

Symbol Error Rate Performance Analysis of Decode and Forward Cooperative Communication System

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Abstract

In this paper, we investigate the performance of decode and forward (DF) in multiple-relay networks. We consider a cooperative diversity system where a source sends information to the destination with the aid of multirelay working in DF relaying protocol and investigate the optimal power allocation (OPA) at both the source and the relay nodes. By taking advantage of the moment generating function (MGF), a closed form expression for the symbol error rate (SER) and signal to noise ratio (SNR) for both, M phase shift keying (MPSK) and M quadrature amplitude modulation (MQAM) signals has been derived to illustrate the asymptotic performance of the DF system where the approximation SER is tight at high SNR. Results show that the proposed system, based on SER lower bound is tight to the theoretical SER upper bound, and the suggested OPA outperforms the equal power allocation (EPA) and at different number of relays.

Keywords: Wireless communication, Cooperative communications, Decode and Forward, Symbol error rate, Optimal power allocation.

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1. Introduction

Recently the best proposed technique to design virtual antenna arrays without using collocated multiple antennas is achieved by using cooperative diversity networks. These networks use the neighbor nodes to support the source by sending the source information to the destination for achieving spatial diversity. The Invention of cooperative communications is not limited only to the physical layer. It is presented in various forms at different higher protocol layers [1].

The most common applications of cooperative diversity are cellular and ad-hoc wireless communication systems [2]. The idea of cooperative diversity has been recently introduced to overcome the problem of space limitations in cellular and ad-hoc networks. Multi-hop transmission is a special case of a broader class of transmission protocols, recently which have been receiving significant attention in various communities [3]. In order to transmit information in wireless ad-hoc networks, cooperative diversity is a novel technique proposed for achieving this process. In [4] and [5], the authors proposed several topologies for cooperative ad-hoc networks to realizing the performance. Some researchers are focused on modifying the interference and noise by applying the ad-hoc configuration to the cooperative nodes [6],[7]. Cooperative communication has been considered as a good method to develop communication quality of service (QoS) in a wireless networks, with mobile ad-hoc networks (MANETs) [8]. The authors in [9] suggested a topology which aims to keep the energy paths efficient and decrease power consumption in the network.

In this paper, we first derived a closed form symbol error rate (SER) formulation for M phase shift keying (MPSK) and M quadrature amplitude modulation (MQAM) signals using the moment generating function (MGF) of the received signal to noise ratio (SNR) at the destination, since the SER formulation is too complicated, we then find a tight lower bound which converges to the same limit as the theoretical upper bound in high SNR. Consequently making use of this result, we develop an optimal power allocation (OPA) method to minimize the SER and illustrate

by simulations the performance development of our new scheme compared to the *equal power allocation* (EPA) scheme for different number of relays used in the system.

The rest of the paper is organized as follows. In section 2, we describe the system model and propose a class of cooperation protocols for multi-node wireless networks. In section 3, we analyze the SER performance by using the concept of MGF and obtained two bounds for the exact SER expression. In section 4, we determine the OPA for the tight SER lower bound and explain it with two types of modulation signals: MPSK and MQAM modulation. The simulation results are presented in section 5. Finally section 6 the conclusions derived from the results are stated.

2. System Model

For the DF strategy, the nodes decode each symbol of the message and transmit the decoded symbol over orthogonal channels; relays apply some form of decoding algorithms to their received signals and re-encode the information into their transmitted signals. In a wireless cooperative communication system, shown in Figure1, the source communicates with the destination through N existing relays, using DF relaying. All terminals are assumed to be supplied with single antenna transmitter and receiver.

