Analysis theorem of unique common fixed point for four maps based on partial –b– metric spaces

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ABSTRACT

A In this paper, An important definitions are to be used to prove the existence of a common fixed point theorem for four mappings incomplete, partial – b – metric spaces, as well as prove a unique common fixed point by assuming another point and getting that, these points are finally equal. We presented an example thus enhancing us the outcome.

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1. INTRODUCTION

In 1989, Bakhtin [1] approved the idea respecting a quasi–metric space as a generalized concept about metric spaces. In 1993, Czerwik [2, 3] expanded abundant upshots concerning for b– metric spaces. In 1994, Matthews [4] found the connotation concerning partial metric space at the self - distance in connection with any point about space might not equal zero. In 1996, O'Neill assured that a connotation for partial metric space through granting negative distances. In 2013, Shukla [5] assured together the connotation about b-metric & partial metric spaces via send in the partial b-metric spaces. For example, researchers explored the concept & its generalizations in several kinds of metric spaces [6-10].

Within this research, we proved a common fixed point theorem for four maps in partial b – metric space and in this paper we generalize both the concepts of b-metric and partial metric spaces by introducing the partial b-metric space. An analog of the common fixed point theorem for four maps in partial b – metric spaces is proved. Some examples are included which illustrate the results obtained in new space. First, we recall some definitions from b-metric and partial metric spaces.

Definition 1.1. [11-13] Let X be a nonempty set and let $s \geq 1$ be a given real number. A function $d: X \times X \rightarrow [0,\infty)$ is called a b-metric if for all $x, y, z \in X$ the following conditions are satisfied:
(i) $d(x, y) = 0$ if and only if $x = y$;
(ii) $d(x, y) = d(y, x)$;
(iii) $d(x, y) \leq s[d(x; z) + d(z; y)]$:
The pair (X, d) is called a b-metric space. The number $s \geq 1$ is called the coefficient of (X, d)

Definition 1.2. [4] Let X be a nonempty set. A function $p: X \times X \rightarrow [0,\infty)$ is called a partial metric if for all x, y, $z \in X$ the following conditions are satisfied:

(i) x = y if and only if p(x, x) = p(x, y) = p(y, y);
(ii) p(x, x) ≤ p(x, y);
(iii) p(x, y) = p(y, x);
(iv) p(x, y) ≤ p(x, z) + p(z, y) - p(z; z):
The pair (X; d) is called a partial metric space.

Remark 1.3 Apparent the partial metric space not necessity be a metric spaces, ∵ in a b - metric space whether v = w , ⟹ d (v, v) = d (v, w) = d (w, w) = 0. in a partial metric space if v = w ⟹p(v, v) = p(v, w) = p(w, w) not necessary be = 0. Thence the partial metric space not necessary be a ь- metric space.

So the else direction, Shukla [5] pressed the connotation of a partial b-metric space as pursue:

Definition 1.4. [5] If V be ≠ ∅ set & Ś ≥ 1 be a givenℝ. function
$P_{ь}: V \times V \to [0, \infty)$ is expressing a partial b – metric if ∀ v, w, z ∈ V the following conditions are convinced:
i: v = w ⟺ $P_{ь}$(v, v) = $P_{ь}$ (v, w) = $P_{ь}$ (w, w);
ii: $P_{ь}$(v, v) ≤ $P_{ь}$(v, w);
iii: $P_{ь}$ (v, w) = $P_{ь}$ (w, v);
iv: $P_{ь}$ (v, w) ≤ Ś [$P_{ь}$ (v, z) + $P_{ь}$ (z, w)] - $P_{ь}$ (z; z):
The (V; $P_{ь}$) is expressing a partial ь-metric space. The amount s ≥1 the parameter is called (V, $P_{ь}$).

Remark 1.5. The kind of partial b-metric space (V, $P_{ь}$) is the most effective way the kind of partial metric space ∵ a partial metric space is a condition shape from a partial b-metric space. (V, $P_{ь}$) while s = 1. Likewise, the kind of partial b-metric space (V, $P_{ь}$) is effective way bigger than the kind from ь-metric space, ∵ a ь-metric space is a private condition from a partial ь-metric space (V, $P_{ь}$) while the same – area p (v; v) = 0.

