known translation in the horizontal and vertical pixel size images using the proposed algorithm to test the experimental data shown in Table 1. In order to facilitate the observation, which brought the symbol of the shift value the translation direction of the moving image, the image origin, in the time domain to be match the image relative to the reference image to the right or pan down, the frequency domain decomposition phase vectora decreasing straight line, the pan value is positive; the contrary, the pan value is negative.

Original	Original	The	The vertical	absolute	absolute
horizontal	vertical	horizontal	displacement	error of	error of
displacement	displacement	displacement	in test	horizontal	vertical
		in test		direction	direction
1	-1	1.0385	-1.0742	0.0385	0.0742
1	-2	1.0189	-1.9758	0.0189	0.0242
1	-3	0.9807	-2.9521	0.0193	0.0479
1	-4	0.9968	-3.9106	0.0032	0.0894
-1	-1	-0.9925	-0.9976	0.0075	0.0024
-1	-2	-0.9877	-1.9909	0.0123	0.0091
-1	-3	-0.9768	-3.0575	0.0232	0.0575
-1	-4	-0.9638	-4.0431	0.0362	0.0431
-1	1	-1.0146	0.9663	0.0146	0.0337
-1	2	-0.9657	2.0528	0.0343	0.0528
-1	3	-0.9638	3.0065	0.0362	0.0065
-1	4	-0.9642	3.9764	0.0358	0.0236
	Original horizontal displacement 1 1 1 1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -	Original horizontal displacement         Original vertical displacement           1         -1           1         -2           1         -3           1         -4           -1         -1           1         -2           1         -3           1         -4           -1         -1           -1         1           -1         -1           -1         -1           -1         -1           -1         -2           -1         1           -1         2           -1         1           -1         2           -1         3           -1         4	Original horizontal         Original vertical         The horizontal           displacement         displacement         displacement           1         -1         1.0385           1         -2         1.0189           1         -3         0.9807           1         -4         0.9968           -1         -1         -0.9925           -1         -2         -0.9877           -1         -3         -0.9768           -1         -4         0.9638           -1         1         -1.0146           -1         2         -0.9657           -1         3         -0.9638           -1         4         -0.9638	Original horizontal         Original vertical         The horizontal         The vertical displacement           displacement         displacement         displacement         in test           1         -1         1.0385         -1.0742           1         -2         1.0189         -1.9758           1         -3         0.9807         -2.9521           1         -4         0.9968         -3.9106           -1         -1         -0.9925         -0.9976           -1         -2         -0.9877         -1.9909           -1         -3         -0.9768         -3.0575           -1         -4         -0.9638         -4.0431           -1         1         -1.0146         0.9663           -1         2         -0.9657         2.0528           -1         3         -0.9638         3.0065           -1         4         -0.9642         3.9764	Original horizontal         Original vertical         The horizontal         The vertical displacement         The horizontal displacement         absolute error of horizontal           1         -1         1.0385         -1.0742         0.0385           1         -2         1.0189         -1.9758         0.0189           1         -3         0.9807         -2.9521         0.0193           1         -4         0.9968         -3.9106         0.0032           -1         -1         -0.9925         -0.9976         0.0075           -1         -2         -0.9877         -1.9909         0.0123           -1         -3         -0.9768         -3.0575         0.0232           -1         -4         -0.9638         -4.0431         0.0362           -1         1         -1.0146         0.9663         0.0146           -1         2         -0.9657         2.0528         0.0343           -1         3         -0.9638         3.0065         0.0362           -1         4         -0.9642         3.9764         0.0358

Table 1. Based on SVD phase correlation algorithm of MEMS m	nicrostructure of planar
movement amplitude measurements pixe	e <b>l</b>

The experimental data show that this method can achieve sub-pixel precision. Standard for measuring the absolute error to represent the uncertainty of the method, calculated horizontal and vertical direction of the uncertainty of 0.02160 and 0.04681, respectively, indicating that the absolute error values are very stable, but also prove the validity of this method.

## 6. Conclusion

Articles relying on MEMS dynamic testing system, the micro-structure of MEMS devices for stroboscopic imaging technique planar motion image sequence processing, the proposed sub-pixel motion estimation algorithm based on the SVD phase, experimental results demonstrate the feasibility of the algorithm and its traditional algorithm embodied in the advantages: 1. image differences due to uneven illumination, gray and other factors the test results will not have an impact, good anti-jamming. 2 direct the plane motion amplitude of MEMS micro-structure of the sub-pixel level without the need for difference and advantages.

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