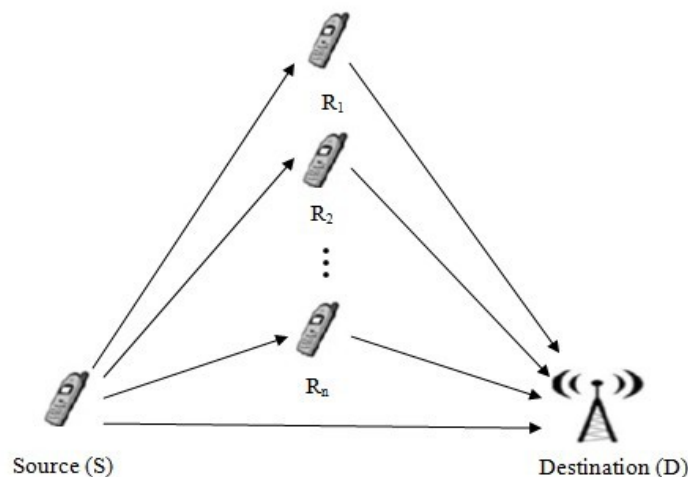


Figure 1. System model with multi relay (R_1, R_2, \dots, R_n)

We suppose that the main channel gains and the channel state information (CSI) are known at the destination. All users transmit signals are through orthogonal channels by using TDMA, FDMA or CDMA scheme.

We divide signal transmission into two phases. In phase 1, the source broadcasts the signal to the destination and to all relay nodes in the network. The received signals at the destination (D) and at the relay nodes (R_i) in the first available time slot are modeled as follows:

$$Y_{SD} = \sqrt{P_S} h_{SD} x + n_{SD} \quad (1)$$

$$Y_{SR_i} = \sqrt{P_S} h_{SR_i} x + n_{SR_i} \quad (2)$$

where x is the transmitted information symbol from the source, Y_{jk} illustrate the received signal from node j to node k , h_{jk} are the fading channel coefficients from node j to node k , and n_{jk} are the corresponding additive white Gaussian noise (AWGN) with variance N_0 from node j to node k .

In phase 2, the source and the relays transmit signal to the destination. Now, the relays correctly decode the received signal. Accordingly, the received signals at the destination terminal are as follows:

$$Y_{R_iD} = \sqrt{P_{R_i}} h_{R_iD} \bar{x} + n_{R_iD} \quad (3)$$

where \bar{x} is the decoded information at the i^{th} relay, P_S and P_{R_i} are the power at the source and at the i^{th} relay respectively. From these expressions, we derive the following relations:

$$\gamma_S = P_S \frac{|h_{SD}|^2}{N_0} \quad (4)$$

$$\gamma_{R_i} = P_{R_i} \frac{|h_{R_iD}|^2}{N_0} \quad (5)$$

where γ_S and γ_{R_i} are the instantaneous SNR of the direct and the i^{th} relay respectively. The variable h_{jk} is defined before, modeled as the independent zero-mean, circularly-symmetric complex Gaussian random variable with variance one. By using the maximum ratio combiner (MRC), the signals from source and i^{th} relay are combined at the destination and the received SNR is:

$$\gamma = \gamma_S + \sum_{i=1}^n \gamma_{R_i} \quad (6)$$

γ_S and γ_{R_i} follow exponential distribution with parameters:

$$\lambda_S = \frac{N_0}{P_S \sigma_{SD}^2} \quad (7)$$

$$\lambda_{R_i} = \frac{N_0}{P_{R_i} \sigma_{R_iD}^2} \quad (8)$$

where σ_{SD}^2 and $\sigma_{R_iD}^2$ are the variance of h_{SD} and h_{R_iD} respectively.

