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The Adaptive Federal Unscented Particle Filter Algorithm with Applications in All-Attitude Integrated BDS/INS Navigation System

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Abstract

In order to improve the attitude accuracy, the thesis establishes the all-attitude integrated BDS/INS navigation nonlinear system model based on the position, velocity, attitude by adding the BDS's attitude measurement information into the measurement equation of the traditional BDS/INS integrated navigation nonlinear system model. Considering the problem that the dynamic navigation system model is difficult to accurately describe the complex navigation environment, the thesis improves the dynamic characteristics of the information distribution of the federal filter algorithm which could timely change based on the eigenvalues ratio of each subsystem's error variance matrix. Then, the adaptive federal unscented particle filter (AFUPF) is proposed. The simulation shows that the proposed algorithm could effectively weaken the impact on the system accuracy of the inaccurate high-dynamic model, and improve the adaptability, the fault tolerance and the accuracy, especially the attitude accuracy.

***Keywords**: BDS/INS integrated navigation system, attitude update, adaptive information distribution, AFUPF*

1. Introduction

The BDS/INS integrated navigation system has become an important development direction of the navigation system, which could overcome the shortcomings of each other, and have higher navigation accuracy than each system to work alone. With the continuous development of satellite attitude determination technology, BDS not only provides the position information and velocity information, but also gives accuracy attitude information [1]**Error! Reference source not found.** for the carrier. The fully integrated BDS/INS navigation system which reflected the BDS attitude information in the integrated navigation system measurement model has become the feasible solutions to improve the accuracy of attitude, which further increases the observability of the model [2].

In practical applications, the integrated navigation system typically uses indirectly kalman filter (KF) algorithm to simplify procedures and improve performance, because the system's error is studied and modeled in advance. At present, the domestic and foreign experts also study the applications of the indirectly KF in the BDS/INS integrated navigation system. However, the BDS's pseudo range-error is worst and the model isn't accurate so that it does not meet the basic premise of the indirect KF, thus affects the filter accuracy of the INS's data integration. The thesis gets the nonlinear model of the all-attitude integrated BDS/INS navigation system [3-5] by the direct method [6]: establishes the state equation based on the mechanics choreography equations and attitude error equation in INS; establishes the measurement equation based on the position message, velocity message of the BDS and the attitude error message of the BDS/INS.

The information fusion has two ways: centralized filter and distributed filter. The distributed federal kalman filter (FKF) based on the KF and the information distribution technology has been taken seriously because of the parallel data processing, flexible design, small amount of calculation, better fault-tolerant performance.

However, the use of the subfilter of the FKF must meet certain ideal conditions:
1. Dynamic model of the system is accurate and linear;

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2. Noise is unrelated to the white noise and has the known statistical properties;
3. Choose the appropriate initial value of the state variables and the noise variance matrix in order to ensure the unbiased estimation results and prevent the filter divergence.

Therefore, the unscented particle filter (UPF) [7, 8] is used in subfilter and the federal unscented particle filter (FUPF) [9] is constituted in the thesis. The UPF is the non-linear non-gaussian filter algorithm and has been used in many fields such as navigation system and target tracking. The UPF directly estimates the parameters of the navigation system, avoides the linearization of the nonlinear state equation and ensures the high precision of the navigation system.

After analysis, it is not difficult to find that the information distribution has always been the key to designing and achieving the FKF, the different values have different federal filter characteristics. In actual application, due to the change of the system environment or other factors, the performance of each subsystem of the integrated navigation system may also change. The predefined information distribution coefficient remaining unchanged during filtering does not accurately reflect the actual situation and affects the estimated accuracy of the system. Meanwhile, taking into account that each component of the state vector has the different observability and convergence, the feedback considering the state variables as a whole is extremely unreasonable.

Because the variance matrix of each subfilter of the FKF could reflect the state estimation accuracy in real-time. Based on the point, a new adaptive federal unscented particle filter (AFUPF) is proposed in the thesis, whose information distribution coefficien could *timely chang based on the eigenvalues ratio of each subsystem's error variance matrix and better satisfy the dynamic state changes under high-dynamic environment so that* improve the estimation accuracy of the federal filter.

The rest of the thesis is organized as follows: In Section 2, the establishment of the all-attitude integrated BDS/INS navigation system model is described. In Section 3, the AFUPF algorithm is discussed. In Section 4, experiments results and analysis are put. Finally, in Section 5, conclusions of the work are made.

2. Establishment of the All-attitude Integrated BDS/INS Navigation System Model

2.1. The Composition of the BDS/INS Integrated Navigation System

In the thesis, the composition of the BDS/INS integrated navigation system is shown in Figure 1. The information fusion system has two navigation sensors which are BDS and INS as the common reference system, which would constitute three subfilters, namely: the position subfilter, the velocity subfilter and the attitude subfilter [10-12] and the main filter. The three subfilters use the UPF algorithm, and their information is sent to the main filter to have the fusion in order to get the navigation parameters and the best attitude estimation as the output of the navigation system.