The next exa. articulate this one a partial ь-metric on V requirement not be a partial metric, neither a b-metric on V, look as well [14], [5].

Example 1.6. [5] Allowed V = [0,1). Realize a function $P_{ь}: V \times V \to [0, \infty)$ S.T.
$P_{ь}$ (v; w) = $[\max.\{v, w\}]^2 + |v - w|^2$, ∀ v, w ∈ V therefor (V, $P_{ь}$) is a partial b-metric metric & also not a partial metric to V.

Definition 1.7. [14] Any partial b-metric $P_{ь}$ is known a b – metric $d_{p_{ь}}$ whosesoever
$d_{p_{ь}}$ (v, w) = $2P_{ь}$ (v; w) –$P_{ь}$ (v, v) – $P_{ь}$ (w, w), ∀ v, w ∈ V.

Definition 1.8. [14] A sequence {v_n} in a partial b-metric space (V, $P_{ь}$) is called:
1- $P_{ь}$–convergent for v ∈ V if $\lim_{n\to\infty} P_{ь}(v, v_n) = P_{ь}(v, v)$
2- $P_{ь}$– Cauchy sequence if $\lim_{n,m\to\infty} P_{ь}(v_n, v_m)$ subsist &is finite;
3- partial b-metric space (V, $P_{ь}$) became $P_{ь}$–complete whether ∀ $P_{ь}$–Cauchy sequence {v_n} in V is $P_{ь}$ converges for v ∈ V, S.T.

$$\lim_{n,m\to\infty} P_{ь}(v_n, v_m) = \lim_{n\to\infty} P_{ь}(v_n, v) = P_{ь}(v, v)$$

Lemma 1.9. [14] A sequence {x_n} is a P_b-Cauchy sequence in a partial b-metric space (X, P_b) if and only if it is a b-Cauchy sequence in the b-metric space (X, d_{P_b}).

Lemma 1.10. [14] A partial b-metric space (X, P_b) is $P_ь$-complete if and only if the b-metric space (X, d_{P_b}) is b-complete. Moreover, $\lim_{n,m\to\infty} d_{P_b}(x_n, x_m) = 0$ if and only if

$$\lim_{n,m\to\infty} P_b(x_m, x) = \lim_{n\to\infty} P_b(x_n, x) = P_b(x, x)$$

Definition 1.11 [15]: A & S two self-maps from a metric space (V, d) are designation weakly compatible if, at coincidence points those commute . Which, in case Av = Sv ⇒ ASv =SAv for v in V.
Presently we demonstrate our essential outcome.

2. MAIN RESULTS

Theorem: 2.1: aLet $(w, p_ь)$ be a partial ь – metric space for the coefficient Ṡ ≥ 1, consign Ⱥ ,Ƀ,Ç ,Đ: V→V be mappings appropriate the next

(2.1.1) $Ṡ.\, p_ь(Ⱥ_V, Ƀ_w) \leq \max. \begin{Bmatrix} p_ь(Ç_V, Đ_W), P_ь(Ç_V, Ⱥ_V), P_ь(Đ_w, Ƀ_w), \\ \frac{1}{2Ṡ}[P_ь(Ç_w, Ƀ_w) + P_ь(Đ_w, Ⱥ_V)] \end{Bmatrix}$

Where K∈ $\left[0, \frac{1}{Ṡ}\right)$, $\forall\, v, w \in V$

(2.1.2) $Ⱥ(V) \subseteq Đ(V), Ƀ(V) \subseteq Ç(V)$

(2.1.3) with regard to Ç(V) or Đ(V) is complete subspace of V.

(2.1.4) the (Ⱥ; Ç) &(Ƀ, Đ) are weakly compatible.
So Ⱥ, Ƀ, Ç & Đ include unique common fixed point in V

Proof : Select $v_0, w_0 \in V$. From (2.1.2) ∃, sequences $\{v_n\}$ & $\{w_n\}$ in V s.t.
$Ⱥv_{2n} = Đv_{2n+1} = w_{2n}$
$Ƀv_{2n+1} = Çv_{2n+2} = w_{2n+1}$ ∀ n=0,1,2,3, ……….

