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# Temperature Control of Liquid Filled Tank System Using Advance State Feedback Controller

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***Abstract***

*In this paper modeling of a temperature measuring tank system has been done and then a Advance state feedback controller have been used for controlling the step responses of the system. The proposed system extends to a three tank system & each tank has same amount of liquid. The results of computer simulation for the system with Advance state Feedback is shown.*

***Keywords:*** *temperature, tank system, control, non-linear control, SFB controller [7,8]*



## 1. Introduction

The Temperature measurement of liquid in a tank can be controlled by classical and advance control algorithm. Here we are considering a three tank non-interacting system. We observed that tank1 affects the dynamic behaviour of tank 2, similarly for tank2 affects the dynamic behavior [2] of tank3 and vice versa, because the flow rate depends on the difference between liquid levels $h_1$and $h_2$.Thus a change in the inlet flow rate affects the liquid level in the tank, which intern affects the temperature of the liquid. Basically it is a thermal process. Various type of temperature sensor RTD, T/C, Thermistor [1], [9-10].In that particular project we used a mercury thermometer as sensor. Mathematical models of three tank method give a third order [6] equation. Each tank give a transfer function of first order system. They make it easy to check whether a particular algorithm is giving the requisite results. A lot of work has been carried out on the temperature control in terms of its stabilization. Many attempts have been made to control the response of temperature measuring system this method is utilized to investigate [3] global properties of the designed controller.

## 2. Mechanical Construction

The system comprises of a mercury-in-glass thermometer placed in a liquid tank to measure the temperature of the liquid which is heated by steam through a coil system. The temperature of the liquid ($T_F$) varies [5] with time. T is the temperature of the mercury in the well of the thermometer. The following assumptions are made to determine the transfer function relating the variation of the thermometer (T) for change in the temperature of the liquid ($T_F$).

(1) The expansion or contraction of the glass walled well containing mercury is negligible (that means the resistance offered by glass wall for heat transfer is negligible)
(2) The liquid film surrounding the bulb is the only resistance to the heat transfer.
(3) The mercury assumes isothermal condition throughout. The system is shown in Figure 1.

## 3. Proposed Mathematical Model

Applying unsteady state heat balance for the bulb, we get input heat rate- output heat rate=Rate of heat accumulation:

$$U\,A\,(T_F - T) - 0 = M\,C_P\,\frac{dT}{dt}$$

$$U\,A\,(T_F - T) = MC_P\,\frac{dT}{dt} \tag{1}$$

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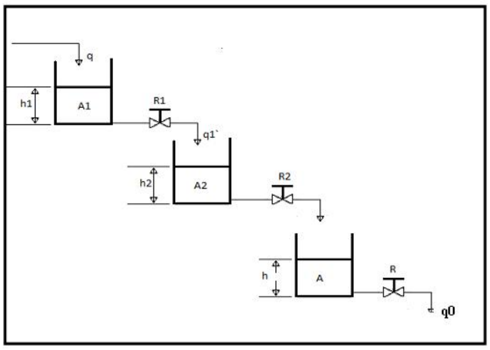


Figure 1. Three tank system

where, A=surface area of the bulb for heat transfer in $m^2$

M=Mass of mercury in the bulb in kg

$C_P$=Heat capacity of the mercury in kj/kg k

U=Film heat transfer coefficient kw/$m^2$k

At steady state, the Equation (1) can be rewritten as:

$$U\,A\,(T_{FS} - T_{F}) = 0 \quad (2)$$

Subtracting Equation (2) from Equation (1):

$$U\,A\,\left[(T_F - T_{FS}) - (T - T_S)\right] = M\,C_P\,\frac{d(T-T_S)}{dt}$$

Defining the deviation variables, $T_F - T_{FS} = T_1$and $T - T_S = T_1$ and substituting in the above equation, we get:

$$U\,A\,(T_{F1} - T_1) = MC_P\frac{dT_1}{dt}$$
$$(T_{F1} - T_1) = \frac{M\,C_P}{UA} \quad (3)$$

Defining time constant $t_p$ for the Thermometer,

$$t_p = M\,\frac{C_P}{U\,A}$$

Equation (3) can be rewritten as:

$$T_{F1} - T_1 = t_p\,\frac{dT_1}{dt} \quad (4)$$

Taking Laplace transform, we get:

$$T_{F1}(S) - T_1(S) = t_p\,s\,T_1(S)$$
$$\frac{T_{1(S)}}{T_{F_1}(S)} = \frac{1}{1+t_p\,s} \quad \text{transfer function of Tank1}$$

Similarly, for tank2 & tank3 we can able to get a first order system. So we can able to say that the entire system is a third order system. Here we can able to construct overall transfer function of the three tank system is:

$$G(S) = G_1(S) \times G_2(S) \times G_3(S)$$
$$= \left(\frac{K_1}{1+t_{p1}s}\right) \times \left(\frac{K_2}{1+t_{p2}s}\right) \times \left(\frac{K_3}{1+t_{p3}s}\right)$$

**4. Transfer Function Modelling**

As per our problem, let us assume:

$t_{p1}$= time constant for tank1=0.5 minute

$t_{p2}$ = time constant for tank2=1.2 minute

$t_{p3}$= time constant for tank3=1.5 minute

$K_1$=$R_1$= 0.25 min

$K_2$=$R_2$= 0.30 min

$K_3$=$R_3$ =0.35 min

$$G(S) = \frac{0.02625}{0.9S^3+3.15S^2+3.2S+1}$$

This transfer function is called plant transfer function. The entire experimental set up is given in Figure 2.

