

Projective synchronization for a class of 6-D hyperchaotic Lorenz system

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Article Info

Article history:

Received Jan 2, 2019

Revised May 3, 2019

Accepted May 25, 2019

Keywords:

6-D hyperchaotic system
Lyapunov's second method
Nonlinear control strategy
Projective synchronization

ABSTRACT

This paper is concerned with the projective synchronization problem for a class of 6-D nonlinear dynamical system which is called hyperchaotic Lorenz system when the parameters of this system are unknown. Based on scaling factor α_i which belong to above strategy, four controller are proposed to achieve projective synchronization between two identical systems via using Lyapunov's direct method and nonlinear control strategy. These scaling factor α_i taken the values 1, -1 ± 1 and 2 for each control respectively. A numerical simulations are used to demonstrate the efficiency of the proposed controller.

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1. INTRODUCTION

Chaos synchronization is an important topic of nonlinear dynamical systems and have great significance in the application of chaos such as physics, secure communication, control theory, etc [1-3]. There are many kinds of synchronization phenomena, for example, complete/full synchronization (CS), anti-synchronization (AS), generalized synchronization (GS), lag synchronization, projective synchronization (PS), generalized projective synchronization (GPS) [4, 5]. The projective synchronization and generalized projective synchronization are based on nonzero constant α_i (scaling factor) and constant scaling matrix α_i respectively, therefore, projective synchronization includes three strategies: full synchronization, anti-synchronization and hybrid synchronization are special cases where the scaling factor $\alpha_i = 1$, $\alpha_i = -1$ and $\alpha_i = \pm 1$ respectively [6-9].

Chaotic system has become an important subject in study of behaviors of dynamical systems [10-13]. But this system has contains one positive Lyapunov exponent only while hyperchaotic system has more than one positive Lyapunov exponent. In secure communication, messages which sent by such simple chaotic systems are not always safe [14]. So, in order to overcome this problem it should be use higher-dimensional hyperchaotic systems, the hyperchaotic may be more useful in some fields such as secure communication [15, 16]. So, it's needed to propose high dimensional nonlinear dynamical systems, these system are characterized as a chaotic system with more than one positive Lyapunov exponent, and have more complex and richer dynamical behaviors than chaotic system. Historical, Rössler system is the first hyperchaotic systems which discover in 1979, Since then, many hyperchaotic systems have been discover [17-22] such as hyperchaotic Liu system, hyperchaotic Chen system, Modified hyperchaotic Pan system, as well as to propose a 5-D hyperchaotic system such as A novel 5-D hyperchaotic Lorenz system (2014), a novel hyperjerk system with two nonlinearities. Currently, a novel 6-D hyperchaotic Lorenz is discover by Yang

which contains four positive Lyapunov Exponents [23, 24].

There are many works deals with synchronization phenomena for various different dimension nonlinear dynamical systems (3-D, 4-D and 5-D) and a few researchers consider system with six dimension. This reason, motivated us to achieve projective synchronization between two identical 6-D hyperchaotic via nonlinear control strategy.

2. STSTEM DESCRIPTION

The Lorenz system is the first 3-D chaotic system which discover in 1963. Later, many system construct from this original system into a 4-D and 5-D hyperchaotic systems by introducing a linear feedback controller. In 2015, Yang constructed a 6-D hyperchaotic system consists of fourteen terms including only three quadratic terms and have four positive Lyapunov Exponents $LE_1 = 1.0034, LE_2 = 0.57515, LE_3 = 0.32785, LE_4 = 0.020937$, and two negative Lyapunov Exponents $LE_5 = -0.12087, LE_6 = -12.4713$. The 6-D system which is described by the following mathematical form [24, 25]:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_4 \\ \dot{x}_2 = cx_1 - x_2 - x_1x_3 + x_5 \\ \dot{x}_3 = -bx_3 + x_1x_2 \\ \dot{x}_4 = dx_4 - x_1x_3 \\ \dot{x}_5 = -kx_2 \\ \dot{x}_6 = rx_2 + hx_6 \end{cases} \tag{1}$$

where a, b, c, d, k, r and h are constant. The Lorenz system has a hyperchaotic attractor when $a = 10, b = \frac{8}{3}, c = 28, d = 2, k = 8.4, r = 1$ and $h = 1$. Figure 1(a) and 1(b) show the 3-D attractor of the system (1).