3. SER Performance Analysis

In this section, we analyze the SER performance of multi relay system to evaluate decode and forward transmission, and determine the SER using MPSK and MQAM signals over the Rayleigh fading channel as follow:

A. MPSK signals

The conditional SER with SNR γ is described in [10], for MPSK modulation signal:

$$P_{SER,MPSK} = \frac{1}{\pi} \int_0^{\frac{M-1}{M}\pi} \exp\left(-\frac{b_1 \gamma}{\sin^2 \theta}\right) d\theta \quad (9)$$

By averaging the conditional SER, represented by (9) over the allocation of γ_S and γ_{R_i} , then the unconditional SER for MPSK signals of the proposed system is given as:

$$P_{SER,MPSK} = \frac{1}{\pi} \int_0^{\frac{M-1}{M}\pi} M_{\gamma_S}(-t_1) \prod_{i=1}^n M_{\gamma_{R_i}}(-t_1) d\theta \quad (10)$$

where $t_1 = b_1 / \sin^2 \theta$ with $b_1 = \sin^2(\pi/M)$, $M_{\gamma_S}(-t_1)$ and $M_{\gamma_{R_i}}(-t_1)$ are the MGF of γ_S and γ_{R_i} respectively. Substituting the MGF's expression, then relation (10) can be given as:

$$P_{SER,MPSK} = \frac{1}{\pi} \int_0^{\frac{M-1}{M}\pi} \left(\frac{1}{1+(b_1/\sin^2 \theta)(1/\lambda_S)} \right) \prod_{i=1}^n \left(\frac{1}{1+(b_1/\sin^2 \theta)(1/\lambda_{R_i})} \right) d\theta \quad (11)$$

Equation (11) represents the exact expression for the SER for MPSK of the proposed system. This expression is not pliable in analysis, so we derive a SER lower bound that is converges to the same limit as a theoretical SER upper bound to apply performance analysis. We set up a tight SER lower bound using the truth that $0 \leq \sin^2 \theta \leq 1$ as following:

$$A \left(\frac{1}{1+(b_1/\lambda_S)} \right) \prod_{i=1}^n \left(\frac{1}{1+(b_1/\lambda_{R_i})} \right) \leq P_{SER} \leq A \left(\frac{\lambda_S}{b_1} \right) \prod_{i=1}^n \left(\frac{\lambda_{R_i}}{b_1} \right) \tag{12}$$

where $A = \frac{1}{\pi} \int_0^{\frac{M-1}{M}\pi} \sin^{2n+2}\theta \, d\theta$.

B. MQAM signals

Now we will derive the exact SER for MQAM signals. The conditional SER with SNR γ is written as:

$$P_{SER, MQAM} = \frac{4K}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{b_2\gamma}{2\sin^2\theta}\right) d\theta - \frac{4K^2}{\pi} \int_0^{\frac{\pi}{4}} \exp\left(-\frac{b_2\gamma}{2\sin^2\theta}\right) d\theta \tag{13}$$

With averaging equation (13) over the allocation of γ_S and γ_{R_i} , then the unconditional SER for MQAM signals of the proposed system is:

$$P_{SER, MQAM} = \left[\frac{4K}{\pi} \int_0^{\frac{\pi}{2}} - \frac{4K^2}{\pi} \int_0^{\frac{\pi}{4}} \right] M_{\gamma_S}(-t_2) \prod_{i=1}^n M_{\gamma_{R_i}}(-t_2) \, d\theta \tag{14}$$

where $K = 1/(1 - (1/\sqrt{M}))$, $t_2 = b_2/2\sin^2\theta$ with $b_2 = 3/(M - 1)$, $M_{\gamma_S}(-t_2)$ and $M_{\gamma_{R_i}}(-t_2)$ are the MGF of γ_S and γ_{R_i} respectively. Substituting the MGF's expression, then equation (14) can be written as:

$$P_{SER, MQAM} = \left[\frac{4K}{\pi} \int_0^{\frac{\pi}{2}} - \frac{4K^2}{\pi} \int_0^{\frac{\pi}{4}} \right] \left(\frac{1}{1+(b_2/\sin^2\theta)(1/\lambda_S)} \right) \prod_{i=1}^n \left(\frac{1}{1+(b_2/\sin^2\theta)(1/\lambda_{R_i})} \right) d\theta \tag{15}$$