Status: (i):-Assume $w_{2n} = w_{2n+1}$ for some n.
Clam: $w_{2n+1} = w_{2n+2}$
Supp. $w_{2n+1} \neq w_{2n+2}$
From (2.1.1) , then

$Ṡ.\, p_ь(w_{2n+1}, w_{2n+2}) = Ṡ.\, p_ь(Ⱥv_{2n+2}, Ƀv_{2n+1})$

$\leq k \max. \begin{Bmatrix} P_ь(Çv_{2n+2}, Đv_{2n+1}), P_ь(Çv_{2n+2}, Ⱥv_{2n+1}), P_ь(Đv_{2n+1}, Ƀv_{2n+1}), \\ \frac{1}{2Ṡ}[P_ь(Çv_{2n+1}, Ƀv_{2n+1}) + P_ь(Đv_{2n+1}, Ⱥv_{2n+1})] \end{Bmatrix}$

$= k \max. \begin{Bmatrix} p_ь(w_{2n+1}, w_{2n}), P_ь(w_{2n+1}, w_{2n+2}), P_ь(w_{2n}, w_{2n+1}), \\ \frac{1}{2Ṡ}[P_ь(w_{2n+1}, w_{2n+1}) + P_ь(w_{2n+1}, w_{2n+2})] \end{Bmatrix}$

$= k \max \begin{Bmatrix} p_ь(w_{2n+1}, w_{2n+1}), P_ь(w_{2n+1}, w_{2n+2}), P_ь(w_{2n+1}, w_{2n+1}), \\ \frac{1}{2Ṡ}[P_ь(w_{2n+1}, w_{2n+1}) + P_ь(w_{2n}, w_{2n+2})] \end{Bmatrix}$

$= k p_ь(w_{2n+1}, w_{2n+2}),$

that is a discrepancy.

$\therefore w_{2n+1} = w_{2n+2}$

Stay in same direction we ability ratiocinate that

$w_{2n} = w_{2n+k}$

∴ $\{w_{2n}\}$ a Cauchy sequence in V

Status (ii):- $w_n \neq w_{n+1}\ \forall\, n.$
From (2.1.1), consider

$Ṡ.\, p_ь(Ⱥv_{2n}, Ƀv_{2n+1}) \leq max \begin{Bmatrix} p_ь(Çv_{2n}, Đv_{2n+1}), P_ь(Çv_{2n}, Ⱥv_{2n}), P_ь(Đv_{2n+1}, Ƀv_{2n+1}), \\ \frac{1}{2Ṡ}[P_ь(Çv_{2n}, Ƀv_{2n+1}) + P_ь(Đv_{2n+1}, Ⱥv_{2n})] \end{Bmatrix}$

$= k \max \begin{Bmatrix} p_ь(w_{2n-1}, w_{2n}), P_ь(w_{2n-1}, w_{2n}), P_ь(w_{2n}, w_{2n+1}), \\ \frac{1}{2Ṡ}[P_ь(w_{2n-1}, w_{2n+1}) + P_ь(w_{2n}, w_{2n})] \end{Bmatrix}$

$$=\text{k max}\begin{Bmatrix} p_{ь}(w_{2n-1,}w_{2n}), P_{ь}(w_{2n}, w_{2n}), P_{ь}(w_{2n}, w_{2n+1}), \\ \frac{1}{2\dot{Ŝ}}[\dot{Ŝ}[P_{ь}(w_{2n-1}, w_{2n}) + P_{ь}(w_{2n}, w_{2n+1})]] \end{Bmatrix}$$

$=\text{k max}\ \{p_{ь}(w_{2n-1}, w_{2n}), p_{ь}(w_{2n}, w_{2n+1})\}$

if $p_{ь}(w_{2n}, w_{2n+1})$ is maximum, then

$\dot{Ŝ}.\, p_{ь}(w_{2n}, w_{2n+1}) \leq k p_{ь}(w_{2n}, w_{2n+1})$

which implies

$p_{ь}(w_{2n}, w_{2n+1}) \leq \frac{K}{\dot{Ŝ}} p_{ь}(w_{2n}, w_{2n+1}) < p_{ь}(w_{2n}, w_{2n+1})$

which is a contradiction.

