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Extended FTOPSIS with Distance and Set Theoretic-Based Similarity Measure

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ABSTRACT

Comparing fuzzy numbers is an essential process in deducing the output of many fuzzy decision making methods. One of the comparison methods commonly used is by using similarity measure. The main advantage of the similarity measure over other approaches is its ability to minimize the loss of information in the computational process. Several similarity measures have been applied effectively in fuzzy decision making methods. In this paper, a new similarity measure based on the geometric distance, the center of gravity, Hausdorf distance and the set theoretic similarity formula known as the Dice similarity index are incorporated into the Extended Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) method particularly in calculating the closeness coefficients. This similarity measure is in favor of others as it is able to discriminate two similar shape fuzzy numbers effectively with two different locations. A validation process is carried out by implementing the proposed procedure of the Extended FTOPSIS with the new similarity measure in solving a supplier selection problem and the ranking outcome is then compared with the Extended FTOPSIS with other existing similarity measure. The result shows that the Extended FTOPSIS with the proposed similarity measure gives a consistent result without reducing any information in the computational process.

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1. INTRODUCTION

Decision making (DM) in fuzzy environment requires extensive use of fuzzy numbers. Rating of alternatives and criteria weights determination are commonly expressed in terms of fuzzy linguistic values defined mathematically by fuzzy numbers. Due to this representation, comparison of fuzzy numbers become a crucial element in solving a DM problem. Similarity measure is a very useful means for the purpose of comparing fuzzy numbers as it has the advantage of minimizing the loss of information in the computational process [1]. Early works on fuzzy similarity measures can be found in [2]and [3]. Various fuzzy similarity measures which take into consideration factors like distance, center of gravity, spread, Jaccard index, Dice similarity index, geometric mean and geometric shape characteristics like height, area and parameter have been introduced in the literature [4-10]. Recently [11] introduced a generalized similarity measure that can measure most types of fuzzy numbers, meanwhile [12] proposed a similarity with multiple features to overcome shortcomings of some existing similarity measures.

Similarity measure has been applied to solve problems in various fields. Similarity measure has been incorporated in the development of a fuzzy knowledge based system [13]. Ref[14] introduced and utilized similarity measure in matching fingerprint image. Risk analysis problems have been solved by several similarity approaches [8,12,15]. A forecasting problem has been studied by [16] where a combined method of fuzzy time series and similarity measure is used to predict the future stock values. In addition, similarity measures have also been integrated in system development which include a recommender system for product classification [17], Fuzzy Logic-Dempster Theory system [18] and a system for determining fuzzy region merging on image segmentation [19]. The applications of similarity measures in ranking decision alternatives in DM methods have also been explored by many researchers, in particular, the FTOPSIS has been the focus of attention. Similarity functions [1,20-21]have also been integrated in variants of FTOPSIS like the Extended FTOPSIS and the Modified FTOPSIS which then were applied to solve supplier selection problems. Ref [22] introduced a similarity measure based on the generalized Łukasiewicz structure in FTOPSIS that enhances patent ranking results.

Distance and set theoretic based similarity measure proposed by [10] has the ability to discriminate two similar shape fuzzy numbers effectively with two different locations. The performance of the measure in determining the similarity degrees of compared generalized trapezoidal fuzzy numbers is found to be comparable with some existing measures. In this paper, the similarity measure is incorporated in the Extended FTOPSIS based decision making [1] specifically in calculating the closeness coefficients of decision alternatives. Implementation of the new decision making procedure in solving a supplier selection problem in a supply chain system adopted from [1]is then carried out to investigate the consistency of the ranking results.

2. PRELIMINARIES

In this section, a new decision making (DM) procedure is proposed whereby the similarity measure by [10] is integrated into the extended FTOPSIS procedure [1]. Some definitions related to the generalized trapezoidal fuzzy numbers (GTFNs) are given as follows.