Equation (15) is the exact expression for the SER for MQAM of the proposed system. As in part A, this expression is not pliable in analysis, so we derive a SER lower bound that converges to the same limit as a theoretical SER upper bound to apply performance analysis. We set up a tight SER lower bound using the truth that $0 \leq \sin^2\theta \leq 1$ as following:

$$B \left(\frac{1}{2+(b_2/\lambda_S)} \right) \prod_{i=1}^n \left(\frac{1}{2+(b_2/\lambda_{R_i})} \right) \leq P_{SER} \leq B \left(\frac{\lambda_S}{b_2} \right) \prod_{i=1}^n \left(\frac{\lambda_{R_i}}{b_2} \right) \tag{16}$$

where $B = \left[\frac{4K}{\pi} \int_0^{\frac{\pi}{2}} - \frac{4K^2}{\pi} \int_0^{\frac{\pi}{4}} \right] (2\sin\theta)^{2n+2} d\theta$, proof sees the appendix.

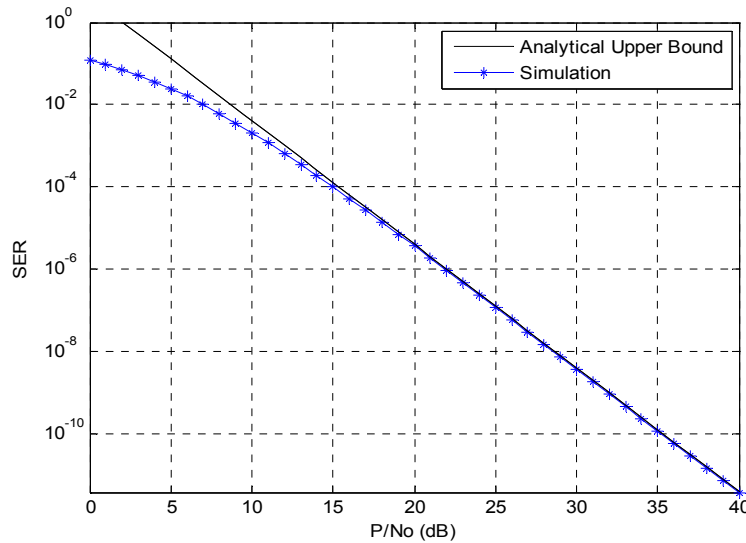


Figure 2. The simulation SER lower bound and analytical upper bound versus SNR for MPSK signal.

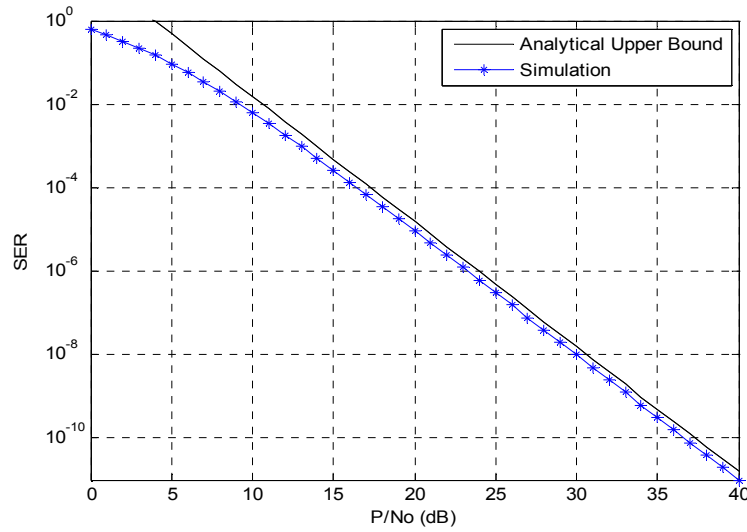


Figure 3. The simulation SER lower bound and analytical upper bound versus SNR for MQAM signal.

