# A New Approach of Detection Algorithm for Reducing Computation Complexity of MIMO Systems

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## Abstract

Multiple-Input Multiple-Output (MIMO) technique is a key technology to strengthen and achieve high-speed and high-throughput wireless communications. In recent years, it was observed that frequent detecting techniques could improve the performance (e.g., symbol error rate 'SER') of different modern digital communication systems. But these systems faced a problem of high complexity for the practical implementation. To solve the problem of high complexity, this work proposed Frequent Improve K-best Sphere Decoding (FIKSD) algorithm with stopping rule depending on the Manhattan metric. Manhattan metric is proposed to use with FIKSD in order to achieve the lowest complexity. FIKSD is a powerful tool to achieve a high performance close to the maximum likelihood (ML), with less complexity. The simulation results show a good reduction in computation complexity with a cost of slight performance degradation within 1dB; the proposed FIKSD requires 0% to 94% and 82% to 97% less complexity than Improved K-best Sphere Decoder (IKSD) and K-best Sphere Decoder (KSD) respectively. This makes the algorithm more suitable for implementation in wireless communication systems.

Keywords: MIMO, MLD, K-best, Sphere Detection, IKSD

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## 1. Introduction

Nowadays, all research works are interested for high-quality and more throughput of communication systems to ensure the demand for high-quality communication devices. The spatial multiplexed MIMO wireless communication systems allow devices to achieve near Shannon's channel capacity and they are promising for high data rates [1-5]. MIMO detector employs the Maximum Likelihood Detection (MLD) techniques to restore the transmit data; MLD is optimal in terms of SER performance, However its complexity exponential growth with the number of symbols detected especially when the number of employed antennas and/or modulation order is increased [6, 7]. Various MIMO detection algorithms have been proposed that can approach the statistically optimal performance of MLD [8, 9]. But the high calculative complexity of these algorithms has made them inconsistentfor widespread adoption in practical MIMO receiver designs when higher-order constellations, and a large number of antennas are used [10].

There are many detection techniques have been proposed in previous research works that achieved close to Maximum likelihood (ML) SER performance with low computational complexity [11, 12], and these techniques can be classified into two categories; The first is Depth First Tree Search (DFTS) algorithms [13] such as Sphere Decoder (SD) [14], and the second are Breadth First Tree Search (BFTS) algorithms [13] such as K-best detection [15]. In K-best algorithms, the computational complexity increases with the value of K where K is the number of paths retained by this algorithm. To achieve SER performance near- ML the K-best algorithm needs a high value of K, but this leads to more computational complexity. Improved K-best sphere decoder (IKSD) [16] has been latterly proposed to make the complexity of K-best lower, and it showed to be efficient in terms of computational complexity. The concept of complexity, defined as the number of floating point operations (additions, multiplications etc.) which are required to compute the estimated transmit vectors or the running time of the detection algorithm when implemented on some specific programs [17].

In the literature, there are several channels ordering schemes to detect symbols in different sequences [18], the SER performance of K-best SD algorithms is known to be sensitive

to the sequence in which symbols are detected [19]. In this work, a frequent detection manner is proposed in order to find the minimum Manhattan metric vector. Anyway, this manner will be useful when the redundant repetitions are reduced in frequent detection. Modification the work of [20] with the use of Manhattan metric instead of Euclidean metric [21] to cut down the redundant frequents when we can know whether the solution it was received at any iteration is near- ML or not. A stopping rule based on Manhattan metric is proposed in our work, this yields a near-ML performance with a lower computational complexity. In this paper, we are proposed to use Manhattan metric to calculate the weight of each candidate node and reduce the number calculation performed. The use of Manhattan metric will be led to eliminate the arithmetic multiplications at the receiver, with a cost of a slight performance degradation.

The remainder of this paper is organized as follows; Section 2 describes the proposed system and channel model. Section 3 gives a description about the proposed frequent detection algorithm has been elucidation by explaining the idea of FIKSD algorithm, stopping rule, and use of Manhattan metric instead of Euclidean metric. At section 4, the frequent detection algorithm with stopping rule was simulated and showed that the use of Manhattan metric is better than Euclidean metric in terms of reducing complexity. Finally, the conclusions are drawn in section 5.

