

# End-to-end performance of DF multihop hybrid RF/FSO system using MPPM coding and MIMO technique under dependent GG turbulence channels

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## Article Info

### Article history:

Received Nov 20, 2019

Revised Jan 26, 2020

Accepted Feb 11, 2020

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### Keywords:

FSO  
GG channel  
Hybrid  
MIMO  
RF

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## ABSTRACT

The paper presents a BER analysis for hybrid Radio Frequency/ multihop Free Space Optics, known as RF/ FSO, communication system benefiting from Multiple Input Multiple Output (MIMO) configuration but impaired by Gamma-Gamma, known as GG, turbulence. Authors deploy Multiple Pulse Position Modulation, known as M-PPM, like a signaling technique. Furthermore, the exact as well as approximated amounts of Symbol-Error Rates, known as SERs, are both judged in this work. A closed form formula related to the whole Probability Density Function, called PDF, for our designed system is derived. Such interesting approach provides to us better quality of service by combating atmospheric turbulence.

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## 1. INTRODUCTION

The FSO communication affords cost-effective means for achieving huge capacity and is used in various telecommunication applications [1-2]. One of the most crucial occurrences that influence FSO system's performance is scintillation [3-4]. This system involves also light signal transmission over atmospheric turbulence channels [5]. GG distributions, eventually, are valid for a larger variety of such turbulence strength from weak to strong [6].

To combat atmospheric turbulence and enhance the transmitted signal quality, the MIMO technique's virtues have motivated the researchers to integrate several transmit and receive sensors in FSO communication scheme [7]. MPPM favours the power efficiency over the bandwidth [8]. With the aim of describing the transmitted signal appropriately at the receiver, many statistical and mathematical channels models are investigated in [9-15]. Moreover, the outage probability's closed-form expression for a MIMO FSO system using binary PPM has been illustrated in [16]. For independent channels, the GG derived signals' result combined summation can be estimated by the  $\alpha - \mu$  distribution [17]. DF relaying is considered in [18-20] whereas Quantize and Forward (QF) one is used in [21].

We are interested to combine such techniques to improve the QoS of FSO communication system by dealing with a new configuration which is RF/MIMO FSO multihop network over GG turbulence correlated channels and using MPPM modulation scheme. Exact and approximated values of SERs are developed for our system with a closed form formula for the PDF expression. This paper is arranged as follow: The scheme model is introduced in subsection 2.1. We also present in this section its statistical model. In subsection 2.2, we express the end to end BER. Section 3 discusses the results with a conclusion in Section 4.

2. RESEARCH METHOD

2.1. System model

Hybrid DF multihop RF/ MIMO FSO network with N optical sources and N optical detectors as shown in Figure 1.

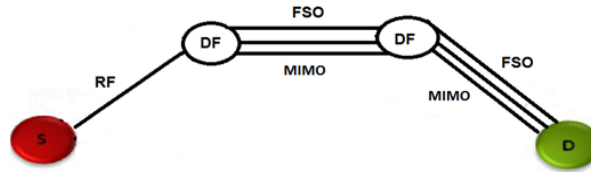


Figure 1. Hybrid DF multihop RF/ MIMO FSO network with N optical sources and N optical detectors

GG channel distribution overview:

The GG distribution’s marginal PDF is introduced like in [6]:

$$g(K_s) = \frac{2(xy)^{\frac{x+y}{2}}}{\Omega \Gamma(x)\Gamma(y)} \left(\frac{K_s}{\Omega}\right)^{\frac{x+y}{2}-1} B_{x-y} \left(2\sqrt{\frac{xyK_s}{\Omega}}\right),$$

where  $K_s \geq 0$ ,

(1)

and knowing that  $\tau(\cdot)$  represents gamma function,  $B_c(\cdot)$  describes cth order case of modified Bessel function related to second kind,  $K_s$  illustrates average received information photon count for each signal slot and  $\Omega = E\{k_s\}$  is  $K_s$ ’s expected mean. We denote  $x$  and  $y$  as GG distribution’s format parameters, associated to refractive as well as diffractive turbulence impacts with large-scale as well as small-scale eddies effective number.  $\chi_{sc}^2 := \text{var}\{k_s\} / E\{k_s^2\}$  denotes scintillation index function of  $(x, y)$ :

$$\chi_{sc}^2 = \frac{1}{x} + \frac{1}{y} + \frac{1}{xy}$$
(2)

Concerning weak towards moderate turbulence,  $\chi_{sc}^2$  belongs to  $[0, 0.75]$ , whereas  $\chi_{sc}^2 > 0.75$  case of strong turbulence [5].

