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A Minimax Polynomial Approximation Objective Function Approach for Optimal Design of Power System Stabilizer by Embedding Particle Swarm Optimization

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Abstract

The paper presents a novel approach based on Minimax approximation and evolutionary tool Particle Swarm Optimization (PSO) to fabricate the parameters of Power System Stabilizers (PSSs) for multi machine power systems. The proposed approach employs PSO algorithm for find the setting of PSS parameters. The worth mentioning feature of this work is the formulation of objective function with the help of swing curves interpolation. A novel transformation known as minimax approximation is used for converting the objective into the polynomials of degree one, two and three. To construct the objective function based on interpolation second order sensitivity analysis is performed. The performance of the PSSs is tested under different topological changes, operating conditions and system configurations. Nonlinear simulation reveals that proposed PSSs are effectively deal with local and interarea modes of oscillations. PSS design obtained through lower order polynomial expression of objective function is able to deal with the oscillatory modes efficiently.

***Keywords**: automatic voltage regulator (AVR), minimax approximation, multimachine power system, particle swarm optimization (PSO), power system stabilizer (PSS).*

1. Introduction

In the modern power system high performance Automatic Voltage Regulators (AVRs) (high gain and fast response) are equipped to ensure transient stability. AVRs of high gain introduce negative damping in the system. To provide a cost effective control PSSs are employed with AVRs. The requirement for modern excitation system exists in the fact that PSS and AVR both are dynamically interlinked [1]. The purpose of the parameter design is to make the PSSs provide proper damping for power system oscillations. PSS parameter estimation problem is an optimization problem, where aim of the optimization process is to maximize the damping of the power network. It is quite empirical to state that there is a tradeoff between synchronizing torque provided by the AVR and damping torque provided by the PSS [2]. Researchers have experimented with the different type of objective functions[]. The PSS parameter estimation problem has been addressed with different optimization algorithms [3-11]. The major contribution reported in literature revolves around overall systems dynamic response i.e. overshoot and settling time, convergence characteristics of proposed methodology, solution quality, time elapsed and comparison of the methodology with conventional techniques [8-11]. The traditional objective function reported in literature considered damping ratios, damping frequencies and weighted combination of these to solve the tuning parameters of PSSs. The inferior modes are shifted to D shaped and fan shaped regions. Sheng kuan wang proposed a new scale which drifted eigenvalues in fan shaped mode with the tip at damping ratio [7].

In this paper, PSO algorithm [12] is employed to solve PSS parameter estimation.The realisa-tion of objective function in 1^{st}, 2^{nd} and 3^{rd} order polynomials are done with the help of sensitivities of derivatives and MATLAB curve fitting tool. For testing results, the proposed approach is applied on two test cases of multi machine power systems. Assertiveness of proposed methodology has been tested on different type of disturbances, loading conditions, and system configurations.

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2. Problem Statement
2.1. Power System Model

Formulation of the power system can be concluded and understand by following set of equations.

$$\dot{x} = f(x,u) \tag{1}$$

Where x is the state variables, u is the vector of input variable. In the PSS the power system is usually linearized and operating equllibrium as the study of the small disturbance comes in small signal stability. Equation (1) can further be transformed as:

$$\dot{x} = A\Delta x + B\Delta u \tag{2}$$

If n is the total no of machines size of A will be $4n \times 4n$, Δx is $4n \times 1$ state vector, while Δu vector is $n_{pss} \times 1$.

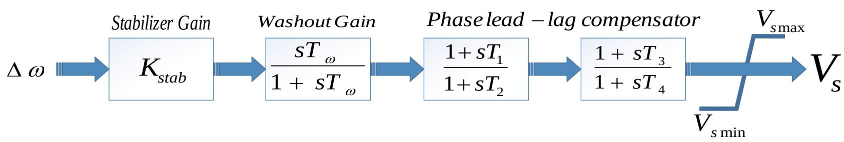

Figure 1. Structure of Power System Stabilizer

2.2. PSS Structure

Conventional lag lead structure of PSS is shown in Figure 1. The structure is used in this work which has transfer function (3). Further the modern excitation system with AVR and PSS is shown in Figure 2.