#### 2. System and Channel Modeling

In this work, the system considering is L-QAM 4×4 MIMO system, with a flat Rayleighfaded channel. We assume that the transmitter and the receiver are equipped with M and Nantennas respectively ( $N \ge M$ ). The received discrete-time complex baseband signal vector  $Y = [y_1, y_2, ..., y_N]^T$  can be written as mathematical model as

$$Y = HX + W \tag{1}$$

where *H* is the channel matrix with  $h_{i,j}$  coefficient of (i, j) component of the  $N \times M$ , represent the *j*-th transmit antenna to the *i*-th receive antenna which is modeled as an independent and identically-distribution (i.i.d.) complex Gaussian variable with zero mean and unit variance. The transmitter sends symbols  $X = \frac{1}{\sqrt{M}} (x_1, \dots, x_M)^T$  chosen from a constellation  $A^L \in \mathbb{C}^M$ , which is defined by symbols in an L-QAM modulation, *W*: is  $(N \times 1)$  i.i.d. (AWGN) vector with zero mean and  $\sigma^2$  variance, the signal to noise ratio (SNR) is given by  $\frac{1}{\sigma^2}$ . The receiver is assumed to have perfect knowledge of the channel state information (CSI).

The performance of existing sequential detection techniques depends mainly on the sequence in which symbols are detected, so we consider a particular sequence like  $(x_{M-i+1}, \dots, x_M, x_1, \dots, x_{M-i})$  for  $i = 0, 1, \dots, (M-1)$ . By shifting the columns of channel matrix H, the received symbol vector can be written as

$$Y = H^i X^i + W \tag{2}$$

where

$$H^{i} = \begin{bmatrix} h_{M-i+1} & \cdots & h_{1,M} & h_{1,1} & \cdots & h_{1,M-i} \\ h_{2,M-i+1} & \cdots & h_{2,M} & h_{2,1} & \cdots & h_{2,M-i} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ h_{N,M-i+1} & \cdots & h_{N,M} & h_{N,1} & \cdots & h_{N,M-i} \end{bmatrix}$$
(3)

By decomposed the shifted channel matrix  $H^i$  using the standard QR decomposition, so it can be expressed as

$$H^i = Q^i R^i \tag{4}$$

where  $Q^i$  is an  $(N \times M)$  unitary matrix and  $R^i$  is an  $(M \times M)$  upper triangular matrix. By premultiplying both sides of (2) by  $Q^{i^H}$ , we can write (2) as

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(5)

$$Z^i = R^i X^i + W^i$$

where  $Z^i = \boldsymbol{Q}^{i^H} Y$  and  $W^i = \boldsymbol{Q}^{i^H} W$ .

The complex valued in (1) is often described by the equivalent real-valued representation, by decomposing the QAM-modulated signal model of the *N*-dimensional complex-valued into a 2*N*-dimensional real-valued, which can be written as

$$\begin{bmatrix} \mathfrak{R}[Y]\\\mathfrak{Z}[Y] \end{bmatrix} = \begin{bmatrix} \mathfrak{R}[H] & -\mathfrak{Z}[H]\\\mathfrak{Z}[H] & \mathfrak{R}[H] \end{bmatrix} \begin{bmatrix} \mathfrak{R}[X]\\\mathfrak{Z}[X] \end{bmatrix} + \begin{bmatrix} \mathfrak{R}[W]\\\mathfrak{Z}[W] \end{bmatrix}$$
(6)

where  $\Re[\cdot]$  and  $\Im[\cdot]$  denote the real and imaginary parts of  $[\cdot]$  respectively [22, 23].