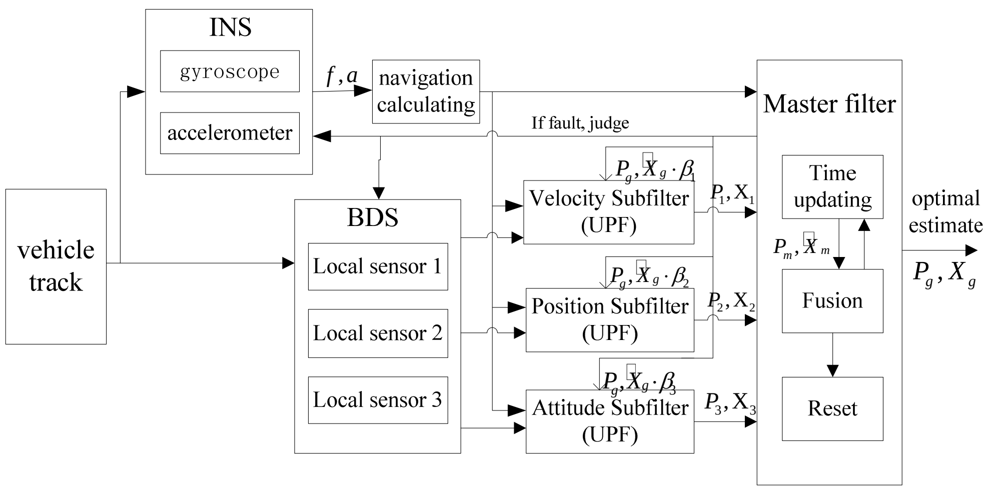


Figure 1. Structure of all-attitude integrated BDS/INS navigation system

2.2. The Model of the BDS/INS Navigation System

The all-attitude integrated BDS/INS navigation system model established by the direct method is adopted in the thesis, which considers the navigation parameters of the INS as the state variable, establishes the measurement equation based on the position message, velocity message of the BDS and the attitude error message of the BDS/INS, and deals with the modified AFUPF in order to get the navigation parameter estimates as the output of the BDS/INS integrated navigation system.

2.2.1. The State Equation of the Integrated Navigation System

The thesis considers the navigation coordinate system to be the geographical coordinate system, establishes the state equation based on the mechanics choreography equations and the INS's attitude error equation considering that the existence of the inclination error of the platform could result in other axial generating specific force components according to the literature [6], and establishes the random constant model for the accelerometer's offset and gyroscope's offset. Therefore, the state variables used in simulation are:

$$x = \left[v_e \; v_n \; v_u \; \lambda \; \phi \; h \; \varphi_e \; \varphi_n \; \varphi_u \; a_e \; a_n \; a_u \; d_e \; d_n \; d_u \right]^T \tag{1}$$

Where, the states are the velocity, position, platform attitude error, accelerometer's biases and gyroscope's range drift.

The state equation of the BDS/INS integrated navigation system is:

$$\dot{x}(t) = F\left[x(t), u(t) \right] \tag{2}$$

Discretize formula (2) with the fourth-order Runge-Kutta method:

$$x_{k+1} = f\left(x_k, u_k\right) \tag{3}$$

Where, $f(\ )$ is the nonlinear function; u_k is the noise variables of system; variance matrix of the system noise is Q_k.

2.2.2. The Observation Equation of the Integrated Navigation System

The velocity for the east (x), north (y), and altitude (z), positions of the BDS and the difference of the attitude between the INS and BDS are selected as the observed variables, namely:

$$y_k = [\upsilon_{EB} \; \upsilon_{NB} \; \upsilon_{UB} \; l_B \; \lambda_B \; h_B \; \delta\theta \; \delta\gamma \; \delta\psi]^T$$

The angle-error of the state equation of the integrated navigation system is the mathematical platform misalignment angle which describes the relationship between the mathematical platform coordinate system and the geographical coordinate system, but the difference of the observed attitude between the INS and BDS describes the relationship between the carrier coordinate system and the geographic coordinate system. In order to make them unified in the physical sense, there should be essentially the relationship between them, namely $C_p^b = C_t^b C_P^t$. Where, p is the platform coordinate system, b is the carrier coordinate system, t is the geographical coordinate system. Based on the relationship, we could obtain an attitude subfilter [11, 12] of the federal filter, whose observation matrix is:

$$h_3 = \frac{1}{\cos\theta} \begin{bmatrix} -\cos\theta\cos\psi & \cos\theta\sin\psi & 0 \\ -\sin\psi & -\cos\psi & 0 \\ -\sin\theta\sin\psi & -\sin\theta\cos\psi & -\cos\theta \end{bmatrix} \tag{4}$$

In the way, the attitude measurement information which comes from the multi-antenna Beidou receiver on the basis of carrier phase difference technology is entered into the observation equation of the traditional BDS/INS integrated navigation nonlinear system model, the purpose of which is to correct the mathematical model error of the BDS/INS integrated navigation system and improve the attitude accuracy.

The observation equation of the integrated navigation system is:

$$y_{k+1} = h\left(x_{k+1}, \upsilon_{k+1}\right) = \begin{bmatrix} h_1 & 0_{3\times3} & 0_{3\times3} & 0_{3\times6} \\ 0_{3\times3} & h_2 & 0_{3\times3} & 0_{3\times6} \\ 0_{3\times3} & 0_{3\times3} & h_3 & 0_{3\times6} \end{bmatrix} x_{k+1} + \upsilon_{k+1} \tag{5}$$

Where, $h_1 = diag(1\ 1\ 1)$; $h_2 = diag\left(1\ 1\ 1\right)$; y_{k+1} is the observed variable; $h(\cdot)$ is linear function; x_k is the state vector value at the k time; v_{k+1} is observation noise, the variance matrix of the measurement noise is R_k.