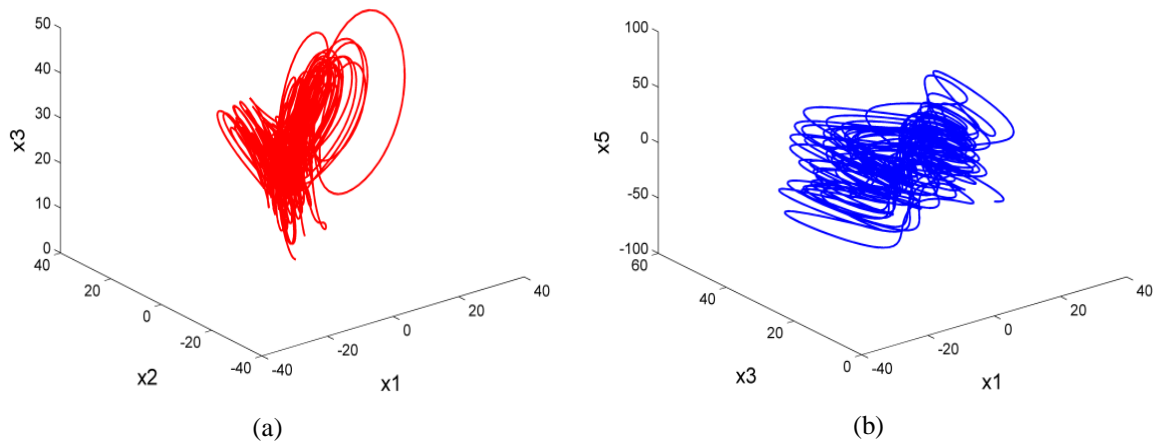


Figure 1. Behavior of system 1 at (a) 3-D attractor in the (x_1, x_2, x_3) space; (b) 3-D attractor in the (x_1, x_3, x_5) space

3. GENERAL PROJECTIVE SYNCHRONIZATION SCHEME

To begin with, the definition of projective synchronization used in this paper is given as For two nonlinear dynamical systems:

$$\dot{X}_i = F_1(X_i) \tag{2}$$

$$\dot{Y}_i = F_2(Y_i) + U(X_i, Y_i) \tag{3}$$

where $X_i = (x_1, x_2, \dots, x_n)^T, Y_i = (y_1, y_2, \dots, y_n)^T \in R^n$ are state variables of the system (2) and system (3), respectively, $F_1, F_2: R^n \rightarrow R^n, i = 1, 2, \dots, n, U(X_i, Y_i)$ is the nonlinear control vector, suppose that system (2) is the drive system, system (3) is the response system. The response system and drive system are said to be chaos synchronized or (projective) synchronized if there exists a nonzero constant α_i which is called scaling factor for projective synchronization such that

$$\forall X_i, Y_i \in R^n, \lim_{t \rightarrow \infty} \|Y_i - \alpha_i X_i\| = 0,$$

4. APPLICATIONS OF THE PROJECTIVE SYNCHRONIZATION SCHEME ON 6-D HYPERCHAOTIC SYSTEM

In this section, we study an engineering application of the 6-D Lorenz hyperchaotic system via nonlinear chaos synchronization of two identical of Lorenz hyperchaotic system with unknown parameters. According to the above definition, assume that the system (1) be the drive system and response systems are given as the following form:

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + y_4 + u_1 \\ \dot{y}_2 = cy_1 - y_2 - y_1y_3 + y_5 + u_2 \\ \dot{y}_3 = -by_3 + y_1y_2 + u_3 \\ \dot{y}_4 = dy_4 - y_1y_3 + u_4 \\ \dot{y}_5 = -ky_2 + u_5 \\ \dot{y}_6 = ry_2 + hy_6 + u_6 \end{cases} \quad (4)$$

where $U = [u_1, u_2, u_3, u_4, u_5, u_6]^T$ is the nonlinear controller to be designed.