For a specified *i* such as *i*=0, the brute-force MLD can be converted into a full tree structure search by using Manhattan metric as

$$X_{MLD(Manhattan metric)} = \arg\min_{X \in A^{L}} |Z^{i} - R^{i}X^{i}| = \arg\min_{X \in A^{L}} \sum_{p=1}^{M} \left| Z_{p}^{i} - \sum_{q=p}^{M} R_{p,q}^{i}X_{q}^{i} \right|$$

$$= \arg\min_{X \in A^{L}} \sum_{p=1}^{M} T_{p}$$
(7)

where  $T_p = |Z_p^i - \sum_{q=p}^M R_{p,q}^i X_q^i|$  is the branch metric for the *p*-th (*p*=1, 2,..., *M*) data layer,  $R_{p,q}^i$  is the (*p*, *q*) component of  $R^i$ , and  $|\cdot|$  denotes the absolute value. The vector  $X_{MLD(Manhattan metric)}$  depends on the constellation size of modulated signal and the number of transmit antennas. The partial Manhattan distance (PMD), i.e., the accumulated branch metric is  $\sum T_p$ . If the all path solution candidates reach the other level, the path with the minimum PMD will be the eventual detection result.

In MIMO systems, the MLD scheme offers an excellent performance, with unacceptable complexity [24]. So we use a sequential detection algorithm like K-best [15] or IKSD [16] frequently from i=0 to i=M-1, and the vector corresponding to the minimum metric over all the repetitions is considered to be the detected vector. Clearly such repetitions can help to improve the SER performance. Nonetheless, the redundant repetitions may lead to a huge computation complexity. Thus, in this paper we suggest to use a stopping rule to get rid of the redundant repetitions by calculating a positive threshold  $\beta$ , and use Manhattan metric (norm-1) instead of Euclidean metric (norm-2), to reduce the number of mathematical calculations performed.

## 3. Frequent Detection Algorithm

The frequent detection scheme has been proposed in this work to achieve a good SER performance, this can be achieved by a frequent detection process to seek to find the minimum Manhattan metric vector. With used of stopping rule to reduce the redundant iterations in frequent detection.

# 3.1. Frequent IKSD (FIKSD)

In this section, we will describe our proposed detection algorithm, which it is used the IKSD frequently, so we called it Frequent IKSD (FIKSD). The main principle of the FIKSD algorithm comes from the tree search process and the traditional KSD algorithm which sorts all the child nodes depending on their partial Manhattan distance (PMD) and selects the K nodes. The IKSD algorithm retains the additional nodes whose costs are close to the cost of *K*-th node. In FIKSD algorithm, the goal of using the detection process frequent to the symbols is to get more accurate results and using the Manhattan metric to reduce the complexity. The complete algorithm is described in Algorithm-1. It can be seen that the repetition stop until  $|Z^i - R^i X^i|$  is greater than the threshold  $\beta$ . So the SER performance and computational complexity of the proposed FIKSD algorithm depend on the choice of  $\beta$  which in turn depends on Manhattan metric.

Algorithm-1: The FIKSD Algorithm

Input: Y, H, K, A, ∆, d, T=∞ Output:X Initialization *i*=0, *T*=0 (the branch metric) and  $X_0$  the root(level *p*=*M*); while i < N do  $H^{i} = circ\_shift(H,i);$  $[Q^i, R^i] = qr\_decomp(H^i);$  $Z^i = Q^{i^H} Y;$ T = 0;while  $k \ge 1$  do for k=1to length (T) do Extend the *k*th node, generate all its successors for all  $PMD_k = \sum T_p$ ; end sort all the components of  $PMD_k$  in an ascending order; if # elements < K Keep all the candidates with  $PMD_k \leq d$  to obtain  $\tau$ ; else Only keep the elements whose cost indexes satisfy  $PMD = PMD_k + \Delta$  in T end Replace the PMD<sub>best</sub> to be the adjusted PMD end Return  $X^i \leftarrow$  the 1<sup>st</sup> element on the tree if  $(|Z^i - R^i X^i| < \beta)$  then  $X = circ\_shift(X^i, M - i)$  and i = Melse if  $T = |Z^i - R^i X^i|$  $X = circ\_shift(X^{i}, M - i)$ end i = i + 1;end

