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A systematic approach for solving mixed constraint fuzzy balanced and unbalanced transportation problem

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ABSTRACT

In present article a mixed type transportation problem is considered. Most of the transportation problems in real life situation have mixed type transportation problem this type of transportation problem cannot be solved by usual methods. Here we attempt a new concept of Best Candidate Method (BCM) to obtain the optimal solution. To determine the compromise solution of balanced mixed fuzzy transportation problem and unbalanced mixed fuzzy transportation problem of trapezoidal and trivial fuzzy numbers with new BCM solution procedure has been applied. The method is illustrated by numerical examples.

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1. INTRODUCTION

The transportation problem was initially developed by Hitchcock [1] in 1941. The transportation problem was assumed that all the parameters is sure about the exact values. Precise nature of the goods is very important in the transport rout. But in real life we can't represent exactness or preciseness. So we have face imprecise conditions. Also the situations are different in different conditions since various parameters not be known precisely. Due to uncontrollable factors we again face to mixed type constaint problems. This type of problems have less information in literature also. There are no efficient methods to solve mixed type problems. Therefore, fuzzy numbers introduced by zadeh [2]. Zimmerman [3] introduced a huge information about fuzzy in his fuzzy set theory and applications. He discussed crispness, vagueness, uncertainty, fuzziness in his work and suitable examples also. Gani [4] solved a mixed constraint transportation problem under fuzzy environment.

Also they highlighted the steps by using a simple flow chart. Mandal and Hussain [5] also studied mixed constraint they described the algorithm to find an optimal more-for-less solution. Gupta and Bari [6] determined the multi objective capacited transportation problem (MOCTP) with mixed constraint. They provide a solution procedure also. Kumar and Hussain [7] proposed a method which is easy to understand and apply for finding intuistionistic fuzzy optimal solution. They used mixed intuitionistic fuzzy transportation problem. Ghadle and Pathade [8] compare balanced and unbalanced fuzzy transportation problem by using hexagonal fuzzy numbers and robust ranking technique. Ahmed and Khan [9] developed a new algorithm for finding an initial basic feasible solution of a transportation problem with fuzzy approach. Prabha and

Vimala [10] used a strategy to solve mixed intuitionistic fuzzy transportation problem by Best Candidate Method. Nidhi et al. [11] used mixed constraint in fuzzy environment. Ghadle and Pathade [12] studied generalized hexagonal and octagonal fuzzy numbers by ranking method. Moreover, properties of the fuzzy transportation have been studied by several authors [13–22].

2. PRELIMINARY

In this section, we collect some basic definitions that will be important to us in the sequel [23–27].

Definition 2..1 *A fuzzy set is characterized by a membership function mapping element of a domain, space, or the universe of discourse* X *to the unit interval* $[0, 1]$ $i.e$ $A = \{(\mu_A(x); x \in X)\}$. *Here* $\mu_A(x) : X \longrightarrow [0, 1]$ *is a mapping called the degree of membership value of* $x \in X$ *in the fuzzy set A. These membership grades are often represented by real numbers ranking from* $[0, 1]$.

Definition 2..2 *A fuzzy set* $\tilde{A}$ *is defined on universal set of real numbers is said to be generalized fuzzy number if its membership function* $\mu_{\tilde{A}}$ *has the following attributes:*

- $\mu_{\tilde{A}}(x) : R \to [0, 1]$ *is continuous;*
- $\mu_{\tilde{A}}(x) = 0$ *for all* $x \in (-\infty, a] \bigcup [d, \infty)$;
- $\mu_{\tilde{A}}(x)$ *is strictly increasing on* $[a, b]$ *and strictly decreasing on* $[c, d]$*;*
- $\mu_{\tilde{A}}(x) = w$ *for all* $x \in [b, c]$, *where* $0 < w \leq 1$.