$$U_i = K_{stabi} \frac{sT_w}{1+sT_w} \cdot \frac{1+sT_{1i}}{1+sT_2} \cdot \frac{1+sT_{3i}}{1+sT_4} U \tag{3}$$

The objective function J is to be minimized with the constraints.

$$J = \min \left\{ \begin{array}{l} \left(\sum_{2}^{t1} .\Delta w_1^2 + \Delta w_2^2 + \ldots + \Delta w_{10}^2 . \right) + \left(\sum_{2}^{t2} .\Delta w_1^2 + \Delta w_2^2 + \ldots + \Delta w_{10}^2 \right) + \ldots \\ \qquad \ldots + \left(\sum_{2}^{t_k} .\Delta w_1^2 + \Delta w_2^2 + \ldots\ldots + \Delta w_{10}^2 \right) \end{array} \right\} \tag{4}$$

Subject to:

$$\left. \begin{array}{r} K_{stabi}^{\min} \le K_{stabi} \le K_{stabi}^{\max} \\ T_{1i}^{\min} \le T_{1i} \le T_{1i}^{\max} \\ T_{3i}^{\min} \le T_{3i} \le T_{3i}^{\max} \end{array} \right\} \tag{5}$$

The objective of the optimization is to find the set of variables K_{stabi}, T_{1i}, T_{3i}, for $i = 1,2,\ldots,n$ to achieve adequate damping in the system. Here n is the size of network or total number of alternators in system.It is assumed that all alternators have incorporated with PSSs. In this work washout time constant $T_w = 10s$ and T_2 & T_4 are considered as $0.05s$. The left over over parameters K_{stab}, T_1 and T_3 are assumed to be the modifiable parameters; hence the number of the parameters for 3 machines will be 9 and for 10 machines system will be 30.

2.3. Construction of Objective Function

The speed deviation based objective function is employed here for the PSS parameter estimation problem. The steps for constructing the objective function are as following.

Step 1. Initialize the optimization process, read system data, select the contingencies/operating conditions 'm' and simulation time steps 'k'.

Step 2. Initialize the PSS data

Step 3. Simulate the system and store the values of speed deviations of the generators for different faults.

Step 4. Apply stopping criterion, if not satisfied then go to step 2.

Step 5. End

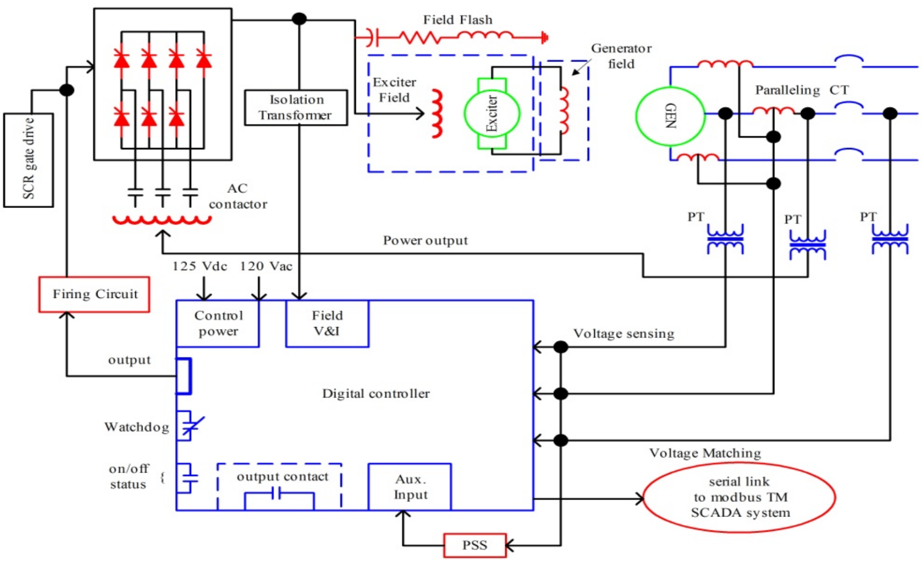

Figure 2. Modern excitation system with AVR & PSS

2.4. Embedding Objective Function in Interpolating Polynomial

The idea is to approximate objective function $f(x)$ by a polynomial $p(x)$ that gives a uniform accurate description in an interval $[a,b]$. Here the $[a,b]$ is the interval of application of the certain disturbance. Let the function $f(x)$ is an approximate continuous function on an interval $[a,b]$. This function with is realized the set of polynomials of degree at most n and let k bounded function defined $[a,b]$, Minimax approximation algorithm suggests that maximum error is minimized [4]. The objective is to find a function $k(x)$ to minimize Equation (6).