4.1. Design Controllers at Scaling Factor $\forall \alpha_i = 1$

The projective synchronization error dynamics between the 6-D hyperchaotic system (1) and system (4) when $\forall \alpha_i = 1$ is defined as

$$e_i = y_i - x_i, \quad i = 1, 2, \dots, 6 \quad (5)$$

The error dynamics is calculated as the following:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + e_4 + u_1 \\ \dot{e}_2 = ce_1 - e_2 + e_5 - e_1e_3 - x_3e_1 - x_1e_3 + u_2 \\ \dot{e}_3 = -be_3 + e_1e_2 + x_1e_2 + x_2e_1 + u_3 \\ \dot{e}_4 = de_4 - e_1e_3 - x_3e_1 - x_1e_3 + u_4 \\ \dot{e}_5 = -ke_2 + u_5 \\ \dot{e}_6 = re_2 + he_6 + u_6 \end{cases} \quad (6)$$

Theorem 1. For the error dynamics system (6) with nonlinear control $U = [u_1, u_2, u_3, u_4, u_5, u_6]^T$ such that

$$\begin{cases} u_1 = -(a+c)e_2 \\ u_2 = ke_5 - re_6 + x_3e_1 \\ u_3 = -x_2e_1 \\ u_4 = -e_1 - 2de_4 + e_1e_3 + x_1e_3 + x_3e_1 \\ u_5 = -e_2 - e_5 \\ u_6 = -2he_6 \end{cases} \quad (7)$$

Then the system (6) can be controlled i.e., system (4) followed to system (1).

Proof. Error dynamics system (6) with controller (7) become

$$\begin{cases} \dot{e}_1 = -ae_1 - ce_2 + e_4 \\ \dot{e}_2 = ce_1 - e_2 + (1+k)e_5 - re_6 - e_1e_3 - x_1e_3 \\ \dot{e}_3 = -be_3 + e_1e_2 + x_1e_2 \\ \dot{e}_4 = -e_1 - de_4 \\ \dot{e}_5 = -(1+k)e_2 - e_5 \\ \dot{e}_6 = re_2 - he_6 \end{cases} \quad (8)$$

The Lyapunov function is defined as

$$V(e) = e^T P e, \quad P = \text{diag}[1/2, 1/2, 1/2, 1/2, 1/2, 1/2] \quad (9)$$

Differentiating $V(e)$ along the error dynamics (6), we obtain

$$\dot{V}(e) = e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 + e_4 \dot{e}_4 + e_5 \dot{e}_5 + e_6 \dot{e}_6 \tag{10}$$

$$\dot{V}(e) = -ae_1^2 - e_2^2 - be_3^2 - de_4^2 - e_5^2 - he_6^2 = -e^T Q e, \quad Q = \text{diag}(a, 1, b, d, 1, h) \tag{11}$$

So $Q > 0$. Therefore, $\dot{V}(e) < 0$. According to the Lyapunov's direct method, the nonlinear controller is achieved.

4.2. Design Controllers at Scaling Factor $\forall \alpha_i = -1$

The projective synchronization error dynamics between the 6-D hyperchaotic system (1) and system (4) when $\forall \alpha_i = -1$ is defined as

$$e_i = y_i + x_i, \quad i = 1, 2, \dots, 6 \tag{12}$$

The error dynamics is calculated as the following:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + e_4 + u_1 \\ \dot{e}_2 = ce_1 - e_2 + e_5 - y_1 e_3 - x_3 e_1 + 2y_1 x_3 + u_2 \\ \dot{e}_3 = -be_3 + e_1 e_2 - x_2 e_1 - x_1 e_2 + 2x_1 x_2 + u_3 \\ \dot{e}_4 = de_4 - y_1 e_3 - x_3 e_1 + 2y_1 x_3 + u_4 \\ \dot{e}_5 = -ke_2 + u_5 \\ \dot{e}_6 = re_2 + he_6 + u_6 \end{cases} \tag{13}$$