## 3.2. Stopping Rule

This section show that our suggestion way to choose  $\beta$  properly so that redundant repetitions can be reduced without a loss in SER performance. Our proposed way is depending on a known mathematical optimization technique called Hill Climbing, which can produce a better result than other algorithms when the amount of time available to perform a search is limited, as simulated to the real-time systems. Now we can consider  $X_1$  as any probably transmit vector with a given channel matrix H, and let  $Y_1$  is an observation vector from which MLD is  $X_1$ . The cost analogical to this MLD is given by using Manhattan metric as  $|Y_1 - HX_1|$ . Let  $X_2$  be the neighbor of transmitting vector  $X_1$  and there is only one element (*i*-th element) different between  $X_1$  and  $X_2$ . When the  $Y_1$  goes farther from  $HX_1$ , the cost will be increased and the farthest point for  $X_1$  which continues to be the MLD will be the mid-point of  $HX_1$  and  $HX_2$  with cost equal to  $|\frac{1}{2}H(X_1 - X_2)|$ . To cut down the redundant repetitions and get a reasonable SER performance, we choose the stopping criteria as the cost  $|\frac{1}{2}H(X_1 - X_2)|$  not greater than it. Since all elements of vector  $X_1$  and  $X_2$  are a same except the *i*-th element, so the cost will be  $\frac{d_{min}}{2\sqrt{M}}|H_i|$  where  $H_i$  is the *i*-th column of H. To minimize the cost we propose the stopping rule as

$$\beta = \frac{d_{min}}{2\sqrt{M}} \min_{i} |H_i| \tag{8}$$

## 3.3. Comparison between Manhattan and Euclidean Metrics

In this section, we explain the difference of using Manhattan metric and Euclidean metric in our proposed FIKSD algorithm. The use of Manhattan metric or Euclidean metric to

calculate the weights of each candidate node [25]. In Euclidean metric, the brute-force MLD can be converted into a full tree structure search by using Euclidean metric such as.

$$X_{MLD(\text{Euclidean metric})} = \arg\min_{X \in A^L} ||Z^i - R^i X^i|| = \arg\min_{X \in A^L} \sum_{p=1}^M \left| Z_p^i - \sum_{q=p}^M R_{p,q}^i X_q^i \right|^2$$
(9)

From (9) the MIMO-MLD searches a candidate  $X^i$  that minimizes the squared Euclidean metric between  $Z^i$  and  $R^i X^i$  that is referred to as the Euclidean metric  $X_{MLD(Euclidean metric)}$ .

The hardware implementation is infeasible due to a logic resource limitation of the target device because there are  $4NL^{M}$ = 1,048,576 real multiplications (for 16-QAM) are required to compute all the Euclidean metric. According to (7) this type of detection algorithm is practically impossible to implement in MIMO systems that utilize high order modulation such as (16-QAM, 64-QAM). So we adopted a practical metric like Manhattan metric to avoid the use of arithmetic multiplications, the Manhattan metric is computed by adding absolute values of  $Z^{i}$  and  $R^{i}X^{i}$ , as in (7).

## 4. Simulation Results

Through our work simulation, we compare the SER performance and the computational complexity of FIKSD proposed with the traditional KSD [15] and IKSD [16]. This simulation considered 4×4 MIMO system with 16-QAM modulation and the proposed stopping rule is based mainly on the Manhattan metric as well on mathematical optimization techniques such as Hill Climbing. The proposed system model assumed the channel with flat Rayleigh faded during each frame and using the equivalent real system model. To get the close ML performance, the simulation parameters will be taken as K=16 for KSD algorithm, K=2,  $\Delta$ =0.25 for IKSD and for FIKSD (K=1,  $\Delta$ =0.2).

Figure 1 shows that the SER performance comparison among our proposed algorithm FIKSD with IKSD and KSD, so we can note that the performance of FIKSD (K=1,  $\Delta$ =0.2) is close to the ML curve with slight performance degradation within 1dB; while the IKSD ( $\Delta$ =0.25) need to set k=2 and traditional KSD need to set K=16 to make the similar SER performance.