$$J = \max_{a \le x \le b} f(x) - k(x) \tag{6}$$

A sensitivity analysis is depicted in Table 1 to Table 9 and indices like first and second derivatives are calculated to ensure the truthfulness of interpolation fit. However it is worth mention here that Chebyshev expansion polynomial of first kind can closely approximate Minimax polynomial [4]. The proposed work transforms the traditional objective function into three types of polynomials of degree $1^{st}, 2^{nd}$ and 3^{rd} order. The parameter tuning is done while optimizing each polynomial with the help of PSO.

2.6. Sensitivity Analysis

To formulate the objective function on the basis of speed deviation, the sensitivity analysis is required. It is interesting to know that how the PSS parameter can be interpolated for constructing the objective function. The objective of the sensitivity problem is to compute the derivative of the function. Suppose a function:

$$f(x) = G(u(x), x)$$

Sensitivity of the function with respect to parameter *x* is given by the following equation:

$$\frac{\partial f}{\partial x} = \frac{\partial u}{\partial x}(u(x), x)\frac{\partial u}{\partial x} + \frac{\partial G}{\partial x}u(x)$$

In this work finite method of difference [4] is used to calculate the derivatives of the various parameters of PSS. The analysis is shown in the Table 1 to Table 9. The operating points between the simulation time steps are selected and the sensitivity of the speed deviation of all 3 generators are calculated. The values of different first order and second order sensitivities are shown in Table 1 to 9. Following points are worth mention here:

a) For the high values of simulation time step i.e. when fault is generated after a long wait than the second derivative of generator 3 for T_1 is less sensitive. It is also observed that for lower values of the simulation time step the second order derivatives of generator 3 are very high. For T_3 parameter the second order derivative of generator 2 is most sensitive. In fact it achieves highest value. The same analogy is followed when the simulation step is delayed and the second order derivative attains higher values for generator 2.
b) Table 1 to Table 9 shows the absolute values of sensitivities for $K_{stab}.T_1$ and T_3. The sensitivity analysis is used to obtain the weights for combination of the effect of the PSS parameters and forming the objective functions linear, polynomial 2 and cubic polynomials.

Table 1. Interpolation Fit for T_1 of Generator 1 (3 Machine System)

x_i	0.6	3.54	6.48	9.42	12.36	15.3	18.2	21.1	24.1	27	30
$f(x_i)$	12.6727	-22.530817	-320.65	-1166.1	-2843.2	-5636.4	-9830.1	-15708	-23556	-33658	-46297
$d'f(x_i)$	0.497	-40.567	-178.362	-412.888	-744.145	-1172.13	-1696.85	-2318.3	-3036.48	-3851.39	-4763.03
$d''f(x_i)$	2.48325	-30.418	-63.319	-96.221	-129.12	-162.02	-194.92	-227.82	-260.72	-293.63	-326.53

Table 2. Interpolation Fit for T_3 of Generator 1 (3 Machine System)

x_i	0.6	3.54	6.48	9.42	12.36	15.3	18.2	21.18	24.12	27.06	30
$f(x_i)$	12.2603	266.941	2028.19	6771.82	15973.6	31109.4	53655	85086.2	126879	180509	247451
$d'f(x_i)$	-2.268	259.183	1022.61	2288.01	4055.39	6324.74	9096.06	12369.4	16144.6	20421.9	25201.1
$d''f(x_i)$	3.5594	174.299	345.039	515.778	686.518	857.257	1028	1198.74	1369.48	1540.22	1710.96