Theorem 2. The above error dynamics system with controller U such that

$$\begin{cases} u_1 = -ce_1 + x_3 e_2 + x_2 e_3 + x_3 e_4 \\ u_2 = -ae_1 - re_6 - 2y_1 x_3 + x_1 e_3 \\ u_3 = y_1 e_2 - e_1 e_2 - 2x_1 x_2 + y_1 e_4 \\ u_4 = -e_1 - 2de_4 - 2y_1 x_3 \\ u_5 = -e_2(1 - k) - e_5 \\ u_6 = -2he_6 \end{cases} \tag{14}$$

Then the system (13) can be controlled.

Proof. Error system (13) with controller (14) become

$$\begin{cases} \dot{e}_1 = -(a+c)e_1 + ae_2 + e_4 + x_3 e_2 + x_2 e_3 + x_3 e_4 \\ \dot{e}_2 = ce_1 - e_2 + e_5 - re_6 - y_1 e_3 - x_3 e_1 + x_1 e_3 \\ \dot{e}_3 = -be_3 - x_2 e_1 - x_1 e_2 + y_1 e_2 + y_1 e_4 \\ \dot{e}_4 = -e_1 - de_4 - y_1 e_3 - x_3 e_1 \\ \dot{e}_5 = -e_2 - e_5 \\ \dot{e}_6 = re_2 - he_6 \end{cases} \tag{15}$$

Differentiating $V(e)$ along the error dynamics (13), we obtain

$$\dot{V}(e) = -ae_1^2 - e_2^2 - be_3^2 - de_4^2 - e_5^2 - he_6^2 = -e^T Q e$$

so, $\dot{V}(e) < 0$, based on the Lyapunov's direct method, the controller is performed.

4.3. Design Controllers at Scaling Factor $\alpha_i = \pm 1$

The projective synchronization error dynamics between the 6-D hyperchaotic system (1) and system (4) when $\alpha_i = \pm 1$ is defined as

$$e_i = y_i \pm x_i, \quad i = 1, 2, \dots, 6 \tag{16}$$

The error dynamics is calculated as the following:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + e_4 - 2ax_2 - 2x_4 + u_1 \\ \dot{e}_2 = ce_1 - e_2 + e_5 - y_1e_3 + x_3e_1 - 2y_1x_3 + 2cx_1 + 2x_5 + u_2 \\ \dot{e}_3 = -be_3 + e_1e_2 - x_2e_1 + x_1e_2 - 2x_1x_2 + u_3 \\ \dot{e}_4 = de_4 - y_1e_3 + x_3e_1 - 2y_1x_3 + u_4 \\ \dot{e}_5 = -ke_2 + 2kx_2 + u_5 \\ \dot{e}_6 = re_2 + he_6 + u_6 \end{cases} \quad (17)$$

Theorem 3. For the error dynamics system (17) with nonlinear control $U = [u_i]^T$ such that

$$\begin{cases} u_1 = -ce_2 + 2ax_2 + 2x_4 - x_3e_2 + x_2e_3 - x_3e_4 \\ u_2 = -ae_1 + ke_5 - re_6 + 2y_1x_3 - 2cx_1 - 2x_5 - x_1e_3 \\ u_3 = y_1e_2 - e_1e_2 + 2x_1x_2 + y_1e_4 \\ u_4 = -e_1 - 2de_4 + 2y_1x_3 \\ u_5 = -e_2 - e_5 - 2kx_2 \\ u_6 = -2he_6 \end{cases} \quad (18)$$

Then the system (17) can be controlled i.e., system (18) followed to system (1).