Hence $p_{ь}(w_{2n-1}, w_{2n})$is maximum. So that

$\dot{Ŝ}.\, p_{ь}(w_{2n}, w_{2n+1}) \leq k p_{ь}(w_{2n-1}, w_{2n})$

implies that

$$p_{ь}(w_{2n}, w_{2n+1}) \leq \frac{K}{S} p_{ь}(w_{2n-1}, w_{2n}) \quad (1)$$

Put $p_{2n} = p_{ь}(w_{2n}, w_{2n+1})$

Then $\{P_{2n}\}$ is decreasing sequence of non-negative ℝ& must converges to some ℝ

$l \geq 0$. (say)

Assume $l > 0$

Letting $n \to \infty$ in (1), we obtain

$$l \leq \frac{k}{s} \cdot e\ l < l$$

Is the antinomy.
⇒ $l = 0$. So

$$\lim_{n\to\infty} p_{ь}(\mathrm{w}_{2\mathrm{n}}, \mathrm{w}_{2\mathrm{n}+1}) = 0 \quad (2)$$

Hence for def.1.4

$$\lim_{n\to\infty} p_{ь}(w_{2n}, w_{2n}) = 0 \quad (3)$$

From (2) and (3) and by definition of d_{P_b}, we get

$$\lim_{n\to\infty} d_{P_{ь}}(w_{2n}, w_{2n+1}) = 0.$$

For m ,n$\in N$ with m > n, we have

$p_ь(w_{2n},w_{2m}) \leq \dot{S}[p_ь(w_{2n},w_{2n+1}) + p_ь(w_{2n+1},w_{2m})] - p_ь(w_{2n+1},w_{2n+1})$

$\leq \dot{S}.\, p_ь(w_{2n},w_{2n+1}) + \dot{S}^2 p_ь(w_{2n+1},w_{2n+2}) + \cdots + \dot{S}^{2m-2n} p_ь(w_{2m-1},w_{2m})$

$\leq \dot{S}.\frac{K^{2n+1}}{\dot{s}^{2n+1}} p_ь(w_0,w_1) + \dot{S}^2 \frac{K^{2n+2}}{\dot{s}^{2n+2}} p_ь(w_0,w_1) + \cdots + \dot{S}^{2m-2n} \frac{K^{2m}}{\dot{s}^{2m}}\, p_ь(w_0,w_1)$

$=\frac{K^{2n}}{\dot{s}^{2n}} [k + k^2 + k^3 + \cdots + k^{2m-2n}].\, p_ь(w_0,w_1)$

As $k \in \left[0, \frac{1}{s}\right)$ & $\dot{S} \geq 1$, it follows from the above then

$$\lim_{n,m\to\infty} p_ь(w_{2n,}w_{2m}) = 0 \tag{4}$$

Then $\{w_{2n}\}$ is a Cauchy sequence in V
Same that we competence likewise evince that$\{w_{2n+1}\}$ is a Cauchy sequence in V.
Subsequently $\{w_{2n}\}$ is a Cauchy sequence in V.
According to Lemma (1.9) , we name it$\{w_{2n}\}$ is a Cauchy sequence in (v, $d_{P_ь}$).
Suppose Ç(v) is a complete subspace of V.

∵ $\{w_{2n+1}\}$ is a Cauchy sequence in complete b-metric space (Ç(v),$d_{P_ь}$).

This is a follow-up $\{w_{2n+1}\}$ converges to w in Ð(V). So

$$\lim_{n,\to\infty} d_{P_ь}(w_{2n+1},w) = 0$$

Some of w$\in$ Ç(V). $\exists\ \alpha \in V$ such that Ç$\alpha \in w$.

∵ $\{w_{2n+1}\}$ is Cauchy sequence & $w_{2n+1} \to w$.