Table 3. Interpolation Fit for K_{stab} of Generator 1 (3 Machine System)

x_i	0.6	3.54	6.48	9.42	12.36	15.3	18.24	21.18	24.12	27.06	30
$f(x_i)$	13.2753	13.5417	13.713	13.8083	13.8464	13.8465	13.8275	13.8083	13.8081	13.8457	13.9402
$d'f(x_i)$	0.108	0.073369	0.044	0.0216	0.00542	-0.004303	-0.00757	-0.0043	0.00528	0.0214	0.043
$d''f(x_i)$	-0.0131	-0.011	-0.0088	-0.0066	-0.0044	-0.0022	-1.10E-05	0.0021	0.00438	0.00658	0.00877

Table 4. Interpolation Fit for T_1 of Generator 2 (3 Machine System)

x_i	0.6	3.5	6.4	9.4	12	15	18	21.1	24.1	27.06	30
$f(x_i)$	12.28	9.713	-26.72	-154.1	-429.7	-910.7	-1654	-2717	-4156.68	-6030.23	-8394.8
$d'f(x_i)$	-1.5938	-3.3955	-24.633	-65.309	-125.42	-204.96	-303.95	-422.36	-560.23	-717.529	-894.26
$d''f(x_i)$	2.69273	-3.91839	-10.5295	-17.1406	-23.7517	-30.3628	-36.9739	-43.5851	-50.1962	-56.8073	-63.4184

Table 5. Interpolation Fit for T_3 of Generator 2 (3 Machine System)

x_i	0.6	3.54	6.48	9.42	12.3	15.3	18.2	21.1	24.1	27	30
$f(x_i)$	17.2717	19721.2	140499	455544	1000000	2400000	3500000	5500000	8200000	16000000	19000000
$d'f(x_i)$	78.8527	18608.3	68836.4	150763	264389	409713	586735	795456	1400000	1300000	1600000
$d''f(x_i)$	911.611	11693.5	22475.3	33257.2	44039	54820.9	65602.7	76384.6	87166.4	97948.3	108730

Table 6. Interpolation Fit for K_{stab} of Generator 2 (3 Machine System)

x_i	0.6	3.54	6.48	9.42	12.3	15.3	18.2	21.1	24.1	27	30
$f(x_i)$	13.7935	13.7092	13.6415	13.5888	13.5495	13.5221	13.5051	13.4969	13.495	13.5006	13.5095
$d'f(x_i)$	-0.0316	-0.0257	-0.0203	-0.0155	-0.0112	-0.0074	-0.0042	-0.0014	0.0007	0.0024	0.00355
$d''f(x_i)$	0.002092	0.001913	0.001731	0.001555	0.001376	0.001197	1.02E-3	0.00083	0.0006	0.00048	0.00030

Table 7. Interpolation Fit for T_1 of Generator 3 (3 Machine System)

x_i	0.6	3.54	6.48	9.42	12.36	15.3	18.24	21.18	24.1	27	30
$f(x_i)$	12.9689	-21.5696	-415.283	-1596.16	-3992.19	-8031.37	-14141.7	-22751.1	-34287.7	-49179.3	-67854
$d'f(x_i)$	0.811412	-48.5694	-243.525	-584.055	-1070.16	-1701.84	-2479.1	-3401.92	-4470.33	-5684.31	-7043.86
$d''f(x_i)$	7.96141	-41.5538	-91.069	-140.584	-190.099	-239.615	-289.13	-338.645	-388.16	-437.676	-487.191

Table 8. Interpolation Fit for T_3 of Generator 3 (3 Machine System)

x_i	0.6	3.54	6.48	9.42	12.36	15.3	18.24	21.18	24.12	27.06	30
$f(x_i)$	13.7248	-74.4346	-693.471	-2366.1	-5615.03	-10963	-18932.7	-30046.8	-44828.1	-63799.3	-87483.1
$d'f(x_i)$	1.03427	-90.639	-360.106	-807.368	-1432.42	-2235.27	-3215.92	-4374.36	-5710.59	-7224.61	-8916.43
$d''f(x_i)$	-0.94429	-61.4185	-121.893	-182.367	-242.841	-303.315	-363.789	-424.264	-484.738	-545.212	-605.686