Figure 1. The SER performance of detected symbols for 4×4 MIMO system with 16- QAM modulation

Figure 2 shows the comparison of the visited nodes FIKSD, IKSD and KSD. We can see that the complexity of the proposed FIKSD is lower than that of IKSD and KSD when achieving the close ML performance. We can observe that the difference between the complexities of three algorithms (KSD, IKSD, and FIKSD) is variable. For example; the comparison between proposed FIKSD and IKSD at minimum SNR=0.6 dB and maximum SNR=25 dB, IKSD (K=2) searches about (66 and 185) nodes, and the FIKSD (K=1) needs (66 and 10) nodes visited respectively. So the proposed FIKSD needs 0% to 97% less complexities than IKSD. The comparison between proposed FIKSD and KSD at minimum SNR=0 dB and maximum SNR=25 dB, the traditional KSD (K=16) searches about (400 and 400) nodes, and the FIKSD (K=1) needs (70 and 10) nodes visited respectively. So the proposed FIKSD needs 82% to 97% less complexities than KSD. Additionally Figure.2 can be shows that the number of visited nodes (10 nodes) represent the lower bound in FIKSD at high SNR.



Figure 2. The average number of nodes visited of detected symbols for 4×4 MIMO system with 16- QAM modulation

Figure 3 shows that the cost of using Manhattan metric instead of Euclidean metric as a slight SER performance degradation within 1dB, but this cost can compensate by reducing the complexity.



Figure 3. The SER performance of detected symbols for 4×4 MIMO system with 16- QAM modulation by FIKSD (K=1, Δ=0.2) using Euclidean metric and Manhattan metric

Figure 4 shows the difference between the complexities of FIKSD algorithm in case of use Euclidean metric and Manhattan metric; we can see that the difference is not constant, and it is minimum at SNR=25 dB; the average visited nodes when use Euclidean metric and Manhattan metric is 108 and 10 respectively. And it is maximum at SNR=12.5 dB; the average visited nodes when use Euclidean metric and Manhattan metric is 934 and 20 respectively. This means for the proposed FIKSD with Manhattan metric requires 90% to 97% less computations than FIKSD with Euclidean metric.



Figure 4. The average number of nodes visited of detected symbols for 4×4 MIMO system with 16- QAM modulation by FIKSD (K=1,  $\Delta$ =0.2) using Euclidean metric and Manhattan metric.

# 5. Conclusion

In this work, a proposed detection algorithm is considered for describe the SER performance in order to achieve a performance close to MLD with low computation complexity for MIMO systems. Our proposed algorithm (FIKSD) considered stopping rule based on a comparison of the distance of the detected vector with a threshold  $\beta$ . The calculated distance of two neighboring transmits vectors depend on Manhattan metric. The aim of using the detection process frequent to the symbols is to get more accurate results. The Manhattan metric reduced the number of mathematical operations by the elimination of multiplication operations in the detection process. In FIKSD, the use of Manhattan metric instead of Euclidean metric reduces the number of calculation performed at a cost of a trivial performance degradation within 1dB. The simulation results show that the proposed FIKSD with stopping rule and Manhattan metric requires less computational complexity than the IKSD and KSD algorithms. Therefore, the FIKSD can be considered a valid alternative for implementation in the future of wireless communication systems.