MIMO-FSO model

Our M-PPM symbol period is composed of  $Q$  intervals (time slots). Besides, an optical power seems solely transmitted during  $w$  times slots, where  $w$  belongs to  $\{1, 2, \dots, Q/2\}$ . We transmit each MPPM frame through all  $N$  transmitters. Using equal gain combining, the average received photon count for every signal slot is described as

$$K_{S_{on}} = \sum_{m=1}^N \sum_{n=1}^N K_{S_{mn}} + K_b$$
(3)

where  $K_{smn}$  represents average received signal photon count concerning  $m$ th source as well as  $n$ th receiver,  $K_b$  denotes average received photon count for every slot because of background noise. Each  $Q$  summation of  $N^2$  GG random variables has a PDF as:

$$Z^j = \sum_{m=1}^N \sum_{n=1}^N Y_{mn}^j, \quad j \in \{1, 2, \dots, Q\}$$
(4)

where  $Y_{mn}^j$  denotes depicted photon count during  $j$ th slot having  $j \in \{1, 2, \dots, Q\}$  having both  $n$  and  $m \in \{1, 2, \dots, N\}$ . MIMO configuration

Having a spacing between detectors often lower than correlation distance,  $(\alpha - \mu)$  distribution is invalid for Z of (4). Thus, the PDF closed-form expression case of equal gain combining dependent MIMO channels in Figure 1 could be mathematically described [22]:

$$f_z(z) = \frac{2}{N^2 \Omega [\det(A)]^{\nu} \Gamma(x)} \sum_{i=1}^{N'} \sum_{q=1}^{y_i} \frac{C_{qi} \lambda_i^{\frac{q-x}{2}}}{\Gamma(q)} \times x^{\frac{q+x}{2}} \left(\frac{z}{N^2 \Omega}\right)^{N^2 y-1-\frac{q-x}{2}} B_{q-x} \left(2 \sqrt{\frac{xz}{N^2 \Omega \lambda_i}}\right) \tag{5}$$

$\{\lambda_i\}_{i=1}^{N^2}$  represent eigenvalues for A=DC, where D mentioned  $N^2 * N^2$  diagonal matrix including small-scale form for the factor y, while C describes  $N^2 * N^2$  positive specific association matrix including  $\sqrt{\rho_{ij}}$  correlation coefficients.  $N_0$  denotes the number case of matrix A's different eigenvalues  $\lambda_i$ . Besides,  $y_i = \mu_A(\lambda_i)y$ , where  $\mu_A(\lambda_i)$  represents eigenvalue's algebraic multiplicity. Thus, the coefficient  $c_{qi}$  could be provided as:

$$c_{qi} = \frac{\sum_{k_1+\dots+k_{N'}=y_i-q} \binom{y_i-q}{k_1, \dots, k_{N'}}}{\prod_{j=1, j \neq i}^{N'} [(-1)^{k_j} (y_j)_{k_j} (d_j - d_i)^{-(y_j+k_j)}} \tag{6}$$

where  $d_j = \frac{z}{N^2 \Omega \lambda_j} \binom{y_i-q}{k_1, \dots, k_{N'}} = \frac{y_i-q}{k_1, \dots, k_{N'}!}$

means that  $k_i$  is deleted from the sequence, and  $\binom{y_j}{k_j}_{k_j = \frac{r(y_j+k_j)!}{r(y_j)}}$  represents Pochhammer symbol

RF link  
The received pulse y(t) could be defined like a function of transmitted one x(t) and channel response h(t) as follows [23]:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau$$

Whereas the AWGN noise can be modeled as in [23]:

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-[\frac{(x-\mu)^2}{2\sigma^2}]}$$

With x: random variable and  $\mu$  its meanwhile  
 $\sigma$  : normalized fluctuation.