It follows that $w_{2n} \to w$

According to Lemma (1.10) & (4), we possess that

$$p_ь(w,w) = \lim_{n,\to\infty} p_ь(w_{2n},\mathrm{w}) = \lim_{n,\to\infty} p_ь w_{2n+1},\mathrm{w}) = 0 \tag{5}$$

Here we evince it $\lim_{n,\to\infty} p_ь(Ꞁ\alpha, w_{2n}) =$

$p_ь(Ꞁ\alpha, w)$

∵ that def. of d_{P_b} ,

$d_{P_ь}(Ꞁ\alpha, w_{2n}) = 2p_ь(Ꞁ\alpha, w_{2n}) - p_ь(Ꞁ\alpha, Ꞁ\alpha) - p_ь(w_{2n}, w_{2n})$

According to def. of d_{P_b} (4) & (5), we possess that

$$d_{P_ь}(Ꞁ\alpha, w) = \lim_{n\to\infty} 2\,.\, p_b(Ꞁ\alpha, w_{2n})$$

implies that

$$\lim_{n,\to\infty} p_ь(Ꞁα, w_{2n}) = p_ь(Ꞁα, w) \tag{6}$$

From, def. (1.4) we have

$$p_ь(Ꞁα, w) \leq Ṡ[p_ь(Ꞁα, w_{2n+1}) + p_ь(w_{2n+1}, w)] - p_ь(w_{2n+1}, w_{2n+1})$$

$= Ṡ[p_ь(Ꞁα, w_{2n+1}) + p_ь(w_{2n+1}, w)]$

Allowing $n \to \infty$,

$\therefore\ p_ь(Ꞁα, w) \leq Ṡ.\lim_{n\to\infty} p_ь(Ꞁα, w_{2n+1})$

$= \lim_{n\to\infty} Ṡ.p_ь(Ꞁα, Ƀv_{2n+1}) \leq \lim_{n\to\infty} k.max\begin{Bmatrix} p_ь(Çα, Đv_{2n+1}), P_ь(Çα, Ꞁα), P_ь(Đv_{2n+1}, Ƀv_{2n+1}), \\ \frac{1}{2Ṡ}[P_ь(Çα, Ƀv_{2n+1}) + P_ь(Đv_{2n+1}, Ꞁα)] \end{Bmatrix}$

$= \lim_{n\to\infty}$ k. max $\begin{Bmatrix} p_ь(w, w_{2n}), P_ь(w, Ꞁα), P_ь(w_{2n}, w_{2n+1}), \\ \frac{1}{2Ṡ}[P_ь(w, w_{2n+1}) + P_ь(w_{2n}, Ꞁα)] \end{Bmatrix}$

=k.$p_ь(Ꞁα, w)$

It is clear that $Ꞁα = w = Çα$.

∵ the pair (Ꞁ, Ç) is a weakly compatible pair, we hold

Ꞁw =Ç w

Here we demonstrate that .Ꞁw=w. Consider

$p_ь(Ꞁw, w) \leq Ṡ[p_ь(Ꞁw, w_{2n+1}) + p_ь(w_{2n+1}, w)] - p_ь(w_{2n+1}, w_{2n+1})$
$\leq Ṡ[p_ь(Ꞁw, w_{2n+1}) + p_ь(w_{2n+1}, w)]$

Allowing $n \to \infty$,

$\therefore\ p_ь(Ꞁw, w) \leq Ṡ.\lim_{n\to\infty} p_ь(Ꞁw, Ƀx_{2n+1})$

$\leq \lim_{n\to\infty} k.max\begin{Bmatrix} p_ь(Çw, Đv_{2n+1}), P_ь(w, Ꞁw), P_ь(Đv_{2n+1}, Ƀv_{2n+1}), \\ \frac{1}{2Ṡ}[P_ь(Çw, Ƀv_{2n+1}) + P_ь(Đv_{2n+1}, Ꞁw)] \end{Bmatrix}$q

$= \lim_{n\to\infty}$ k. max $\begin{Bmatrix} p_ь(Ꞁw, w_{2n}), P_ь(Ꞁw, Ꞁw), P_ь(w_{2n}, w_{2n+1}), \\ \frac{1}{2Ṡ}[P_ь(Ꞁw, w_{2n+1}) + P_ь(Ꞁw, w_{2n})] \end{Bmatrix}$

$= kp_ь(Ꞁw, w)$.

It is clear that Ꞁw=w .

∴ w is common fixed point of Ꞁ & Ç.