3. The AFUPF Algorithm

3.1. The Unscented Particle Filter

In the BDS/INS integrated navigation system, the particle filter (PF) [13] could solve the nonlinear nongaussian problem. The PF relies on the importance sampling and the proposal distribution which could approximate the posterior distribution reasonably. Because it is hard to design such proposal distribution so that the PF generally selectes the priori distribution as the proposal distribution, which would ignore the observation information of the current moment, and make the state estimation seriously depend on the system's model. If the model is not accurate or the measurement noise increases suddenly, the way could not effectively represent the real distribution of the probability density function. Meanwhile, under the distribution, the weights are calculated without considering the influence of the model noise, which would affect the filtering accuracy.

Therefore, R. Merwe proposes the UPF [7] which uses the UKF approximation to generate the proposal distribution for the PF. But the UPF does not satisfy the fault-tolerant requirement of BDS/INS system.

3.2. The Principle of the Federal Kalman Filter

The federal kalman filter [14] based on the theory of the decentralized filter is a kind of two-stage filter structure, whose basic idea is to parallel deal with each subfilter and obtain their estimation $\hat{x}_i$ and variance matrix $P_i\left(i=1,\cdots,nn\right)$, then send them into the main filter to get the periodic fusion processing and obtain the global optimal estimation $\hat{x}_g$ and the variance matrix P_g, and then send the $\hat{x}_g$ and the enlarged variance matrix $\beta_i^{-1}\cdot P_g$ into each subfilter to reset each subfilter, that is:

$$\begin{cases} P_i\left(k\right) = \beta_i^{-1} * P_g\left(k\right) \\ Q_i\left(k\right) = \beta_i^{-1} * Q_g\left(k\right) \\ \hat{x}_i\left(k\right) = \hat{x}_g\left(k\right) \\ \sum_{i=1}^{n} \beta_i = 1 \end{cases} \tag{6}$$

Each subfilter correctes the time and the observation$\left(y_i\left(k+1\right)\right)$ and gets the estimation $\hat{x}_i\left(k+1\right)$ and the variance matrix $P_i\left(k+1\right)$. According to the following formula, we can obtain the global optimal estimation and the variance matrix of the main filter.

$$\begin{cases} P_g^{-1}(k+1) = \sum_{i=1}^{n} P_i^{-1}(k+1) \\ \hat{x}_g(k+1) = P_g(k+1)\sum_{i=1}^{n} P_i^{-1}(\text{k+1})\cdot \hat{x}_i(k+1) \end{cases} \quad (7)$$

Where, $x(k)$ is the state variables of system; $y(k+1)$ is the observed variable; $P(k)$ is the mean square error matrix (MSE) for the k time; $Q(k)$ is the noise matrix for the k time; β_i is the information distribution coefficient; i means the ith subfilter; g means the main filter.

3.3. The Information Distribution Method of the Federal Filter Algorithm

The key to research and design the federal filter is to determine the information distribution coefficient [14, 15] which directly affects the precision and the fault tolerance performance. After a lot of simulation, the global estimation of the federal filter has the best accuracy when the information distribution coefficient is $\beta_m = 0, \beta_i = 1/nn, (i = 1, 2, \cdots, nn)$.

But in the actual high dynamic navigation environment, the performance and the estimated quality of each subfilter is constantly changing. Therefore, it is hoped that the overall performance of the filter can always be close to the optimal subsystem, in other words, the information distribution coefficient can follow the performance of the subfilter to make timely changes [15].

According to the information distribution way, the essence of the federal filter algorithm [14] is to magnify the mean square error matrix (MSE) β_i^{-1} times, and assign them to the different subfilters. The KF could automatically make the use of different weights according to the merits of the quality of information, the higher the information quality of the subfilter is, the smaller the information distribution coefficient is, and then the higher the utilization of the fusion information of the subfilter is.

We could determine the information distribution coefficient of each subfilter according to the variance matrix which contains the estimated error message. According to this method, the higher the precision of the filter is, the smaller the information distribution coefficient is, which is equivalent to expanding the role of sensor with higher accuracy. The adaptive federal filter algorithm proposed in the thesis is as follows:

Schur decomposition of P_i.

$$P_i = U\Lambda_i U^T \quad (8)$$

Therefore, the information distribution coefficient of the i subfilter is:

$$\begin{cases} \beta_{ij} = \dfrac{1}{\lambda_{i,j}} \Big/ \sum_{i=1}^{nn} \dfrac{1}{\lambda_{i,j}} \\ \beta_i = \begin{bmatrix} \beta_{i1} & & & \\ & \beta_{i2} & & \\ & & \ddots & \\ & & & \beta_{imm} \end{bmatrix} = diag\{\beta_{i1}, \beta_{i2}, \cdots, \beta_{imm}\} \end{cases} \quad (i = 1, \cdots, \text{nn}; j = 1, \cdots, \text{mm}) \quad (9)$$

Where, $\Lambda_i = diag(\lambda_{i,1}, \lambda_{i,2}, \cdots, \lambda_{i,mm})$, $\lambda_{i,1}, \lambda_{i,2}, \cdots, \lambda_{i,mm}$ is the eigenvalues of P_i; U is the unitary matrix; nn is the total number of the subfilters, mm is the dimension of the variance matrix of the subfilter, β_i is the assignment matrix, which is determined by the proportion of each respective component $\lambda_{i,j}$ on the diagonal of the variance matrix of each subfilter to the sum of each respective component $\sum_{i=1}^{nn}\lambda_{i,j}$ on the diagonal of the variance matrix of each subfilter, which is used to make the j component of the i subfilter have its own information distribution coefficient β_{ij}, avoid treating the features, the estimation accuracy and the convergence velocity of each component in the same way, thereby amplifying the same multiples so as to result in the pathological cases, and improve calculation accuracy.