Figure 2 and 3 represent the SER obtained from our system in relation with SNR (P/N_0) of DF multi-relay system with MPSK and MQAM signals respectively. In our system, the variances of the channel coefficients are equal to one. Assume: $P_S = P_{R_1} = P_{R_2} = \dots = P_{R_n}$ and $N_0 = 1$. From the two figures, we can observe that the simulation of SER lower bound and analytical upper bound are very close especially in the high SNR regime. In this result, the exact SER expression is bound with two tight bounds that is can be considered as important.

4. Optimal Power Allocation

In this section, we aim to illustrate the asymptotic performance of the system by finding the OPA to the tight SER lower bound. The main idea is that we try to find the OPA for the multi-node system that minimizes the SER lower bound. By using the total fixed transmission power denoted by P as in [11] to find the optimal power at the source P_S and the i^{th} relay P_{R_i} . In the following analysis, we consider the two types of modulation signals: MPSK and MQAM signal.

A. MPSK signals

In the proposed cooperative system, we consider the variance of the noise is unit, by substituting λ_S and λ_{R_i} by their values in the lower bound expression in (12), then we get the following optimization problem for MPSK modulation signal:

$$\begin{cases} \min_{(P_S, \{P_{R_i}\}_{i=1}^n)} \left(A \frac{1}{1+bP_S} \prod_{i=1}^n \frac{1}{1+bP_{R_i}} \right) \\ P_S + \sum_{i=1}^n P_{R_i} = P \end{cases} \quad (17)$$

After applying lagrange multiplier approach into formula (17), and setting $x_i = P_{R_i}/P_S$, then the next function is formed:

$$G(P_S, x_i, \lambda) = A \left(\frac{1}{1+bP_S} \right) \prod_{i=1}^n \left(\frac{1}{1+bP_S x_i} \right) - \lambda \left(1 + \sum_{i=1}^n x_i - \frac{P}{P_S} \right) \quad (18)$$

We derive the following functions by employing of the logarithm function in (18):

$$\partial G / \partial P_S = A \left(\frac{-b}{1+bP_S} - \sum_{i=1}^n \frac{bx_i}{1+bP_S x_i} \right) - \lambda \frac{P}{P_S^2} = 0 \quad (19)$$

$$\partial G / \partial x_i = A \left(\sum_{i=1}^n \frac{-bP_S}{1+bP_S x_i} \right) - \lambda = 0 \quad (20)$$

$$\partial G/\partial \lambda = -\left(1 + \sum_{i=1}^n x_i - \frac{P}{P_S}\right) = 0 \quad (21)$$

Accordingly we have to solve the formation in (19), (20) and (21). Assume all relays have the same power, that $x_1 = x_2 = \dots = x_n = x$, and verify the equation as:

$$N^3(N-1)x^3 + (3N^2(N-1) + NbP(N^2 - N - 1))x^2 + (3N(N-1) + NP(2N-b) - bP)x + (N-1) + NbP = 0 \quad (22)$$

B. MQAM signals

In the proposed cooperative system, we consider the variance of the noise is unit, by substituting λ_S and λ_{R_i} by their values in the lower bound expression in (16), then we get the following optimization problem for MQAM modulation signal:

$$\begin{cases} \min_{(P_S, \{P_{R_i}\}_{i=1}^n)} \left(B \frac{1}{2+bP_S} \prod_{i=1}^n \frac{1}{2+bP_{R_i}} \right) \\ P_S + \sum_{i=1}^n P_{R_i} = P \end{cases} \quad (23)$$

With applying lagrange multiplier approach into equation (23), then we get:

$$G(P_S, x_i, \lambda) = B \left(\frac{1}{2+bP_S} \right) \prod_{i=1}^n \left(\frac{1}{2+bP_S x_i} \right) - \lambda \left(1 + \sum_{i=1}^n x_i - \frac{P}{P_S} \right) \quad (24)$$