Table 9. Interpolation Fit for K_{stab} of Generator 3 (3 Machine System)

x_i	0.6	3.54	6.48	9.42	12.36	15.3	18.24	21.18	24.12	27.06	30
$f(x_i)$	15.2778	14.9018	14.5859	14.3247	14.1131	13.946	13.8181	13.7243	13.6593	13.618	13.5951
$d'f(x_i)$	-0.13867	-0.11738	-0.09785	-0.08011	-0.06410	-0.04987	-0.0374	-0.02671	-0.01778	-0.0106	-0.00521
$d''f(x_i)$	0.00754	0.006940	0.006340	0.005740	0.005139	0.004539	3.94E-3	0.003338	0.002738	0.00213	0.001537

3. Case 1: Three Machine Power System

The case taken over here to understand the response of different polynomials is the 3 machine 9-bus system [13], the minute observations on the system shows that without installing PSSs on generating machine, system get unstable for various perturbations.

Table 10. Generating Conditions

Base Case		Case-A		Case-B		Case-C	
P	Q	P	Q	P	Q	P	Q
0.716	0.270	1.527	0.249	1.3283	0.2393	0.5077	0.3029
1.63	0.0665	1.00	-0.003	1.00	-0.006	1.85	0.1134
0.85	-0.108	0.65	-0.117	0.850	-0.12	0.85	-0.094

Table 11. Loading Conditions

Base Case		Case-A		Case-B		Case-C	
P	Q	P	Q	P	Q	P	Q
1.25	0.50	0.75	0.39	1.50	0.90	0.65	0.55
0.90	0.30	0.90	0.30	1.20	0.80	0.45	0.35
1.00	0.35	1.00	0.35	1.00	0.50	0.50	0.25

Table 10 and Table 11 give different operating conditions related with generation as well as load side. As stated in literature these operating conditions are considered as hard as far as system stability is concerned [7].

System response is judged with load patterns and following perturbations:

a) A 6-cycle fault disturbance at bus 6 at the end of line 5-6 with Case A, Case B. The fault has been cleared without tripping
b) A 6-cycle fault is cleared by tripping the line 5-6 with Case C.

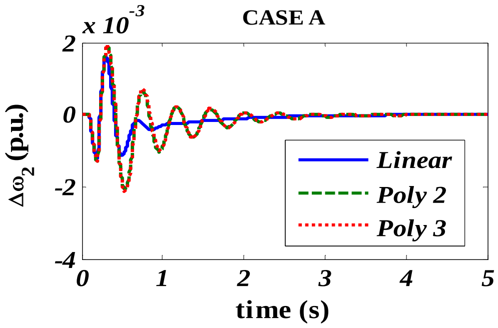

Figure 4. Speed Deviation of Generator 2

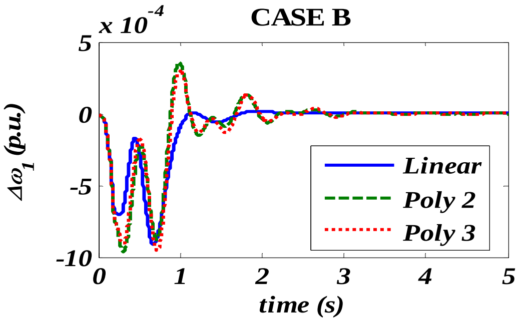

Figure 5. Speed Deviation of Generator 1

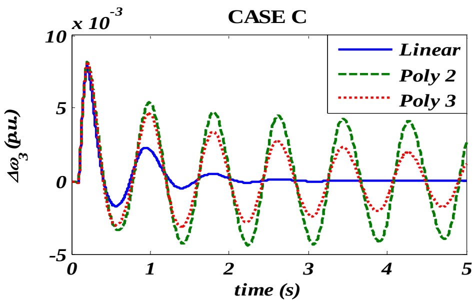

Figure 6. Speed Deviation of Generator 3

4. Case 2: New England Power System

In this case, the 10 machine 39 bus system [14] is considered. The system is comparatively larger than 3 generator system and dynamic as interarea oscillation is considered.