Definition 2..3 *A fuzzy number* $\tilde{A} = (a, b, c, d)$ *is said to be a trapezoidal fuzzy number if its membership function is given by the following expression:*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 0, & otherwise \end{cases} \tag{1}$$

Definition 2..4 *A fuzzy number* $\tilde{A} = (a, b, c, d)$ *is said to be a trivial trapezoidal fuzzy number if and only if*

- *a=b=c=d*
- *it's membership function is given by,*

$$\mu_{\tilde{A}}(x) = \begin{cases} 1, & x = a \\ 0, & otherwise \end{cases} \tag{2}$$

Definition 2..5 ***Properties of Trapezoidal Fuzzy Number***

1. *A trapezoidal fuzzy number* $\tilde{A} = (a, b, c, d)$ *is said to be non-negative (non-positive) trapezoidal fuzzy number i.e.* $\tilde{A} \geq 0 (\tilde{A} \leq 0)$ *if and only if* $a \geq 0$ $(c \leq 0)$. *A trapezoidal fuzzy number is said to be positive (negative) trapezoidal fuzzy number i.e.* $\tilde{A} > 0$ $(\tilde{A} < 0)$ *if and only if* $a > 0$ $(c < 0)$.
2. *Two trapezoidal fuzzy number* $\tilde{A}_1 = (a, b, c, d)$ *and* $\tilde{A}_2 = (a, b, c, d)$ *are said to be equal.*
3. *A zero trapezoidal fuzzy number is denoted by* $\tilde{O} = (0, 0, 0, 0)$.
4. *A trapezoidal fuzzy number* $\tilde{A} = (a, b, c, d)$ *is a traingular fuzzy number if* $b = c$.

Definition 2..6 *Let* $\tilde{A} = (a, b, c, d)$ *be a trapezoidal fuzzy number. Then its* α*-cut is given by* $(\alpha_\alpha^L, \alpha_\alpha^U) = \big[(b-a)\alpha + a, d - (d-c)\alpha\big]$, *where* $\alpha \in [0, 1]$.

Definition 2..7 *A trapezoidal fuzzy number* $\tilde{A} = (a, b, c, d)$ *is said to be a fuzzy zero if* $R(\tilde{A}) = 0$.

Definition 2..8 *Fuzzy numbers have no natural order but in the decision making process fuzzy numbers must be systematic. With the help of systematic order of numbers we make good decision in fuzzy environment. Here we use Robust ranking function. If* $\tilde{A} = (a, b, c, d)$ *is a fuzzy number then the Robust ranking is defined by,* $R(\tilde{A}) = \int_0^1 (0.5)(\alpha_\alpha^L, \alpha_\alpha^U) d\alpha$, *where* $(\alpha_\alpha^L, \alpha_\alpha^U)$ *is the* α*-cut of the trapezoidal fuzzy number* $\tilde{A}$. $R(\tilde{A}) = \frac{a+b+c+d}{4}$ *For any two fuzzy numbers,* $\tilde{A}_1$, $\tilde{A}_2$ *we have the following comparision,*

(a) $\tilde{A}_1 < \tilde{A}_2$ *if and only if* $R(\tilde{A}_1) < R(\tilde{A}_2)$,
(b) $\tilde{A}_1 > \tilde{A}_2$ *if and only if* $R(\tilde{A}_1) > R(\tilde{A}_2)$,
(c) $\tilde{A}_1 = \tilde{A}_2$ *if and only if* $R(\tilde{A}_1) = R(\tilde{A}_2)$.

3. NEW BEST CANDIDATE METHOD

Step 1: Must check the matrix is balanced, if the total supply equal to the total demand then the matrix is balanced and if the total supply is not equal to the total demand. So the transportation cost to this row or column assigned to zero.
Step 2: The best candidates are selected by choosing minimum cost for minimization problem and maximum cost for maximization problems.
Step 3: Assign as much as possible to the cell with the smallest unit cost (or highest) in the entire tableau. If there is a tie then choose arbitrarily.
Step 4: Check each row (and column) has atleast one best candidate.
Step 5: Identify the row with the smallest cost candidate from the chosen combination. Then allocate the demand and the supply as much as possible to the variable with the least unit cost in the selected row or column.
Step 6: Also, we should adjust the supply and demand by crossing out the row/column to be then assigned to zero. If the row or column not assigned to zero, then we check the selected row if it has an element in the chosen combination, then we elect it.
Step 7: Elect the next least cost from the chosen combination and repeat Step 5 until all columns and rows is exhausted.