Then with employing of the logarithm function in (24) we derive the following relations:

$$\partial G/\partial P_S = B \left(\frac{-b}{2+bP_S} - \sum_{i=1}^n \frac{bx_i}{2+bP_S x_i} \right) - \lambda \frac{P}{P_S^2} = 0 \quad (25)$$

$$\partial G/\partial x_i = B \left(\sum_{i=1}^n \frac{-bP_S}{2+bP_S x_i} \right) - \lambda = 0 \quad (26)$$

$$\partial G/\partial \lambda = -\left(1 + \sum_{i=1}^n x_i - \frac{P}{P_S}\right) = 0 \quad (27)$$

After solving the above relations, then we get:

$$2N^3(N-1)x^3 + (6N^2(N-1) + NbP(N^2 - N - 1))x^2 + (6N(N-1) + bP(2N^2 - N - 1))x + 2(N-1) + NbP = 0 \quad (28)$$

Finally, by solving equations (22) and (28) by matlab and making use of (21) and (27), we have the optimum power values with multi relay system for both MPSK and MQAM modulation signals:

$$P_S = P/(1 + nx) \quad (29)$$

By substituting the relation of P_S , P_{R_i} and x_i , we have:

$$P_{R_1} = P_{R_2} = \dots = P_{R_n} = xP/(1 + nx) \quad (30)$$

5. Simulation Results

In this section, we represent the OPA results using MPSK and MQAM modulation signals to authenticate the mathematical terms in section 4. In both types of modulations, we show the tightness of the analytical expression along with the simulation curves at high SNR. This behavior is in accordance with the fact that at adequately high SNR, the obtained lower and upper bounds converge to the same limit as declared in the previous analytical results.

We compare the SER performance of our proposed OPA technique (P_S and P_{R_i} that are substituted by equations (29) and (30) respectively) with respect to EPA ($P_S = P_{R_i} = P/(n+1)$) at different number of relays in relation to the SNR. From figure 4 and 5, it is clear that the

proposed OPA obtains better performance than the EPA method due to the performance going parallel along the theoretical SER upper bound at different numbers of relays.

According to the obtained results, we can see that the behavior of our proposed OPA is the same when we use one relay, but when we increase the number of relays, the performance become better than EPA especially when we use number of relays equal 3, and that is the reason that prompt us to suggest use of multi-relay in our proposed model.

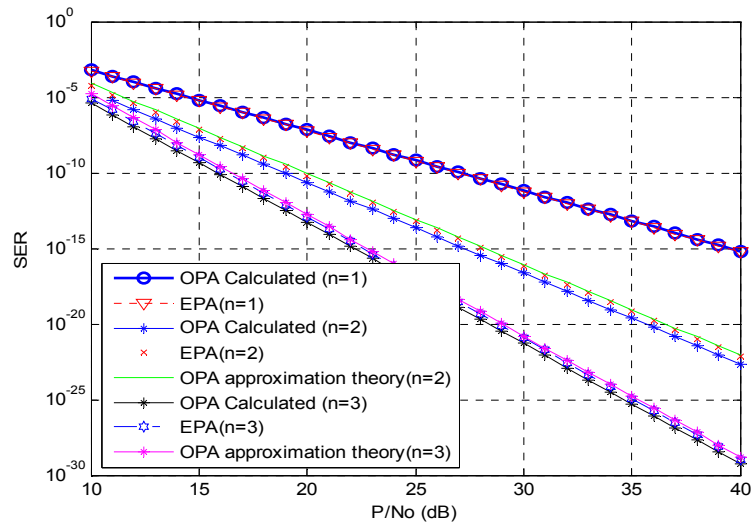


Figure 4. SER comparison between suggested OPA and EPA for MPSK modulation.