Proof. The error dynamics system (19) with controller (20) become

$$\begin{cases} \dot{e}_1 = -ae_1 + (a - c)e_2 + e_4 - x_3e_2 + x_2e_3 - x_3e_4 \\ \dot{e}_2 = (c - a)e_1 - e_2 + (1 + k)e_5 - re_6 - y_1e_3 + x_3e_1 - x_1e_3 \\ \dot{e}_3 = -be_3 - x_2e_1 + x_1e_2 + y_1(e_2 + e_4) \\ \dot{e}_4 = -e_1 - de_4 - y_1e_3 + x_3e_1 \\ \dot{e}_5 = -(1 + k)e_2 - e_5 \\ \dot{e}_6 = re_2 - he_6 \end{cases} \quad (19)$$

Differentiating $V(e)$ along the error dynamics (17), we obtain

$$\dot{V}(e) = -ae_1^2 - e_2^2 - be_3^2 - de_4^2 - e_5^2 - he_6^2$$

Therefore, $\dot{V}(e) < 0$, the controller is performed.

4.4. Design Controllers at Scaling Factor $\forall \alpha_i = 2$

The projective synchronization error dynamics between the 6-D hyperchaotic system (1) and system (4) when $\forall \alpha_i = 2$ is defined as

$$e_i = y_i - 2x_i \quad , \quad i = 1, 2, \dots, 6 \quad (20)$$

The error dynamics is calculated as the following:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + e_4 + u_1 \\ \dot{e}_2 = ce_1 - e_2 + e_5 - e_1e_3 - 2x_3e_1 - 2x_1e_3 - 2x_1x_3 + u_2 \\ \dot{e}_3 = -be_3 + e_1e_2 + 2x_2e_1 + 2x_1e_2 + 2x_1x_2 + u_3 \\ \dot{e}_4 = de_4 - e_1e_3 - 2x_3e_1 - 2x_1e_3 - 2x_1x_3 + u_4 \\ \dot{e}_5 = -ke_2 + u_5 \\ \dot{e}_6 = re_2 + he_6 + u_6 \end{cases} \quad (21)$$

Theorem 4. For the error dynamics system (21) with nonlinear control $U = [u_1, u_2, u_3, u_4, u_5, u_6]^T$ such that

$$\begin{cases} u_1 = -ce_2 + 2x_3e_2 - 2x_2e_3 + 2x_3e_4 \\ u_2 = -ae_1 + ke_5 - re_6 + 2x_1x_3 - 2x_1e_3 \\ u_3 = 2x_1e_2 - 2x_1x_2 + e_1e_4 + 2x_1e_4 \\ u_4 = -e_1 - 2de_4 + 2x_1x_3 \\ u_5 = -e_2 - e_5 \\ u_6 = -2he_6 \end{cases} \quad (22)$$

Then the system (21) can be controlled.

Proof. The error dynamics system (21) with controller (22) become

$$\begin{cases} \dot{e}_1 = -ae_1 + (a - c)e_2 + e_4 + 2x_3e_2 - 2x_2e_3 + 2x_3e_4 \\ \dot{e}_2 = (c - a)e_1 - e_2 - e_1e_3 + (1 + k)e_5 - re_6 - 2x_3e_1 - 4x_3e_4 \\ \dot{e}_3 = -be_3 + e_1e_2 + 2x_2e_1 + 4x_1e_2 + e_1e_4 + 2x_1e_4 \\ \dot{e}_4 = -e_1 - de_4 - e_1e_3 - 2x_3e_1 - -2x_1e_3 \\ \dot{e}_5 = -(1 + k)e_2 - e_5 \\ \dot{e}_6 = re_2 - he_6 \end{cases} \tag{23}$$

Differentiating $V(e)$ along the error dynamics (21), we obtain of the Lyapunov function $V(e)$ as

$$\dot{V}(e) = -ae_1^2 - e_2^2 - be_3^2 - de_4^2 - e_5^2 - he_6^2$$

Therefore, $\dot{V}(e)$ is negative definite. The nonlinear controller is performed.

4.5. Numerical Simulation

For simulation, the MATLAB is used to solve the differential equation of controlled error dynamical system (6), system (15) and system (19) based on fourth-order Runge-Kutta scheme with time step 0.001 and the and the initial values of the drive system and the response system are following $(2, 1, 8, 6, 12, 4)$ and $(-18, -9, -1, -5, -20, 15)$ respectively. We choose the parameters $a = 10, b = \frac{8}{3}, c = 28, d = 2, k = 8.4, r = 1$ and $h = 1$.