# References

- [1] Hochwald, Bertrand M., Stephan Ten Brink. "Achieving near-capacity on a multiple-antenna channel". *IEEE Transactions on Communications*. 2003; 51(3): 389-399.
- [2] Wu, Chao Qun, Dan Zhao, and Jing Peng Gao. "Signal Detection Based on Particle Swarm Optimization for MIMO-OFDM System". *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2014; 12(5): 3761-3768.
- [3] Zhen, Han Yao, and Hairong Xiao. "RBFNN Variable Structure Controller for MIMO System and Application to Ship Rudder/Fin Joint Control". *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2014; 12(12): 8166-8174.
- [4] Nuwanpriya, Asanka, Siu-Wai Ho, Chung Shue Chen. "Indoor MIMO Visible Light Communications: Novel Angle Diversity Receivers for Mobile Users". 2015.
- [5] Nuwanpriya, Asanka, Siu-Wai Ho, Chung Shue Chen. "Angle diversity receiver for indoor MIMO visible light communications". *Globecom Workshops (GC Wkshps), 2014.* IEEE. 2014.
- [6] Ueng, Fang-Biau, Yu-Kuan Chang, Ying-Mu Yang. "Low-Complexity Turbo Receiver for MIMO SC-FDMA Communication Systems". Proceedings of the World Congress on Engineering. 2015; 1.
- [7] Choi, Junil, Jianhua Mo, Robert W. Heath Jr. "Near maximum-likelihood detector and channel estimator for uplink multiuser massive MIMO systems with one-bit ADCs". arXiv preprint arXiv: 1507.04452. 2015.
- [8] Viterbi, Andrew J. "Error bounds for convolutional codes and an asymptotically optimum decoding algorithm". *IEEE Transactions on Information Theory*. 1967; 13(2): 260-269.
- [9] Chu, Xuezheng, John McAllister. "Software-defined sphere decoding for FPGA-based MIMO detection". *IEEE Transactions on Signal Processing*. 2012; 60(11): 6017-6026.
- [10] Viterbo, Emanuele, Joseph Boutros. "A universal lattice code decoder for fading channels". *IEEE Transactions on Information Theory*. 1999; 45(5): 1639-1642.
- [11] Barbero, Luis G, John S Thompson. "Fixing the complexity of the sphere decoder for MIMO detection". *IEEE Transactions on Wireless Communications*. 2008; 7(6): 2131-2142.
- [12] Rachid, Mansour, and Babak Daneshrad. "A low-complexity iterative mimo sphere decoding algorithm". *Signal Processing Conference, 2009 17th European.* IEEE. 2009.
- [13] Jia, Yugang, et al. "Depth-first and breadth-first search based multilevel SGA algorithms for near optimal symbol detection in MIMO systems". *IEEE Transactions on Wireless Communications*. 2008; 7(3): 1052-1061.
- [14] Hassibi, Babak, Haris Vikalo. "On the sphere-decoding algorithm I. Expected complexity". *IEEE Transactions on Signal Processing*. 2005; 53(8): 2806-2818.
- [15] Guo, Zhan, Peter Nilsson. "Algorithm and implementation of the K-best sphere decoding for MIMO detection". *IEEE Journal on Selected Areas in Communications*. 2006; 24(3): 491-503.
- [16] Han, Shuangshuang, Tao Cui, Chintha Tellambura. "Improved K-best sphere detection for uncoded and coded MIMO systems". *Wireless Communications Letters, IEEE*. 2012; 1(5): 472-475.
- [17] Elshokry, Auda M. Complexity and performance evaluation of detection schemes for spatial multiplexing mimo systems. Diss. Faculty of Engineering, Islamic University Gaza. 2010
- [18] Thian, Boon Sim, Andrea Goldsmith. "A reduced-complexity MIMO receiver via channel ordering". Global Telecommunications Conference, 2009. GLOBECOM 2009. IEEE. IEEE. 2009.
- [19] Damen, Mohamed Oussama, Hesham El Gamal, Giuseppe Caire. "On maximum-likelihood detection and the search for the closest lattice point". *IEEE Transactions on Information Theory*. 2003; 49(10): 2389-2402.

- [20] Ma, Zheng, et al. "Stopping criterion for complexity reduction of sphere decoding". *Communications Letters, IEEE.* 2009; 13(6): 402-404.
- [21] Koike, Toshiaki, et al. "FPGA prototyping of MIMO detector for over-1Gbps wireless transmissions". FPGA prototiping of MIMO ditector for over-1Gbps wireless transmissions GA 2005. 2005.
- [22] Jaldén, Joakim, Björn Otterste. "On the complexity of sphere decoding in digital communications". *IEEE Transactions on Signal Processing*. 2005; 53(4): 1474-1484.
- [23] Hassibi, Babak, and Baris Vikalo. "On the expected complexity of integer least-squares problems". 2002 IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP). IEEE, 2002; 2.
- [24] Kim, Jin-Sung, Sung-Hyun Moon, and Inkyu Lee. "A new reduced complexity ML detection scheme for MIMO systems". *IEEE Transactions on Communications*. 2010; 58(4): 1302-1310.
- [25] Castillo-León, Juan Francisco, Ulises Pineda-Rico, and Enrique Stevens-Navarro. "MIMO Detector for Uncoded Symbols based on Complex Sphere Decoder and Manhattan Metric". Procedia Technology. 2013; 7: 54-60.