4.1. Test System

The system is tested over different perturbations and configurations which is extremely hard for system stability [12]

4.2. PSS Design

To design the proposed PSS by using minimax approximation interpolation polynomials, three different operating conditions and critical line outages are considered which are the enormously rigid from the stability point of view. They can be considered as:

a) Base Case; No outage of line
b) Case A; outage of line 22-23
c) Case B; outage of line 1-39

Speed deviation curve of generator 9 is shown to demonstrate the effectiveness of the proposed PSSs as it is the nearest with the fault location (line 14-15), another speed deviation curve of generator 3 is shown as the generator location is also a key derivative considering the above given conditions.

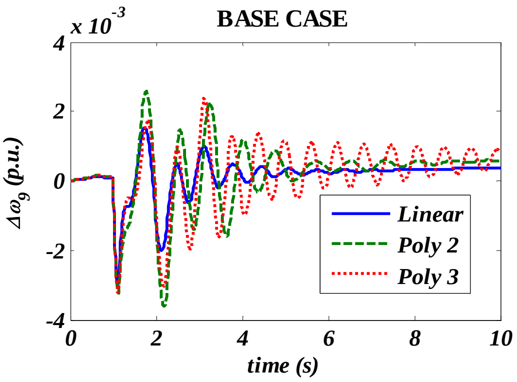

Figure 7. Speed Deviation of Generator 9

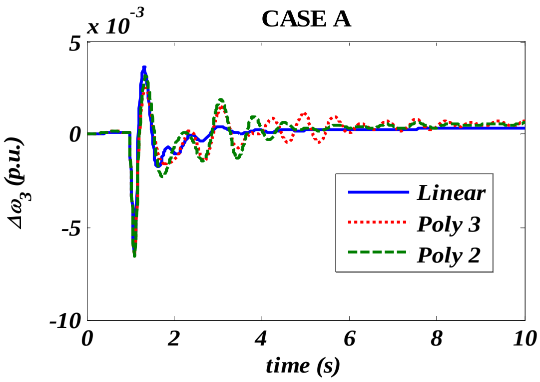

Figure 8. Speed Deviation of Generator 3

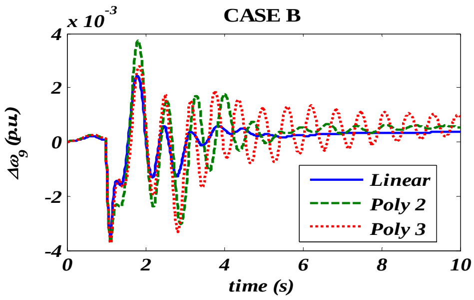

Figure 9. Speed Deviation of Generator 9

4.3. Discussions

1) It is observed from the speed deviation curves of different generators from Figure (3) to Figure (6) that PSS designed through linear polynomial objective function gives best solution as far as the overall system stability is concerned. However, it is worth mention here that system gets unstable while using the PSSs parameters obtained from either polynomial 2^{nd} order or polynomial 3^{rd} order.

2) On New England System, the critical operating condition (Case B) reveals the efficacy of the proposed linear PSS prominently. PSS designed from other than linear polynomial fit show a poor dynamic response in this operating condition.

3) While observing the speed deviations related with the polynomial fit of order 1^{st}, 2^{nd} and 3^{rd}; it is observed that the maximum error term is minimized with using linear i.e. 1^{st} order fit.

5. Conclusion

Work presented in this paper is to transform the traditional objective function into minimax polynomials. Table 1 to 9 shows various interpolation statistics while optimizing the PSSs parameter i.e. time constants and PSSs Gains sensitivity with respect to the objective function. Higher values of 2^{nd} derivative shows that it presents a poor fit to the objective function; however values for 2^{nd} derivatives is zero in case of linear polynomials. The technique used for optimization is PSO. The response obtained under different operating conditions shows that linear fit is the most suitable fit for obtaining the PSSs parameters. By using linear objective function the small signal stability can be enhanced.

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