Therefore:

$$P_i(k)=\beta_i^{-1}P_g(k)=\begin{bmatrix}\beta_{i1}^{-1} & & & \\ & \beta_{i2}^{-1} & & \\ & & \ddots & \\ & & & \beta_{imm}^{-1}\end{bmatrix}\begin{bmatrix}P_{g11} & P_{g12} & \cdots & P_{g1mm} \\ P_{g21} & P_{g22} & \ddots & P_{g2mm} \\ \vdots & \vdots & \ddots & \vdots \\ P_{gmm1} & P_{gmm2} & \cdots & P_{gmmmm}\end{bmatrix}$$
$$=\begin{bmatrix}\beta_{i1}^{-1}P_{g11} & \beta_{i1}^{-1}P_{g12} & \cdots & \beta_{i1}^{-1}P_{g1mm} \\ \beta_{i2}^{-1}P_{g21} & \beta_{i2}^{-1}P_{g22} & \ddots & \beta_{i2}^{-1}P_{g2mm} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{imm}^{-1}P_{gmm1} & \beta_{imm}^{-1}P_{gmm2} & \cdots & \beta_{imm}^{-1}P_{gmmmm}\end{bmatrix} \tag{10}$$

Where, known that $\beta_{i1}^{-1}\neq\beta_{i2}^{-1}$ and $P_g(k)=P_g^T(k)$, we could know $P_i(k)\neq P_i^T(k)$, which means that the variance matrix does not satisfy the requirement of the symmetry, which could destroy the convergence and stability of the filter, resulting in lower estimation accuracy, even divergent, losing the true meaning of the algorithm. Therefore, in order to ensure the symmetry of the variance matrix of the subfilter of the FKF, the paper improves information distribution method described above.

First, split the information distribution coefficient. Set up:

$$B_i=\begin{bmatrix}\sqrt{\beta_{i1}} & & & \\ & \sqrt{\beta_{i2}} & & \\ & & \ddots & \\ & & & \sqrt{\beta_{imm}}\end{bmatrix} \tag{11}$$

Therefore:

$$\beta_i=\begin{bmatrix}\beta_{i1} & & & \\ & \beta_{i2} & & \\ & & \ddots & \\ & & & \beta_{imm}\end{bmatrix}=B_i\cdot B_i=\begin{bmatrix}\sqrt{\beta_{i1}} & & & \\ & \sqrt{\beta_{i2}} & & \\ & & \ddots & \\ & & & \sqrt{\beta_{imm}}\end{bmatrix}\cdot\begin{bmatrix}\sqrt{\beta_{i1}} & & & \\ & \sqrt{\beta_{i2}} & & \\ & & \ddots & \\ & & & \sqrt{\beta_{imm}}\end{bmatrix} \tag{12}$$

Get the new formula, that is:

$$P_i^{-1}(k)=B_iP_g^{-1}(k)\cdot B_i=\begin{bmatrix}\sqrt{\beta_{i1}} & & & \\ & \sqrt{\beta_{i2}} & & \\ & & \ddots & \\ & & & \sqrt{\beta_{imm}}\end{bmatrix}\cdot P_g^{-1}(k)\cdot\begin{bmatrix}\sqrt{\beta_{i1}} & & & \\ & \sqrt{\beta_{i2}} & & \\ & & \ddots & \\ & & & \sqrt{\beta_{imm}}\end{bmatrix} \tag{13}$$

Thus:

$$P_i(k) = P_i^{-1}(k)^{-1} = B_i^{-1} \cdot P_g(k) \cdot B_i^{-1}$$
$$= \begin{bmatrix} P_{g11}/\sqrt{\beta_{i1}\beta_{i1}} & P_{g12}/\sqrt{\beta_{i1}\beta_{i2}} & \cdots & P_{g1mm}/\sqrt{\beta_{i1}\beta_{imm}} \\ P_{g21}/\sqrt{\beta_{i2}\beta_{i1}} & P_{g22}/\sqrt{\beta_{i2}\beta_{i2}} & \ddots & P_{g2mm}/\sqrt{\beta_{i2}\beta_{imm}} \\ \vdots & \vdots & \ddots & \vdots \\ P_{gmm1}/\sqrt{\beta_{imm}\beta_{i1}} & P_{gmm2}/\sqrt{\beta_{imm}\beta_{i2}} & \ddots & P_{gmmmm}/\sqrt{\beta_{imm}\beta_{imm}} \end{bmatrix} \tag{14}$$

Where, $P_i(k) = P_i^T(k)$, which means that the variance matrix satisfies the requirement of symmetry. Therefor, we can get the adaptive information distribution coefficient expressed in vector form. With the adaptive allocation strategy, we could reset the state estimation $\hat{x}_i(k)$, the variance matrix $P_i(k)$, and the variance matrix of the process noise $Q_i(k)$ for each subfilter in accordance with the formula (15).

$$\begin{cases} P_i^{-1}(k) = \sqrt{\beta_i} * P_g^{-1}(k) * \sqrt{\beta_i} \\ Q_i^{-1}(k) = \sqrt{\beta_i} * Q_g^{-1}(k) * \sqrt{\beta_i} \\ \hat{x}_i(k) = \hat{x}_g(k) \\ \sum_{i=1}^{n} \beta_i = 1 \end{cases} \tag{15}$$