**2.2. Performance analysis**

MIMO-FSO portion

SER:

Having a received vector described like

$$Z = (Z^1, Z^2, \dots, Z^Q) \tag{7}$$

where  $Z_j$ , whose  $j \in \{1, 2, \dots, Q\}$ , denotes a summation over jth slot. In addition to such Q slots, we have w ON ones for carrying data while  $Q - \omega$  OFF ones for no information. Let

$$P_{MIN} = p_1(z_{min})^m (1 - P_1(z_{min}))^{w-m} \quad (8)$$

Knowing that  $z_{min}$  represents lowest photon amount per symbol signal time slots. Whereas,  $p_1(\cdot)$  represents probability of photon count for signal time slots.  $P_1(\cdot)$  denotes its cumulative statistical distribution. If a number of one or else more among OFF slots has count  $\geq$  to that case of ON slots, we have a transmission error. The SER case of MPPM coding in absence of turbulent atmosphere path is introduced as in [24]:

$$SER = \sum_{z_{min}=0}^{\infty} \sum_{l=1}^{Q-w} \sum_{m=1}^w \binom{w}{m} \binom{Q-l}{l} \times P_{MIN} P_0(z_{min} - 1)^{Q-w-l} [(1 - P_0(z_{min}))^l + p_0(z_{min})^l \left( 1 - \frac{1}{\binom{l+m}{m}} \right)] \quad (9)$$

Where  $p_0(\cdot)$  represents probability of photon count for non-signal slots. Whereas,  $P_0(\cdot)$  represents its cumulative statistical distribution. Adopting Poisson distribution, the above probabilities are described as:

$$P_0(k) = \frac{K_b^k}{k!} e^{-K_b}, P_1(k) = \frac{(z+K_b)^k}{k!} e^{-(z+K_b)} \\ P_0(k) = \sum_{j=0}^k \frac{K_b^j}{j!} e^{-K_b}, P_1(k) = \sum_{j=0}^k \frac{(z+K_b)^j}{j!} e^{-(z+K_b)} \quad (10)$$

Having  $k$  a positive integer,  $Z$  an average received data photon amount by ON time slot. Regular SER might be determined as a result of averaging (9) considering  $z$  value. Assuming that solely PMIN in (9) depends on our channel statistical distribution, we can obtain average SER from replacing PMIN by its average  $P_2(z_{min})$  in (9):

$$P_2(z_{min}) = \int_0^{\infty} p_1(z_{min})^m (1 - P_1(z_{min}))^{w-m} f_Z(z) dz \quad (11)$$

which after algebraic manipulations can be expressed as:

$$P_2(z_{min}) = \sum_{j=(w-m)(z_{min}+1)}^{\infty} \sum_{B=0}^{j+mz_{min}} \binom{j+mz_{min}}{B} r(j) \times \frac{e^{-wk_b} k_b^{j+mz_{min}-B}}{z_{min}!^m} \int_0^{\infty} z^B e^{-wz} f_Z(z) dz \quad (12)$$

Whose  $r(j)$ , such as  $j \geq (w-m)(Z_{min} + 1)$ , is described like

$$r(j) = \sum_{(S_1, S_2, \dots, S_{w-m}) \in \chi(j)} \frac{1}{S_1! S_2! \dots S_{w-m}!}$$

over the set of vectors  $\chi(j)$ , where

$$\chi(j) = (S_1, S_2, \dots, S_{w-m}) \in N^{w-m} : \sum_{i=1}^{w-m} S_i = j \text{ and } \forall \ell \in \{1, 2, \dots, w-m\}, z_{min} + 1 \leq S_\ell \leq j - (w-m-1)(z_{min} + 1)$$

Exact SER:

By replacing (5) and (6) into (12), the average SER of MIMO-FSO dependent channels could be rewritten from [25]:

$$\begin{aligned}
 P_2(z_{min}) &= \sum_{j=(w-m)(z_{min}+1)}^{+\infty} \sum_{B=0}^{j+mz_{min}} \sum_{i=1}^{N'} \sum_{q=1}^{y_i} \\
 &\sum_{k_1+\dots+k_{N'}=y_i-q} \binom{y_i-q}{k_1+\dots+k_{N'}} \times \binom{j+mz_{min}}{B} \\
 &\exp\left(\frac{x}{2N^2\Omega\lambda_i\omega} - \omega K_b\right) \omega^{-(B+N^2y+\frac{q+x}{2}-\sum_{j=1}^{N'} y_j-\frac{1}{2})} \\
 &\times \frac{\Gamma(B+N^2y+x-\sum_{j=1}^{N'} y_j)\lambda_i^{\frac{q-x+1}{2}} x^{\frac{q+x-1}{2}} K_b^{j+mz_{min}-B}}{(y_i-q)!z_{min}!m(N^2\Omega)^{(N^2y+\frac{q+x}{2}-\sum_{j=1}^{N'} y_j-\frac{1}{2})}} \\
 &\times \frac{r(j)\Gamma(B+N^2y+q-\sum_{j=1}^{N'} y_j)}{[det(A)]^y\Gamma(q)\Gamma(x)} \\
 &\times \prod_{j=1, j \neq i}^{N'} \left[ (-1)^{k_j} (y_j)_{k_j} \left(\frac{1}{\lambda_j} - \frac{1}{\lambda_i}\right)^{-(y_j+k_j)} \right] \\
 &\times W_{-\left(B+N^2y+\frac{q+x}{2}-\sum_{j=1}^{N'} y_j-\frac{1}{2}\right), \left(\frac{q-x}{2}\right)}\left(\frac{x}{N^2\Omega\lambda_i W}\right)
 \end{aligned} \tag{13}$$

having  $W(\cdot, \cdot)$  as Whittaker function

Approximate SER:

Through Gauss-Laguerre method for quadrature law [5]. We can rewrite (11) like:

$$\begin{aligned}
 P_2(z_{min}) &= \frac{e^{(-wk_b)}}{z_{min}!m} \int_0^\infty e^{(-wz)} (z+k_b)^{mz_{min}} \times [e^{(z+k_b)} \\
 &\quad - \sum_{B=0}^{j+mz_{min}} \frac{(z+k_b)^j}{j!}]^{w-m} f_z(z) dz
 \end{aligned} \tag{14}$$

This integration might be estimated due to Gauss-Laguerre approach:

$$\begin{aligned}
 P_2(z_{min}) &\approx \sum_{i=1}^c \Lambda_i \frac{e^{(-wk_b)}}{z_{min}!m} (V_i+k_b)^{mz_{min}} \\
 &\quad \times [e^{(V_i+k_b)} \\
 &\quad - \sum_{j=0}^{z_{min}} \frac{(V_i+k_b)^j}{j!}]^{w-m} f_z(V_i) dz
 \end{aligned} \tag{15}$$

where  $V_i$  represents  $i$ -th basis for the Laguerre polynomial  $L_c(x)$  where  $c > 1$ .  $\Lambda_i$  denotes the corresponding weighting coefficient.

RF portion

BER:

Our bit error function could be defined [23]:

$$\overline{P(e)} = Q\left(\left[2\left(\frac{N_0}{E_S}\right) + 4B\Delta\left(\frac{N_0}{E_S}\right)^2\right]^{-\frac{1}{2}}\right)$$

with  $E_S$ : received energy per symbol

$Q(x)$ : null mean Gaussian distribution function

$N_0$ : AWGN's single-sided power spectral density

$B$ : signals bandwidth

$T$ : integration time.

The End to End BER

We have, in our model, 3 independent symmetric portions with an error probability  $P_i$  for the  $i$ th hop

( $i \in \{1, 3\}$ ). We obtain an E2E SEP ( $P_e^t$ ) as:

$$P_e^t = 1 - (1 - P_1)(1 - P_2)(1 - P_3) = 1 - (1 - P(e))(1 - P_2(z_{min}))^2 \tag{16}$$

### 3. RESULTS AND DISCUSSION

In this section, we assumed that  $M=N=2$ , Laguerre polynomial degree  $c=100$  and having 8PPM modulation scheme under weak and strong GG atmospheric turbulence (respectively  $\sigma_R = \{0.25, 0.75\}$ ). In Figure 2, we remark that average SER (ASER) diminishes when average SNR (ASNR) augments. Besides, when we have greater correlation coefficient, we have lower system quality. When  $\rho \rightarrow 1$  (high correlation), having channels affected by identical fading and then receiving identical signals, there is no difference between MIMO and SISO configurations in term of QoS. Nevertheless, when  $\rho \rightarrow 0$  (low correlation), all channels are nearly independent and thus the system takes complete virtue of MIMO diversity leading to better QoS. Figure 3 showed in fact that when the number of hops increased which means that the distance between resource and destination was growing; the ASER seemed to be little worse but didn't affect so much the whole system performance. Thanks to multihop configuration, we could send data for far distances.

For example, for single hop for moderate turbulence, we had  $ASER = 10^{-11}$  when the  $ASNR = 100$  while for three-hop we had  $10^{-11} < SER < 10^{-10}$  which means that  $SER_{3hops} \approx SER_{1hop}$  under same turbulence and for the same SNR. Indeed, we gained 2Km (the distance between two nodes = 1Km) more having almost the same quality of service i.e. the same data rate. In addition, we found in Figure 4 that the all RF links seemed the most reliable one with less ABER than hybrid one which is better than all FSO one. To note here that FSO looks more important to repair last mile breakdown at the absence of fiber-optic communications like complement for RF networks.