Definition 1 [23]

A generalized trapezoidal fuzzy number (GTFN) $\tilde{A} = (a_1, a_2, a_3, a_4 : \omega_{\tilde{A}})$ is a fuzzy set defined by a membership function $\mu_{\tilde{A}}(x) : \Re \rightarrow [0,1]$ where

$$\mu_{\tilde{A}}(x) = \begin{cases} \omega_{\tilde{A}}\left(\dfrac{x - a_1}{a_2 - a_1}\right) & , \quad a_1 \le x \le a_2 \\ \omega_{\tilde{A}} & , \quad a_2 \le x \le a_3 \\ \omega_{\tilde{A}}\left(\dfrac{x - a_4}{a_3 - a_4}\right) & , \quad a_3 \le x \le a_4 \\ 0 & , \quad otherwise \end{cases} \tag{1}$$

such that $a_1, a_2, a_3, a_4 \in \Re$, $a_1 \le a_2 \le a_3 \le a_4$ and $\omega_{\tilde{A}} \in [0,1]$.

In particular, for $\omega_{\tilde{A}} = 1$, $\tilde{A}$ is called a trapezoidal fuzzy number if $a_1 < a_2 < a_3 < a_4$, a triangular fuzzy number if $a_1 < a_2 = a_3 < a_4$, and is a singleton if $a_1 = a_2 = a_3 = a_4$. Figure 1 shows a graphical representation of a GTFN.

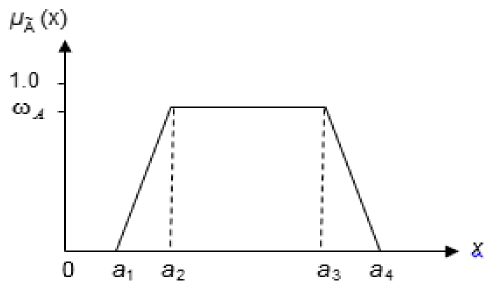

Figure 1. The Graphical Representation of GTFN

Definition 2[24]

Operations on two GTFNs $\tilde{A}=\left(a_1,a_2,a_3,a_4:\omega_{\tilde{A}}\right)$ and $\tilde{B}=\left(b_1,b_2,b_3,b_4;\omega_{\tilde{B}}\right)$.

a) Addition: $\left(\tilde{A}\oplus\tilde{B}\right)=\left(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4;min(\omega_{\tilde{A}},\omega_{\tilde{B}}\right)$

b) Multiplication: $\left(\tilde{A}\otimes\tilde{B}\right)=\left(a_1b_1,a_2b_2,a_3b_3,a_4b_4;min(\omega_{\tilde{A}},\omega_{\tilde{B}})\right)$.

Definition 3[10]

Given a continuous universe U = [0,1] and a set of generalized fuzzy numbers over U, $FS(U)$. Let $\tilde{A}=\left(a_1,a_2,a_3,a_4:\omega_{\tilde{A}}\right)$ and $\tilde{B}=\left(b_1,b_2,b_3,b_4:\omega_{\tilde{B}}\right)$ be two generalized trapezoidal fuzzy numbers in $FS(U)$ and $S:FS(U)\times FS(U)\to[0,1]$. The similarity measure between $\tilde{A}$ and $\tilde{B}$ is defined as

$$S\left(\tilde{A},\tilde{B}\right)=\left(1-\frac{1}{4}\sum_{i=1}^{4}|a_i-b_i|\right)\left(1-\left|\hat{x}_{\tilde{A}}-\hat{x}_{\tilde{B}}\right|\right)^{B\left(S_{\tilde{A}},S_{\tilde{B}}\right)}$$

$$\left(\frac{2\omega_{\tilde{A}}\omega_{\tilde{B}}\left[(a_1+a_2)(b_1+b_2)+(a_3+a_4)(b_3+b_4)\right]}{(\omega_{\tilde{A}}^{2})\left((a_1+a_2)^2+(a_3+a_4)^2\right)+(\omega_{\tilde{B}}^{2})\left((b_1+b_2)^2+(b_3+b_4)^2\right)}\right)$$

$$\left(\frac{1}{1+\left(max\left\{|a_1-b_1|,|a_2-b_2|,|a_3-b_3|,|a_4-b_4|\right\}+\left|\omega_{\tilde{A}}-\omega_{\tilde{B}}\right|\right)}\right)$$

where $\hat{x}_{\tilde{A}}$ and $\hat{y}_{\tilde{A}}$ are the horizontal center of gravity (COG) of $\tilde{A}$ and $\tilde{B}$ calculated as