3.4. The Improved Adaptive Federal UPF Algorithm

In this work, the AFUPF is proposed to overcome the FKF's information distribution limitation and satisfy the fault-tolerant requirement of the BDS/INS system. Figure 1 shows the structure of the AFUPF algorithm. Based on the all-attitude integrated BDS/INS navigation system completed with the nonlinear state Equation (3) and linear observation equation (5) of the all-attitude integrated BDS/INS navigation system, the improved AFUPF algorithm can be adopted to solve the problem of information fusion, which could be concluded as follows:

(1) Initialize the state parameters: x_{g0}, P_{g0}, Q_{g0}, β_{n0} ,and distribute them for each subfilter according to the formula (6).

For $n = 1, \cdots, nn,$

For $i = 1, \cdots, \mathrm{N},$

Sample the i th particle from the prior $p(x_0)$ in the n th subfilter and set up:

$$\hat{x}_{0|0} = [x_k^T \ 0 \ 0]^T \ , \ P_{0|0} = \begin{bmatrix} P_k & 0 & 0 \\ 0 & Q_k & 0 \\ 0 & 0 & R_k \end{bmatrix} \tag{16}$$

Compute $(2n_x + 1)$ sigma points and their weights:

$$x_0 = \hat{x}_{0|0}, P_0 = P_{0|0}$$

$$x_k^i = \left[x_0 , \ x_0 + \left(\sqrt{(n_x + \lambda) P_0^i} \right)_j , x_0 - \left(\sqrt{(n_x + \lambda) P_0^i} \right)_j \right] \tag{17}$$

$$\begin{cases} W_0^m = \dfrac{\lambda}{(n_x + \lambda)}, j = 0 \\ W_0^c = \dfrac{\lambda}{(n_x + \lambda)} + (1 - \alpha^2 + \beta), j = 0 \\ W_j^c = W_j^m = W_j = 1/2(n + \lambda), j = 1, 2, \cdots 2n_x + 1 \end{cases} \tag{18}$$

Where, nn is the number of the subfilters, and N is the number of the sampled particles, n_x is the dimension of state vector.

(2) Calculated using the UPF [9] for each subfilter at the $k = 1, \cdots, TT$ time.

Importance sampling step of the UPF in the n th subfilter at the $k = 1, \cdots, TT$ time.

For $k = 1, \cdots, TT$,

For $n = 1, \cdots, nn$,

For $i = 1, \cdots, \mathrm{N}$, update the particles with the UKF [16-18] in the n th subfilter in time k .

(a) Calculating sigma points

$$x_{k-1}^i = \left[\hat{x}_{k-1}^i , \hat{x}_{k-1}^i + \left(\sqrt{(n + \lambda) P_{k-1}^i} \right)_j , \hat{x}_{k-1}^i - \left(\sqrt{(n + \lambda) P_{k-1}^i} \right)_j \right] \tag{19}$$

(b) Updating time

$$x_{j,k/k-1}^i = f\left(x_{j,k-1}^i \right) \tag{20}$$

$$\begin{cases} \overline{x}_{k/k-1}^i = \sum_{j=0}^{2n} W_j^m x_{j,k/k-1}^i \\ P_{k|k-1} = \sum_{j=0}^{2n} W_j^c \left[x_{j,k/k-1}^i - \overline{x}_{k/k-1}^i \right] \cdot \left[x_{j,k/k-1}^i - \overline{x}_{k/k-1}^i \right]^T + Q_{k-1} \end{cases} \tag{21}$$

$$z_{j,k/k-1}^i = h\left(x_{j,k|k-1}^i \right) \tag{22}$$

$$\overline{z}_{k/k-1}^i = \sum_{j=0}^{2n} W_j^m z_{j,k/k-1}^i \tag{23}$$

(c) Updating measurement with latest observation

$$\begin{cases} P_{z_k z_k} = \sum_{j=0}^{2n} W_j^c \left[z_{j,k/k-1}^i - \overline{z}_{k/k-1}^i \right] \cdot \left[z_{j,k/k-1}^i - \overline{z}_{k/k-1}^i \right]^T + R_k \\ P_{x_k z_k} = \sum_{j=0}^{2n} W_j^c \left[x_{j,k/k-1}^i - \overline{x}_{k/k-1}^i \right] \cdot \left[z_{j,k/k-1}^i - \overline{z}_{k/k-1}^i \right]^T \end{cases} \tag{24}$$

$$K_k = P_{x_k z_k} P_{z_k z_k}^{-1} \tag{25}$$

$$\hat{x}_k^i = \overline{x}_{k/k-1}^i + K_k \left(z_k - \overline{z}_{k/k-1}^i \right) \tag{26}$$

$$P_k^i = P_{k|k-1}^i - K_k P_{z_k z_k} K_k^T \tag{27}$$

(d) Sampling
Sample:

$$q\left(x_k^i / x_{k-1}^i, z_k \right) = N\left(\hat{x}_k^i, P_k^i \right) \tag{28}$$

For $i = 1, \cdots, \mathrm{N},$ evaluate and normalize the importance weights:

$$w_k^i = w_{k-1}^i \frac{p\left(z_k \left| x_k^i \right. \right) p\left(x_k^i \left| x_{k-1}^i \right. \right)}{q\left(x_k^i \left| x_{k-1}^i, z_k \right. \right)} \quad , \quad \tilde{w}_k^i = w_k^i \Big/ \sum_{i=0}^{n} w_k^i \tag{29}$$