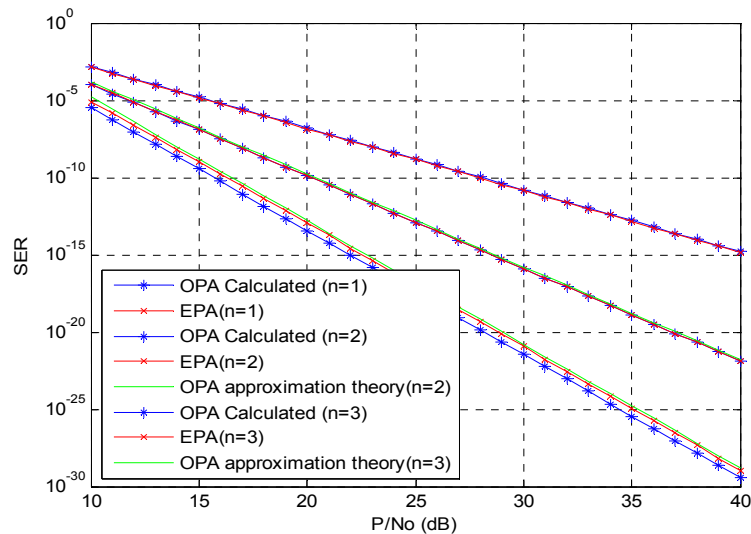


Figure 5. SER comparison between suggested OPA and EPA for MQAM modulation.

6. Conclusion

This paper presents a structure for enhancing the SER performance of DF multi-relay cooperative transmission over Rayleigh fading channels. We demonstrate that the SER performance can be significantly improved by the proper relay strategy. We derive a closed expression for the SER, by using the concept of MGF for MPSK and MQAM modulation signals and compare it with theoretical SER upper bound. An OPA scheme is investigated to minimize the SER. It can be seen from the simulation results how our proposed OPA outperforms the EPA at different number of relays.

Appendix: proof of relation (16)

As we have $0 \leq \sin^2 \theta \leq 1$, we can derive the following inequalities for MQAM signals:

$$\begin{aligned} 0 \leq 2\sin^2 \theta \leq 2 &\implies \frac{b_2}{\lambda_S} \leq 2\sin^2 \theta + \frac{b_2}{\lambda_S} \leq 2 + \frac{b_2}{\lambda_S} \\ &\implies \frac{2\sin^2 \theta}{2 + \frac{b_2}{\lambda_S}} \leq \frac{2\sin^2 \theta}{2\sin^2 \theta + \frac{b_2}{\lambda_S}} \leq \frac{2\sin^2 \theta}{\frac{b_2}{\lambda_S}} \end{aligned} \quad (31)$$

$$\prod_{i=1}^n \left(\frac{2\sin^2 \theta}{2 + \frac{b_2}{\lambda_{R_i}}} \right) \leq \prod_{i=1}^n \left(\frac{2\sin^2 \theta}{2\sin^2 \theta + \frac{b_2}{\lambda_{R_i}}} \right) \leq \prod_{i=1}^n \left(\frac{2\sin^2 \theta}{\frac{b_2}{\lambda_{R_i}}} \right) \quad (32)$$

Multiplying (31) and (32), then we obtain:

$$\frac{2\sin^2 \theta}{2 + \frac{b_2}{\lambda_S}} \prod_{i=1}^n \left(\frac{2\sin^2 \theta}{2 + \frac{b_2}{\lambda_{R_i}}} \right) \leq \frac{2\sin^2 \theta}{2\sin^2 \theta + \frac{b_2}{\lambda_S}} \prod_{i=1}^n \left(\frac{2\sin^2 \theta}{2\sin^2 \theta + \frac{b_2}{\lambda_{R_i}}} \right) \leq \frac{2\sin^2 \theta}{\frac{b_2}{\lambda_S}} \prod_{i=1}^n \left(\frac{2\sin^2 \theta}{\frac{b_2}{\lambda_{R_i}}} \right) \quad (33)$$

Now subsequently we take the integral of (33), then we prove the relation (16). We can use the same as the above procedure for MPSK signals.

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