4. NUMERICAL EXAMPLES

4.1. Example 1.

Consider the following mixed fuzzy transportation problem. Here the availability of the problem:
Solution: The given mixed fuzzy transportation problem is a balanced fuzzy transportation problem. Here the cost matrix contains ve real numbers and rest are trapezoidal fuzzy numbers. Here the availability of the product available at the three origins and the demand of the product available at the four destinations, unit cost of the product from each origin to each destination is represented by trapezoidal fuzzy numbers shown in Table 1.

Table 1. Mixed constraint balanced fuzzy transportation

	d_1	d_2	d_3	d_4	Supply
s_1	2.5	(1, 3, 4, 6)	(9, 11, 12, 14)	(5, 7, 8, 11)	(1, 6, 7, 12)
s_2	1.75	0.5	(5,6,7,8)	1.5	(0, 1, 2, 3)
s_3	(3,5,6,8)	8.5	(12,15,16,19)	(7,9,10,12)	(7,10,12,17)
Demand	(5, 7, 8, 10)	(1, 5, 6, 10)	(1, 3, 4, 7)	(1,2,3,5)	

We used the definition of trivial trapezoidal fuzzy numbers to convert the real values as trapezoidal fuzzy numbers shown in Table 2. Ranking of trapezoidal fuzzy numbers is important task to solve the problem. So we used the robust ranking method to rank the trapezoidal and trivial trapezoidal fuzzy numbers. We applied the robust ranking method and reduced table is shown in Table 3.

Table 2. Trivial trapezoidal transportation

	d_1	d_2	d_3	d_4	Supply
s_1	(2.5,2.5,2.5,2.5)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,12)
s_2	(1.75,1.75,1.75,1.75)	(0.5,0.5,0.5,0.5)	(5,6,7,8)	(1.5,1.5,1.5,1.5)	(0,1,2,3)
s_3	(3,5,6,8)	(8.5,8.5,8.5,8.5)	(12,15,16,19)	(7,9,10,12)	(7,10,12,17)
Demand	(5,7,8,10)	(1,5,6,10)	(1,3,4,7)	(1,2,3,5)	

Table 3. Ranking of mixed trapezoidal fuzzy transportation

	d_1	d_2	d_3	d_4	Supply
s_1	2.5	3.5	11.5	7.75	6.5
s_2	5.5	0.5	6.5	1.5	1.5
s_3	5.5	8.5	15.5	9.5	11.5
Demand	7.5	5.5	3.75	2.75	

After ranking the numbers we shall determine the whole tableau and able to used the new best candidate method shown in Table 4.

Table 4. Selection of Best candidates in transportation

	d_1	d_2	d_3	d_4	Supply
s_1	2.5	3.5	11.5	7.75	6.5
s_2	5.5	0.5	6.5	1.5	1.5
s_3	5.5	8.5	15.5	9.5	11.5
Demand	7.5	5.5	3.75	2.75	

It is obvious from the Table 5. that the optimal solution obtained by new best candidate method. We calculate the cost and get the result as:

Table 5. Apply new best candidate method

	d_1	d_2	d_3	d_4	Supply
s_1	2.5	$^{2}3.5$	$^{3.75}11.5$	$^{2.75}7.75$	6.5
s_2	5.5	$^{1.5}0.5$	6.5	1.5	1.5
s_3	$^{7.5}5.5$	$^{4}8.5$	15.5	9.5	11.5
Demand	7.5	5.5	3.75	2.75	

$= (11.5)(3.75) + (7.75)(2.75) + (0.5)(1.5) + (5.5)(7.5) + (8.5)(4)$
$= 140.4375$

4.2. Example 2.

Consider the following mixed fuzzy transportation problem. Here the availability of the product available at the three origins and the demand of the product at three destinations. Unit cost of the product from each origin to each destination is represented by mixed trapezoidal fuzzy numbers shown in Table 6.