- **For scaling factor $\alpha_i = 1$.** Figure 2 show the complete/full synchronization of the hyperchaotic Lorenz system (1) and system (4) with control (7).
- **For scaling factor $\alpha_i = -1$.** Figure 3 show the anti-synchronization of the hyperchaotic Lorenz system (1) and system (4) with control (14).
- **For scaling factor $\alpha_i = \pm 1$.** Figure 4 show the hybrid synchronization of the hyperchaotic Lorenz system (1) and system (4) with control (18).
- **For scaling factor $\alpha_i = 2$.** Figure 5 show the projective synchronization of the hyperchaotic Lorenz system (1) and system (4) with control (22).

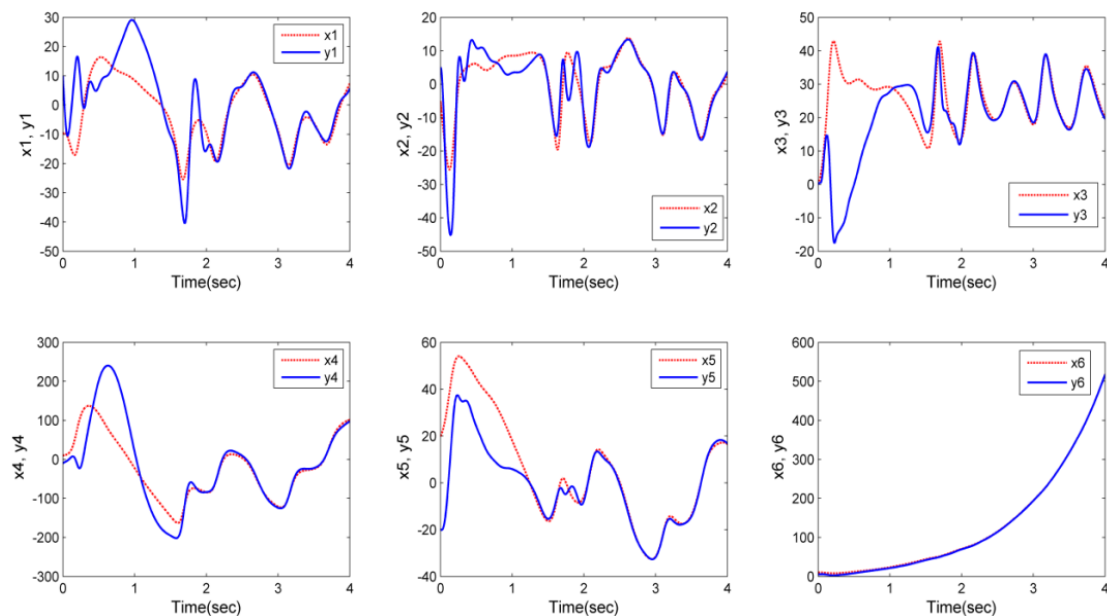


Figure 2. The state variables with control (7) at scaling factors $\alpha_i = 1$

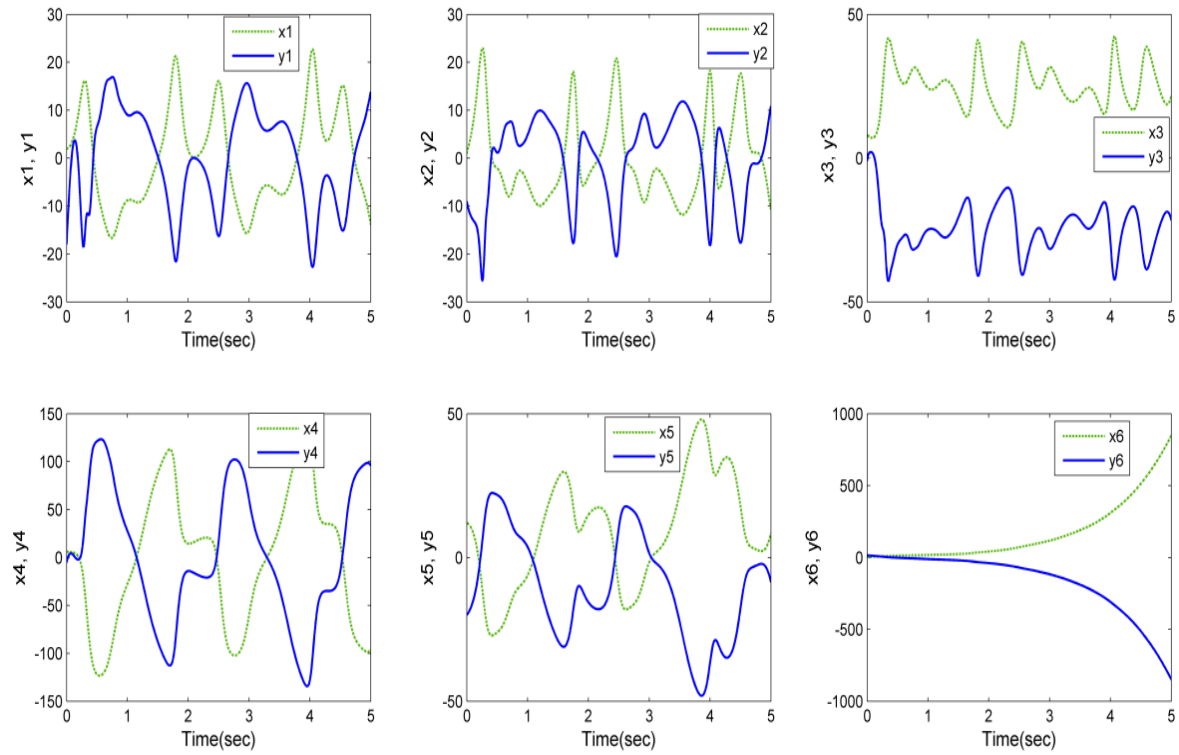


Figure 3. The state variables with control (14) at scaling factors $\alpha_i = -1$