$$\hat{x}_{\tilde{A}}=\frac{\hat{y}_{\tilde{A}}(a_2+a_3)+\left(\omega_{\tilde{A}}-\hat{y}_{\tilde{A}}\right)(a_1+a_4)}{2\omega_{\tilde{A}}},$$

$$\hat{y}_{\tilde{A}}=\begin{cases}\dfrac{\omega_{\tilde{A}}}{6}\left(\dfrac{a_3-a_2}{a_4-a_1}+2\right) & if\quad a_1\neq a_4\ and\ 0<\omega_{\tilde{A}}\leq 1,\\ \dfrac{\omega_{\tilde{A}}}{2} & if\quad a_1=a_4\ and\ 0<\omega_{\tilde{A}}\leq 1\end{cases}$$

and $B\left(S_{\tilde{A}},S_{\tilde{B}}\right)=\begin{cases}1 & if\ S_{\tilde{A}}+S_{\tilde{B}}>0\\ 0 & if\ S_{\tilde{A}}+S_{\tilde{B}}=0\end{cases}$ such that $S_{\tilde{A}}=a_4-a_1$ and $S_{\tilde{B}}=b_4-b_1$.

The similarity measure $S\left(\tilde{A},\tilde{B}\right)$ satisfies the following properties:

(P1) Two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are identical if and only if $S\left(\tilde{A},\tilde{B}\right)=1$.

(P2) $S\left(\tilde{A},\tilde{B}\right)=S\left(\tilde{B},\tilde{A}\right)$

(P3) If $\tilde{A}=\left(a,a,a,a:\omega_{\tilde{A}}\right)$ and $\tilde{B}=\left(b,b,b,b:\omega_{\tilde{B}}\right)$ are real numbers, then

$$S\left(\tilde{A},\tilde{B}\right)=\left(\frac{2\omega_{\tilde{A}}\omega_{\tilde{B}}ab}{\left(\omega_{\tilde{A}}\right)^2a^2+\left(\omega_{\tilde{B}}\right)^2b^2}\right)\left(\frac{1-|a-b|}{1+|a-b|+\left|\omega_{\tilde{A}}-\omega_{\tilde{B}}\right|}\right).$$

The above similarity measure embeds four elements in the formula which are the geometric distance, the center of gravity, Hausdorff distance, and Dice similarity index that are important and favorable in similarity measurement. The measure has the advantage of discriminating two similar shape fuzzy numbers effectively with two different locations [10].

3. EXTENDED FTOPSIS USING DISTANCE AND SET THEORETIC-BASED SIMILARITY MEASURE

An extended FTOPSIS procedure incorporating a similarity measure by [10] particularly in calculating the closeness coefficients of the decision alternatives is presented as follows.

Step 1: Set up a committee of *K* decision makers to determine the importance weights of *n* criteria and to rate *m* alternatives based on the criteria. Linguistic terms and the corresponding trapezoidal fuzzy numbers used for these purposes are as shown in Table 1. In the following steps, let $\tilde{w}_j^k = (w_{jk1}, w_{jk2}, w_{jk3}, w_{jk4})$ and $\tilde{x}_{ij}^k = (a_{ijk}, b_{ijk}, c_{ijk}, d_{ijk})$ represent the weights of criteria C_j and ratings of alternatives A_i given by the *k*-th decision-maker, D_k, respectively, with $i = 1,2,...,m$, $j = 1,2,...,n$ and $k = 1,2,...,K$.

Table 1. Linguistic Terms for Criteria Weights and Rating of Alternatives

Criteria Weight	Fuzzy Number	Linguistic Terms	Fuzzy Number
Very low (VL)	(0.0, 0.0, 0.1, 0.2)	Very Poor (VP)	(0, 0, 1, 2)
Low (L)	(0.1, 0.2, 0.2, 0.3)	Poor (P)	(1, 2, 2, 3)
Medium Low (ML)	(0.2, 0.3, 0.4, 0.5)	Medium Poor (MP)	(2, 3, 4, 5)
Medium (M)	(0.4, 0.5, 0.5, 0.6)	Fair (F)	(4, 5, 5, 6)
Medium High (MH)	(0.5, 0.6, 0.7, 0.8)	Medium Good (MG)	(5, 7, 8, 9)
High (H)	(0.7, 0.8, 0.8, 0.9)	Good (G)	(7, 8, 8, 9)
Very High (VH)	(0.8, 0.9, 1.0, 1.0)	Very Good (VG)	(8, 9, 10, 10)