Since,$Ꞁ(V) \subseteq Đ(V)$ we have that w = Ꞁw = Đ , $\forall\ t \in V$. From (2.1.1), ⟹

$Ṡ.p_ь(Ꞁw, Ƀt) \leq$k. max $\begin{Bmatrix} p_ь(Çw, Đt), P_ь(Çw, Ꞁw), P_ь(Đt, Ƀt), \\ \frac{1}{2Ṡ}[P_ь(Çw, Ƀt) + P_ь(Đt, Ꞁw)] \end{Bmatrix}$

$= \text{k. max}\begin{Bmatrix} p_{ь}(w,w), P_{ь}(w,w), P_{ь}(w, Ƀt), \\ \frac{1}{2Ś}[P_{ь}(w, Ƀt) + P_{ь}(w,w)] \end{Bmatrix}$

$= kp_{ь}(w, Ƀt)$

It apparent this $Ƀt = w = Đt$.

∵ (Ƀ, Đ) is weakly compatible , so that $Ƀw = Đw$.

Again (2.1.1), ⟹

$Ś. p_{ь}(Ⱥw, Ƀw) \le$k. max. $\begin{Bmatrix} p_{ь}(Çw, Đw), P_{ь}(Çw, Ⱥw), P_{ь}(Đw, Ƀw), \\ \frac{1}{2Ś}[P_{ь}(w, Ƀw) + P_{ь}(Đw, Ⱥw)] \end{Bmatrix}$

$= \text{k. max}\begin{Bmatrix} p_{ь}(w, Ƀw), P_{ь}(w,w), P_{ь}(Ƀw, Ƀw), \\ \frac{1}{2Ś}[P_{ь}(Çw, Ƀw) + P_{ь}(w, Ƀw)] \end{Bmatrix}$

$= kp_{ь}(w, Ƀw)$

It is clear that $w = Ƀw = Đw$.

∴ *w* is common fixed point of Ⱥ, Ƀ, Ç & Đ.

Now we demonstrate that *w* is unique common fixed point in V. Let us assume *z* is other common fixed point of Ⱥ,Ƀ,Ç & Đ.

Claim : $w = z$.

From (2.1.1), ⟹
$Ś. p_{ь}(w, z) \le Ś. p_{ь}(Ⱥw, Ƀz)$

$\le k.max\begin{Bmatrix} p_{ь}(Çw, Đz), P_{ь}(Çw, Ⱥw), P_{ь}(Đz, Ƀz), \\ \frac{1}{2Ś}[P_{ь}(Çw, Ƀz) + P_{ь}(Đz, Ⱥw)] \end{Bmatrix}$

$= \text{k. max}\begin{Bmatrix} p_{ь}(w,z), P_{ь}(w,w), P_{ь}(z,z), \\ \frac{1}{2Ś}[P_{ь}(w,z) + P_{ь}(z,w)] \end{Bmatrix}$

$\le k. p_{ь}(w, z)$.

It is clear that $w = z$.

Hence *w* is the unique common fixed point of Ⱥ, Ƀ, Ç& Đ. The next example Clear up our substantial Theorem 2.1.

Example 2.2: Authorize $w = [0,1)$ be partial b-metric space with. $P_{ь}: V \times V \to [0, \infty)$ realize b
$P_{ь}(v, w) = [max.\{v, w\}]^2, \forall\, v, w \in V$. Clearly $(V, P_{ь})$ is partial b-metric space with Ś=2.
Realize the mapping $Ⱥ, Ƀ, Ç, Đ: V \to V$ by

a. $Ⱥ(v) = \frac{V^2}{2\sqrt{1+V}}$, $Ƀ(V) = \frac{V^2}{4\sqrt{1+V}}$
b. $Ç(v) = \frac{V}{2}$, $Đ(V) = \frac{V}{2}$.

So Ⱥ, Ƀ, Ç& Đ content with every stipulation of theorem (2.1) & 0 is the unique fixed point of Ⱥ , Ƀ, Ç & Đ

3. CONCLUSION

In this paper, we gave a newly fixed point theorems for Partial b-metric space. We hope that our study contributes to the development of these results by other researchers.

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