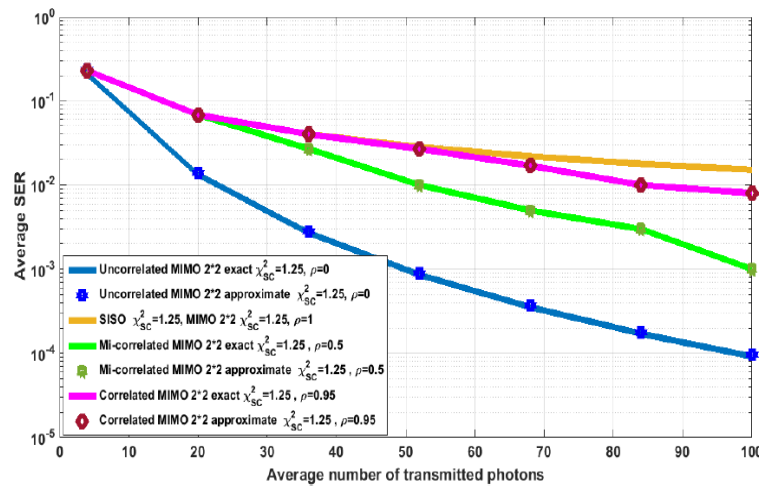


Figure 2. ASER versus ASNR for different correlated MIMO channels parameters under strong turbulence

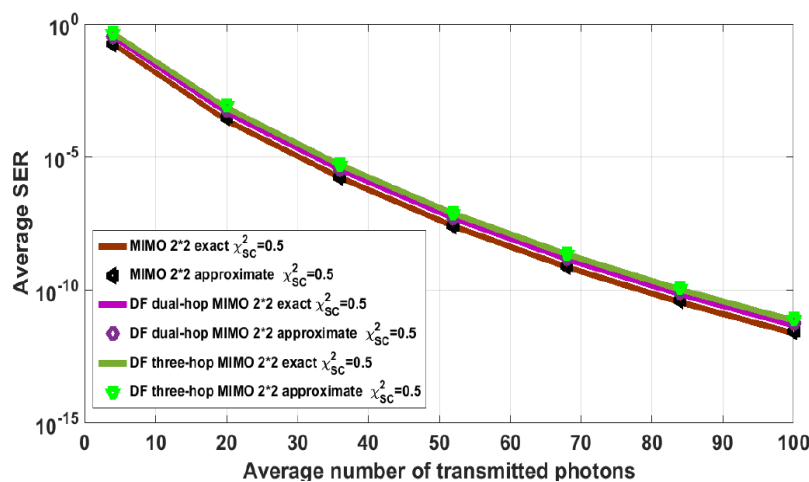


Figure 3. Multihop MIMO-FSO system QoS under moderate turbulence where  $\rho = 0.1$

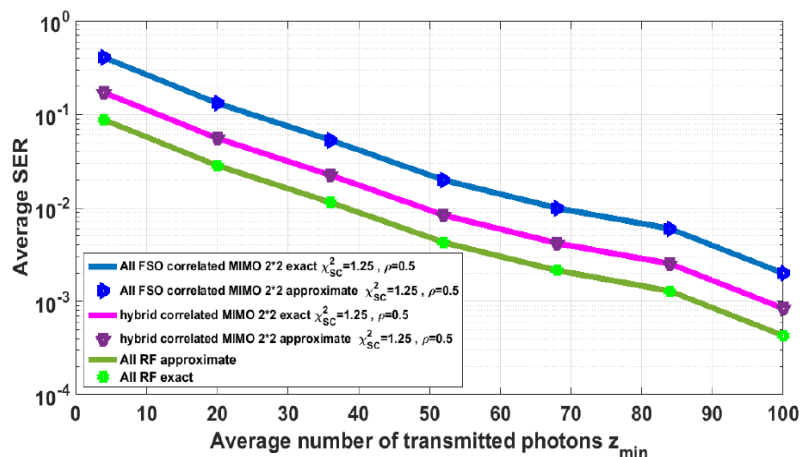


Figure 4. Comparison between multihop DF relaying only MIMO FSO or only RF or hybrid RF/MIMO FSO

#### 4. CONCLUSION

We have formulated a closed-form expression for BER of Hybrid RF/ SO system. Thanks to the use of hybrid configuration, MIMO and MPPM signaling, the effect of many impairments had been combatted. The numerical results helped to corroborate such result. In fact, the great problem for our multihop FSO system is the complexity of serial AF multihop mathematics analysis. It is now required to go deeper into the analysis and into the study of the boost of its performance. Moreover, the considering error pointing effect would be another outlook for this work

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