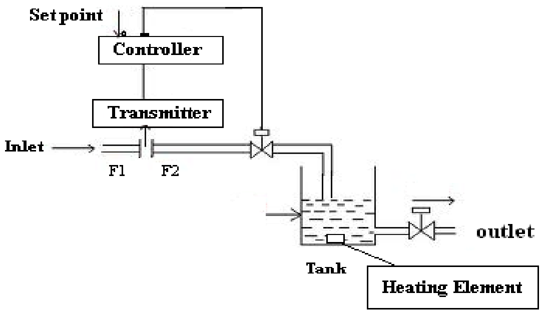


Figure 2. Proposed Experimental Set up

**5. Advance State Feedback Controller Design**

**Conditions:**

Closed loop system has an overshoot of 10% and settling time of 1 sec.

**Equations:**

Overshoot ($M_P$)=$e^{-\xi\pi/\sqrt{(1-\xi^2)}}$=0.1

Taking natural log on both sides

$\ln(e^{-\xi\pi/\sqrt{(1-\xi^2)}})=\ln(0.1)$

or, $-\xi\pi/\sqrt{(1-\xi^2)}=\ln 0.1$

Squaring both sides,

$\xi^2\pi^2/(1-\xi^2)=(\ln 0.1)^2$

or, $\xi^2\pi^2=(\ln 0.1)^2(1-\xi^2)$

or, $\xi^2\pi^2+ (\ln 0.1)^2\ \xi^2= (\ln 0.1)^2$

or, $\xi^2\{\pi^2+ (\ln 0.1)^2\}=(\ln 0.1)^2$

or, $\xi^2=(\ln 0.1)^2/\{\pi^2+ (\ln 0.1)^2\}$

Therefore, $\xi =\pm\ln 0.1/\sqrt{\{\pi^2+ (\ln 0.1)^2\}}$

$=\pm [-2.3025/\sqrt{(9.8596+5.3018)}]$

$=\pm(-2.3025/3.8937)$

$=\pm(-0.591328)$

Therefore, $\xi =-(-0.591328)$

$= 0.591328$

$t_s$=1 sec

or, $4/ \xi w_n=1$

or, $\xi w_n=4$

or, $w_n=4/ \xi$

$=4/(0.591328)$

$=6.7644$ rad/sec

The dominant poles are at

$-\xi w_n\pm jw_n\sqrt{(1- \xi^2)}$

$=-3.9999 \pm j*6.7644*\sqrt{(1-0.3496)}$
$=-3.9999 \pm j\,5.45531 \cong -4 \pm j5.45531$

The third pole is placed 10 times deeper into the s-plane than the dominant poles. Hence the desired characteristics equation is:

$(s+39.999)(s+3.9999+j\,5.45531)(s+3.9999-j\,5.45531)=0$
Or, $(s+39.999)\{(s+3.9999)^2+5.45531^2\}=0$
$(s^2+39.999)(s^2+16+8s+29.7604)=0$
$s^3+48s^2+365.7604s+1830.416=0$ (5)

Let $k=[k_1\ \ k_2\ \ k_3]$

$$\text{Now, } [SI-A]=S\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}-\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1.111 & -3.5556 & -3.5000\end{bmatrix}$$
$$=\begin{bmatrix}S & -1 & 0\\ 0 & S & -1\\ 1.111 & 3.5556 & S+3.5000\end{bmatrix}$$

Closed loop characteristics equation:

$S^3+(3.500)s^2+(3.555)s^1+1.1111=0$ (6)

Comparing the coefficient of Equation (5) & (6). Therefore,
$k=[k_1\ \ k_2\ \ k_3]$
$k=[(1830.416-1.1111)\ \ (365.7604-3.555)\ \ (48-3.500)]$
$=[1829.3049\ \ 362.2054\ \ 44.5]$

Similarly we take 10 results to observe if the poles are less or more deeper in s plane then what changes seen in their initial and step response.

## 6. Graphical Result

### 6.1. Step Response of the Plant with SFB Controller

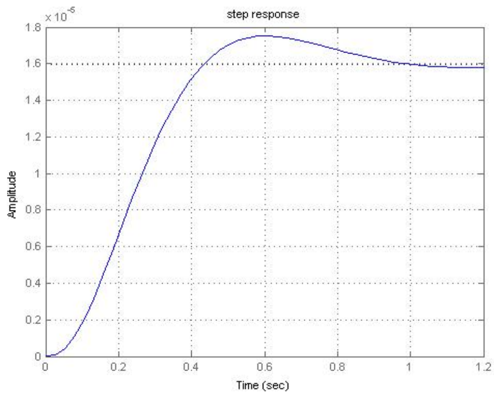


Figure 3. Response of state feedback controller considering step condition in MATLAB

### 6.2. Step Response of the Plant with Initial SFB Controller

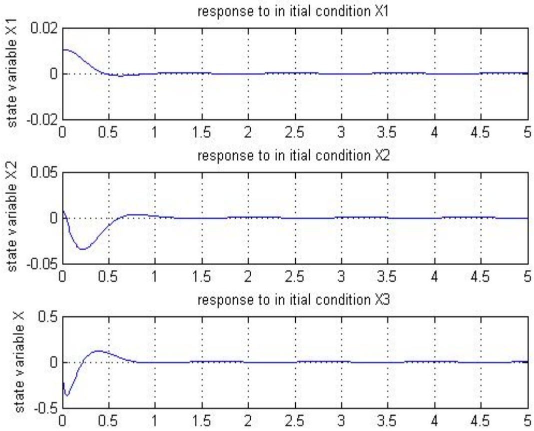


Figure 4. Response of state feedback controller considering initial condition in MATLAB

## 7. Conclusion

Modeling of three tank temperature measuring system shows that system is unstable for a certain range. That's why we tried to design a conventional controller strategy process so that we can minimize the steady state error & maximize the settling time. In future we may use Genetic Algorithm for designing the controller.

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