Step 2: Obtain the aggregated criteria weight $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$ with respect to C_j where

$$w_{j1} = \min_k \{w_{jk1}\},\ w_{j2} = \frac{1}{K}\sum_{k=1}^{K} w_{jk2},\ w_{j3} = \frac{1}{K}\sum_{k=1}^{K} w_{jk3},\ w_{j4} = \max_k \{w_{jk4}\}.$$

Step 3: Obtain the aggregated fuzzy ratings, $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$, of the *i*-th alternative with respect to C_j, $j = 1,2,...,n$ such that

$$a_{ij} = \min_k \{a_{ijk}\},\quad b_{ij} = \frac{1}{K}\sum_{k=1}^{K} b_{ijk},\quad c_{ij} = \frac{1}{K}\sum_{k=1}^{K} c_{ijk},\quad d_{ij} = \max_k \{d_{ijk}\}.$$

Step 4: Construct a fuzzy decision matrix, $D = [\tilde{x}_{ij}]_{mxn}$, and the normalized fuzzy decision matrix, $\tilde{R} = [\tilde{r}_{ij}]_{m\times n}$ where

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{d_j^*}, \frac{c_{ij}}{d_j^*}, \frac{d_{ij}}{d_j^*}\right),\ d_j^* = \max_i \{d_{ij}\},\ j \in B;$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right),\ a_j^- = \min_i \{a_{ij}\},\ j \in C$$

with *B* and *C* representing benefit and cost criteria, respectively.

Step 5: Obtain the weighted normalized fuzzy decision matrix, $\tilde{V}$, by multiplying the normalized fuzzy decision matrix $\tilde{R} = [\tilde{r}_{ij}]_{m\times n}$ and the column criteria weight matrix, $\tilde{W} = [\tilde{w}_1, \tilde{w}_2, ..., \tilde{w}_n]$. We have $\tilde{V} = [\tilde{v}_{ij}]_{m\times n}$ where

$$\tilde{v}_{ij} = \tilde{r}_{ij} \otimes \tilde{w}_j = (v_{ij1}, v_{ij2}, v_{ij3}, v_{ij4}).$$

Step 6: Determine the Fuzzy Positive Ideal Solution (FPIS), $A^* = \{A_{T1}^*, A_{T2}^*\}$ and the Fuzzy Negative Ideal Solution (FNIS), $A^- = \{A_{T1}^-, A_{T2}^-\}$ such that

$$A_{T1}^* = (\tilde{v}_1^*, ..., v_j^*, ..., \tilde{v}_n^*) \text{ with } \tilde{v}_j^* = \left(\max_i\{v_{ij4}\}, \max_i\{v_{ij4}\}, \max_i\{v_{ij4}\}, \max_i\{v_{ij4}\}\right),$$

$$A_{T1}^- = (\tilde{v}_1^-, ..., \tilde{v}_j^-, ..., \tilde{v}_n^-) \text{ with } \tilde{v}_j^* = \left(\min_i\{v_{ij1}\}, \min_i\{v_{ij1}\}, \min_i\{v_{ij1}\}, \min_i\{v_{ij1}\}\right);$$

$A^*_{T2} = \left(\tilde{v}^*_1, ..., v^*_j, ..., \tilde{v}^*_n\right)$ with $\tilde{v}^*_j = \left(\max_i\{v_{ij1}\}, \max_i\{v_{ij2}\}, \max_i\{v_{ij3}\}, \max_i\{v_{ij4}\}\right)$,

$A^-_{T2} = \left(\tilde{v}^-_1, ..., \tilde{v}^-_j, ..., \tilde{v}^-_n\right)$with $\tilde{v}^*_j = \left(\min_i\{v_{ij1}\}, \min_i\{v_{ij2}\}, \min_i\{v_{ij3}\}, \min_i\{v_{ij4}\}\right)$.