Table 6. Mixed constraint unbalanced fuzzy transportation

	d_1	d_2	d_3	Supply
s_1	4	(2,4,5,6)	(1,5,6,7)	(1,2,3,4)
s_2	4.75	2.75	(0,1,2,3)	(1,4,5,5)
s_3	(5,6,8,9)	(1,5,6,7)	5.5	(1,3,5,7)
Demand	(1,2,4,6)	(0,1,1,2)	(3,5,7,8)	

Solution: The given mixed fuzzy transportation problem is a unbalanced fuzzy transportation problem. Here the cost matrix contains four real numbers and rest are trapezoidal fuzzy numbers. Here the availability of the product available at the three origins and the demand of the product available at the three destinations, unit cost of the product from each origin to each destination is represented by trapezoidal fuzzy numbers shown in Table 6. The above problem is unbalanced so we make it balanced shown in Table 7.

Table 7. Trivial trapezoidal transportation

	d_1	d_2	d_3	Supply
s_1	(4,4,4,4)	(2,4,5,6)	(1,5,6,7)	(1,2,3,4)
s_2	(4.75,4.75,4.75,4.75)	(2.75,2.75,2.75)	(0,1,2,3)	(1,4,5,5)
s_3	(5,6,8,9)	(1,5,6,7)	(5.5,5.5,5.5,5.5)	(1,3,5,7)
s_4	0	0	0	0.5
Demand	(1,2,4,6)	(0,1,1,2)	(3,5,7,8)	

We used the definition of trivial trapezoidal fuzzy numbers to convert the real values as trapezoidal fuzzy numbers shown in above Table. We used the robust ranking method to rank the trapezoidal and trivial trapezoidal fuzzy numbers we used the robust ranking method which is also applied in trivial trapezoidal fuzzy numbers also and the reduced table is shown in Table 8. After ranking the numbers has been determined by the whole tableau and able to used the new best candidate method shown in Table 9, then apply new best candidate method Table 10.

Table 8. Ranking of mixed trapezoidal fuzzy transportation

	d_1	d_2	d_3	Supply
s_1	4	4.25	4.75	2.5
s_2	4.75	2.75	1.5	3
s_3	7	4.75	5.5	4
s_4	0	0	0	0.5
Demand	3.25	1	5.75	

Table 9. Selection of best candidates in transportation

	d_1	d_2	d_3	Supply
s_1	4	4.25	4.75	2.5
s_2	4.75	2.75	1.5	3
s_3	7	4.75	5.5	4
s_4	0	0	0	0.5
Demand	3.25	1	5.75	

Table 10. Apply new best candidate method

	d_1	d_2	d_3	Supply
s_1	$^{2.5}4$	4.25	4.75	2.5
s_2	4.75	$^{1}2.75$	$^{2}1.5$	3
s_3	$7^{0.75}$	4.75	$5.5^{3.25}$	4
s_4	0	0	$0^{0.5}$	0.5
Demand	3.25	1	5.75	

We calculate the cost and get the result as,
$= (4)(2.5) + (2.75)(1) + (1.5)(2) + (7)(0.75) + (5.5)(3.25) + (0)(0.5)$
$= 38.875$

5. CONCLUSION

In this paper new method is proposed for finding the optimal solution of mixed trapezoidal fuzzy transportation problem. The balanced and unbalanced mixed trapezoidal fuzzy transportation problem are discussed and a numerical example is solved to illustrate the proposed method. The proposed method is easy to apply for solving the fuzzy transportation problem of mixed type. In real life situations this type of new ideas are necessary to face the problems because we can apply the discussed methods.

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