(3) Resampling of the subfilter

(a) Evaluating N_{eff}

$$N_{eff} = 1 \Big/ \sum_{i=1}^{N} \left(\tilde{w}_k^i \right)^2 \tag{30}$$

(b) Residual resampling when $N_{eff} < N_{th}$, for $i = 1, \cdots, N$, set:

$$\left\{ x_k^i, \omega_k^i \right\}_{i=1}^{N} \rightarrow \left\{ x_k^i, 1/N \right\}_{i=1}^{N} \tag{31}$$

(c) Subfilter's output
The posterior distribution is approximated as follows.

$$p\left(x_k \left| z_{1:k} \right. \right) = \sum_{i=1}^{N} \omega_k^i \delta\left(x_k - x_k^i \right) \tag{32}$$

One obtaines the following estimate as:

$$\hat{x}_k = \sum_{i=1}^{N} \omega_k^i x_k^i \tag{33}$$

(4) Adaptive information distribution matrix and total output

Fusing results of all subfilters to generate the adaptive information distribution matrix of each subfilter according to the formula (9) and the final total estimation result according to the formula (7).

4. Res Simulation results

In this section the numerical results obtained by applying simulated data to the filters are presented. There are three different cases used in the testing in order to verify the performanceof the improved AFUPF algorithm with the all-attitude integrated BDS/INS navigation system, which are:

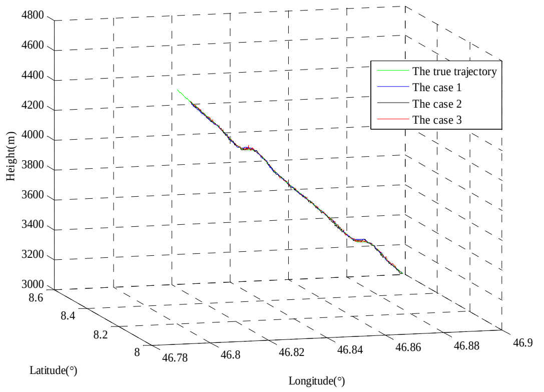
Case 1: the traditional FUPF algorithm based on the BDS/INS integrated navigation system model established by the direct method according to the literature [3];

Case 2: the traditional FUPF algorithm based on the all-attitude integrated BDS/INS navigation system model established by the direct method in the thesis;

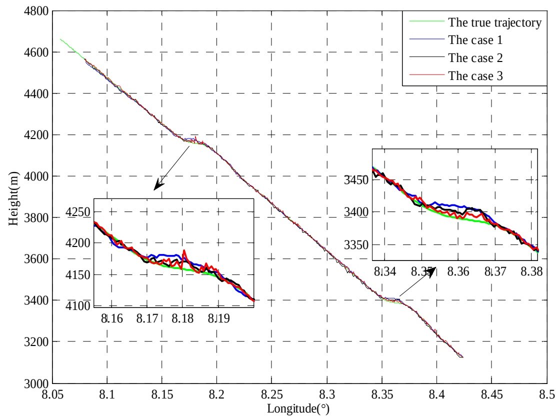
Case 3: the improved AFUPF algorithm based on the all-attitude integrated BDS/INS navigation system model established by the direct method in the thesis.

Simulation computer is: Pentium (R) Dual-Core E5200 CPU; 2.50GHz frequency; 2.00GB RAM; Windows XP Professional OS. The Matlab 7.1 simulator runs in a Monte Carlo fashion. In the simulation, the sampling frequency of IMU is 10Hz, the update cycle of the BDS's data is 1s, and the simulation time is 500s. Set up: the random constant of gyroscope is $0.1^{\circ}/h$, the driving white noise of the first-order Markov process of gyroscope is $0.1^{\circ}/h$, the white noise of gyroscope is $0.05^{\circ}/h$, the related time is 60s; the driving white noise of the first-order Markov process of gyroscope is $2\times10^{-4}g$, the related time is 60s; the position, velocity and attitude precision of the BDS receivers which is used in the system is 10m, 0.2 m/s and 1'.

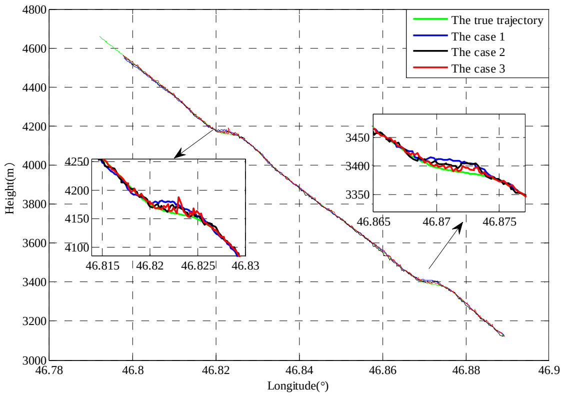
The results from each case are compared in terms of trajectory, velocity, position and attitude error. The comparisons of trajectory under different filter algorithm are showed in Figure 2. The error curves of the position estimation in the three cases are showed in Figure 3 to 5. The error curves of the velocity estimation in the three cases are showed in Figure 6 to 8. The error curves of the attitude estimation in the three cases are showed in Figure 9 to 10.