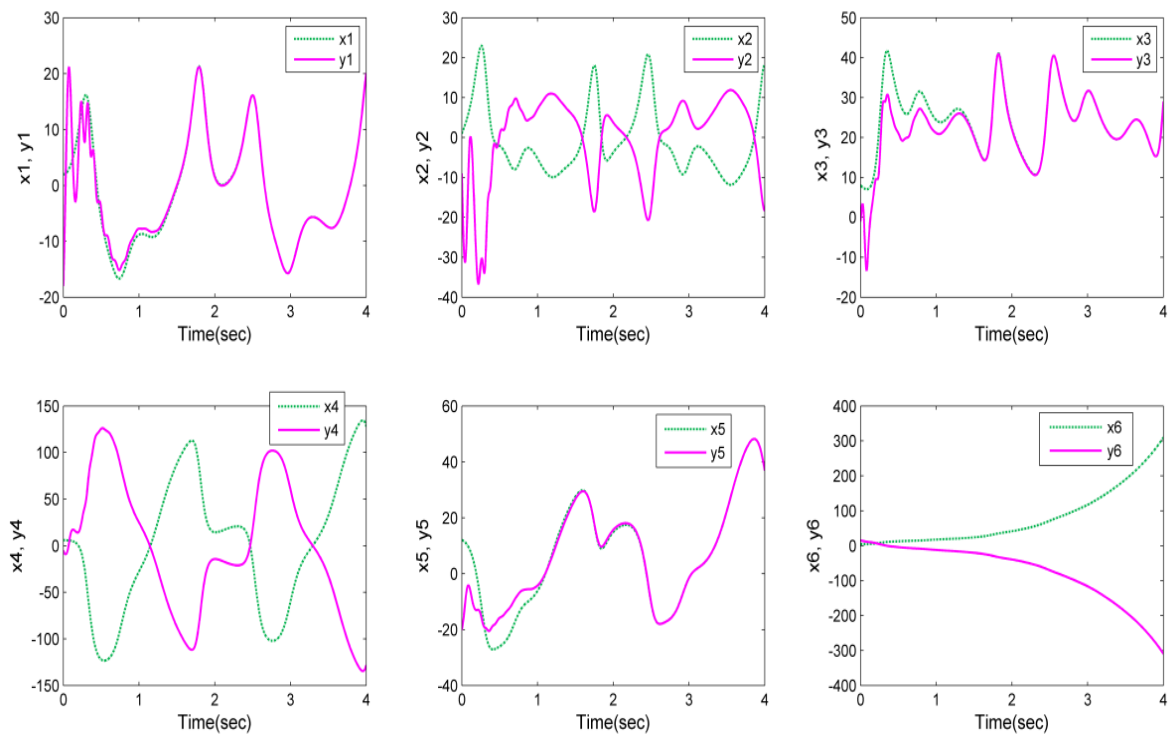


Figure 4. The state variables with control (18) at scaling factors $\alpha_i = \pm 1$

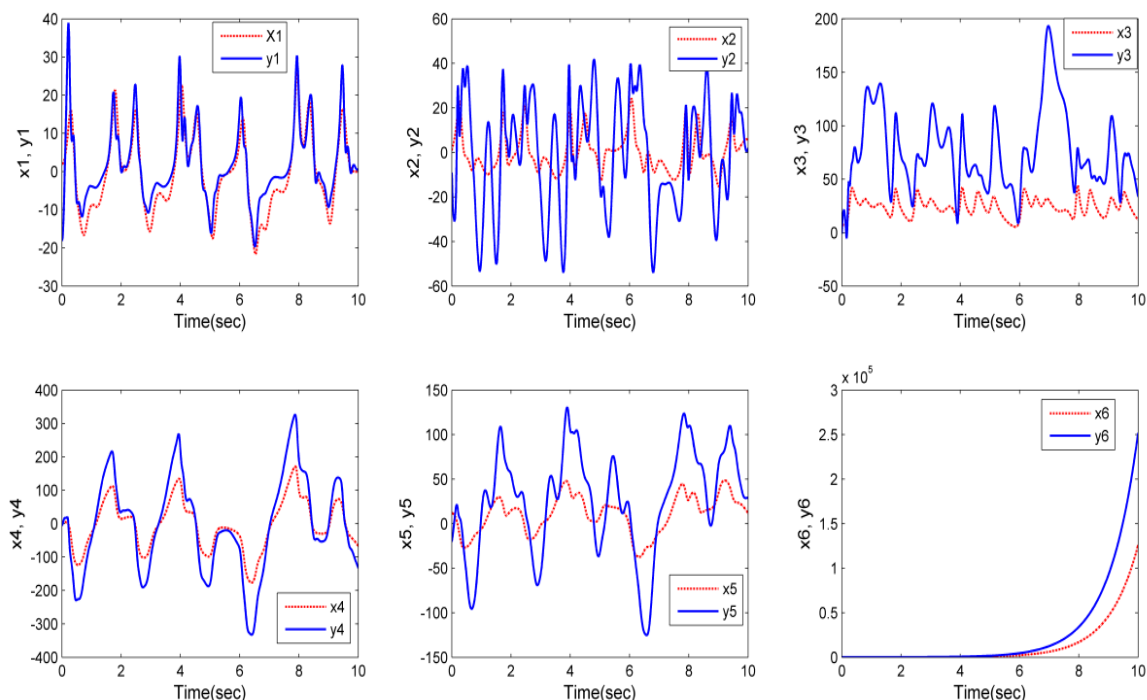


Figure 5. The state variables with control (24) at scaling factors $\alpha_i = 2$

5. CONCLUSION

Based on scaling factor α_i , a four projective synchronization error have been constructed, and a four controller have been proposed for achieving projective synchronization between two identical 6-D hyperchaotic Lorenz systems with unknown parameters based on the nonlinear control strategy and Lyapunov's direct method. Obviously from this projective synchronization, we achieved complete synchronization, anti-synchronization and hybrid synchronization through this phenomenon. The effectiveness of these proposed control strategies was validated by numerical simulation results.

ACKNOWLEDGEMENTS

The authors thank the University of Mosul for support this research under the College of Computer Sciences and Mathematics.

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