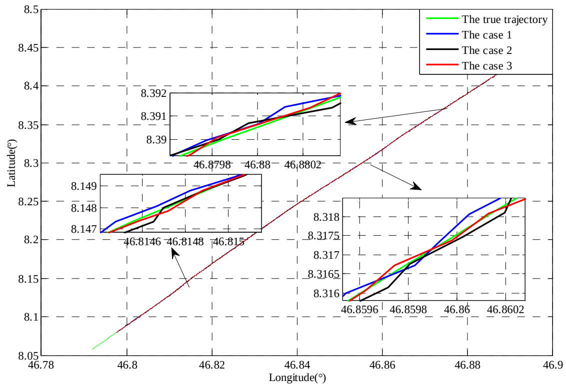
(a) Comparison of estimated and real trajectory



(b) Comparison of trajectory in height-latitude plane



(c) Comparison of trajectory in height-longitude plane



(d) Comparison of trajectory in latitude-longitude plane

Figure 2. Comparison of trajectory under different filter algorithm

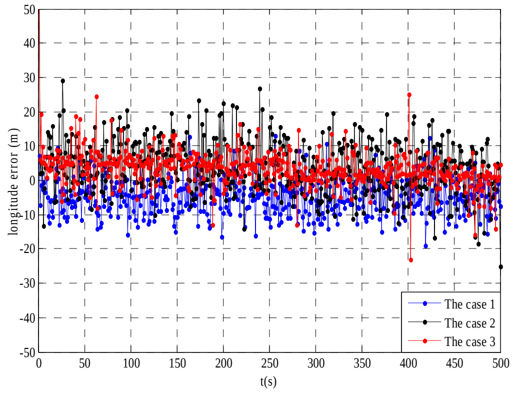


Figure 3. The error curve of longitude in the three cases

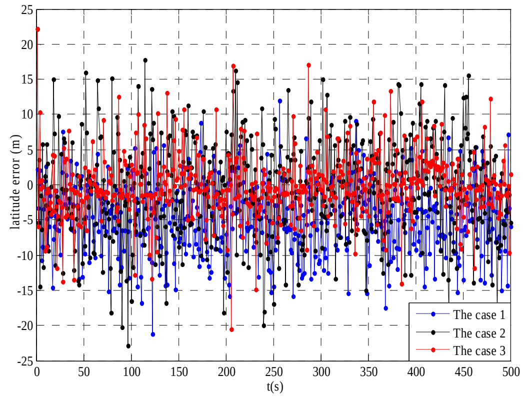


Figure 4. The error curve of latitude in the three cases

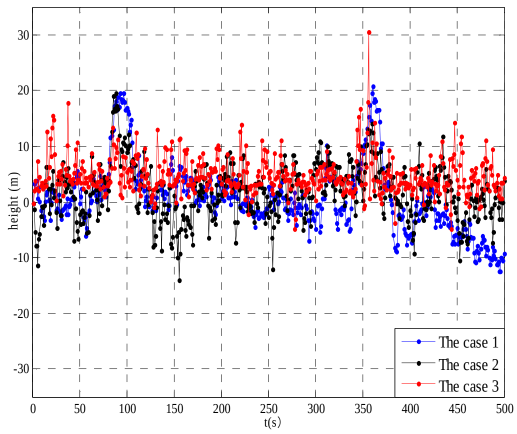


Figure 5. The error curve of height in the three cases

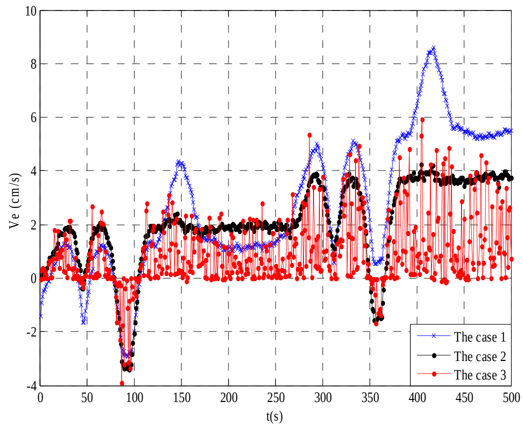


Figure 6. The error curve of east velocity in the three cases

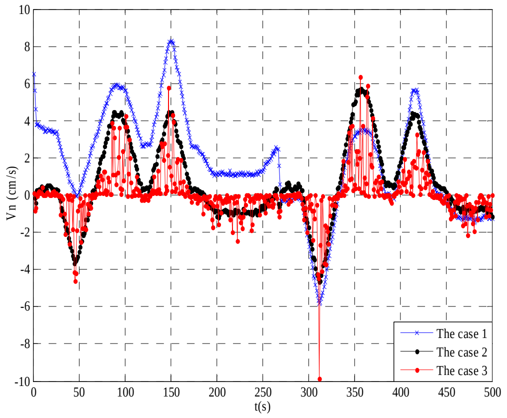


Figure 7. The error curve of north velocity in the three cases

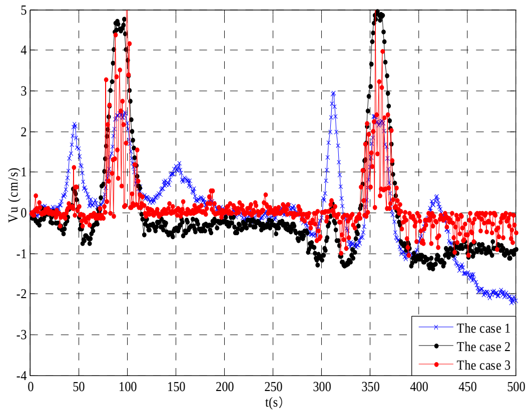


Figure 8. The error curve of altitude velocity in the three cases

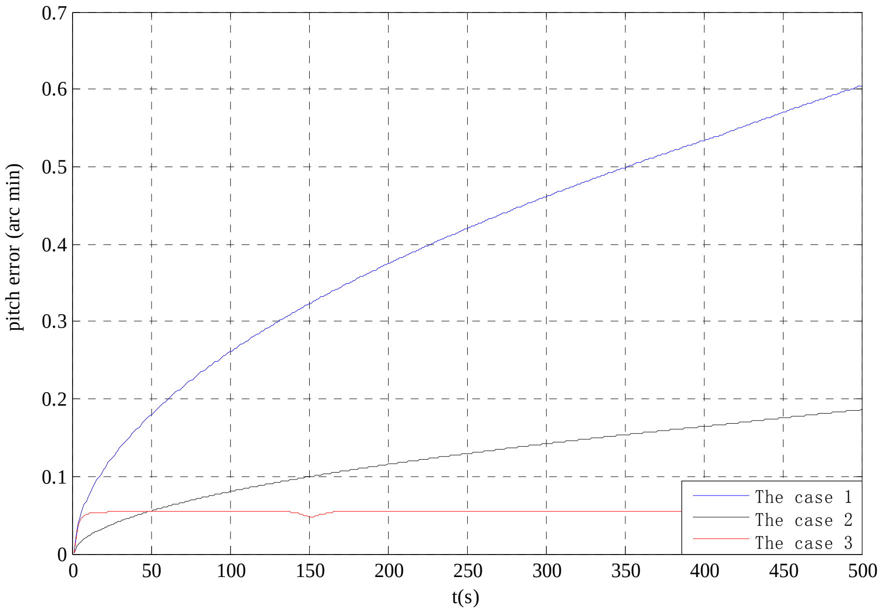


Figure 9. The error curve of the pitch/roll in the three cases

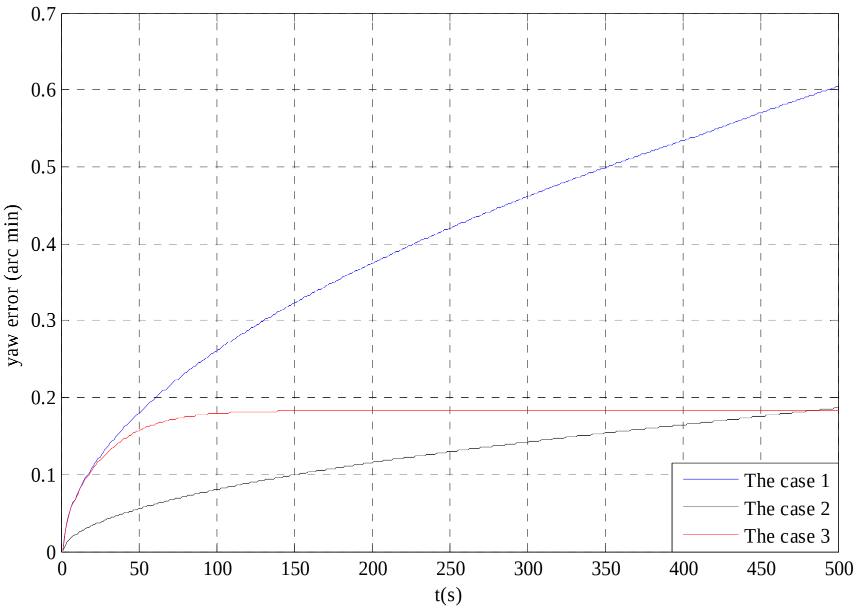


Figure 10. The error curve of the yaw in the three cases

The results and analysis:

Figure 2 displays the comparison of estimated and real trajectory of carrier. From figures (a) ~ (d), it is observed that the AFUPF has best tracking ability, most accurate, best stability.

Figure 3-5 showes the comparison of the error curve of the positions (latitude, longitude and height) of three different cases, and the means of the AFUPF's errors are is closer to the zero compared with that of the case 2, the means of position errors of case 1 is worst.

Figure 6-8 displays the comparison of the error curve of the velocities of three different cases. It is observed that the velocity-error of the AFUPF is lower than that of the other two cases, especially under dynamic conditions, such as near the 100s and 360s.

Figure 9-10 showes the comparison of the error curve of the attitudes of three different cases. As expected, the attitude-error of the AFUPF is obviously less than that of the other two cases.

From Figure 2-10, the AFUPF has better performance than the FUPF on an average by comparing the case 2 and the case 3; the influence of attitude measurement information on the accuracy of attitude of the BDS/INS integrated navigation system is enormous by comparing the case 1 and the case 2.

5. Conclusion

The thesis establishes the nonlinear all-attitude integrated BDS/INS navigation system model in order to improve the attitude accuracy of INS and proposes a new adaptive federal unscented particle filter in order to enhance system adaptability to the environment. Through simulation, you can get the following conclusions:

1) Joining the BDS attitude observation information can be effective in improving the system observability and attitude accuracy.

2) The adaptive information distribution coefficient can be changed to follow the subfilter performance, thereby change the proportion of the subfilter estimation information in the global estimation information and ensure the overall performance of the federal filter can always be close to the performance of the optimal subsystem where to make the positioning results more accurate.

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