Step 7: Using Definition 3, calculate the similarity values S^*_i and S^-_i for the *i*-th alternative from FPIS, $A^* = \{A^*_{T1}, A^*_{T2}\}$ and FNIS, $A^- = \{A^-_{T1}, A^-_{T2}\}$ where

$$S^*_i\left(A_i, A^*\right) = \sum_{j=1}^{n} S_i\left(\tilde{v}_{ij}, \tilde{v}^*_j\right), \quad S^-_i\left(A_i, A^-\right) = \sum_{j=1}^{n} S_i\left(\tilde{v}_{ij}, \tilde{v}^-_j\right), \quad i = 1, 2, ..., m.$$

Step 8: Calculate two types of closeness coefficients for the *i*-th alternative, $CC_{T1}(A_i)$ and $CC_{T2}(A_i)$, $i = 1, 2, ..., m$ where

- $$CC_{T1}(A_i) = \frac{S^*_i(A_i, A^*_{T1})}{S^*_i(A_i, A^*_{T1}) + S^-_i(A_i, A^-_{T1})},$$
- $$CC_{T2}(A_i) = \frac{S^*_i(A_i, A^*_{T2})}{S^*_i(A_i, A^*_{T2}) + S^-_i(A_i, A^-_{T2})}.$$

Step 9: Determine the ranking position of the *i*-th alternatives according to the CC_{T1} and CC_{T2} values. The higher the value, the higher is the ranking position.

4. RESULTS AND ANALYSIS

In this section, the performance of the similarity measure by [10] in the context of decision making (DM) is investigated by implementing the proposed DM procedure in solving a supplier selection problem adopted from [1] in which the data are presented in Table 2 and Table 3.

Numerical Example: A high-technology manufacturing company desires to select a suitable material supplier to purchase new products. After preliminary screening, five candidates, $A_i, i = 1,2,...,5$ are shortlisted for further evaluation. A committee of decision makers, $D_k, k = 1,2,3$ rate the shortlisted suppliers based on five benefit criteria $C_j, j = 1,2,...,5$ shown in Table 1.

Table 2. Decision Criteria

Decision criteria	C_j
Profitability of supplier	C_1
Relationship closeness	C_2
Technological capability	C_3
Conformance quality	C_4
Conflict resolution	C_5

The weights of criteria and rating of alternatives by the decision makers are presented in Table 3 and 4, respectively.

Table 3. Importance Weights of Criteria C_j by Decision Makers D_k

C_1			C_2			C_3			C_4			C_5		
D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
H	H	H	VH	VH	VH	VH	VH	H	H	H	H	H	H	H

Table 4. Rating of alternatives A_i , based on criteria C_j by decision maker D_k

	C_1			C_2			C_3			C_4			C_5		
	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
A_1	MG	MG	MG	MG	MG	VG	G	G	G	G	G	G	G	G	G
A_2	G	G	G	VG	VG	VG	VG	VG	VG	G	VG	VG	VG	VG	VG
A_3	VG	VG	G	VG	G	G	VG	VG	G	VG	VG	VG	G	VG	G
A_4	G	G	G	G	G	MG	MG	MG	G	G	G	G	G	G	G
A_5	MG	MG	MG	MG	G	G	MG	MG	MG	MG	MG	G	MG	MG	MG

Table 5 shows the weighted normalized fuzzy decision matrix obtained by performing Step 3 to Step 5 in the procedure presented in Section 3.

Table 5. Weighted Normalized Fuzzy Decision Matrix

	C_1	C_2	C_3	C_4	C_5
A_1	(0.35,0.48,0.56,0.72)	(0.35,0.53,0.64,0.90)	(0.49,0.64,0.64,0.81)	(0.49,0.64,0.64,0.81)	(0.49,0.64,0.64,0.81)
A_2	(0.56,0.72,0.80,0.90)	(0.64,0.72,1.00,1.00)	(0.64,0.72,1.00,1.00)	(0.56,0.72,0.93,1.00)	(0.64,0.72,1.00,1.00)
A_3	(0.49,0.69,0.87,1.00)	(0.49,0.69,0.81,1.00)	(0.49,0.69,0.87,1.00)	(0.56,0.69,0.93,1.00)	(0.49,0.69,0.81,1.00)
A_4	(0.49,0.64,0.64,0.81)	(0.35,0.59,0.61,0.81)	(0.35,0.53,0.59,0.81)	(0.49,0.64,0.64,0.81)	(0.49,0.64,0.69,0.90)
A_5	(0.35,0.48,0.56,0.72)	(0.35,0.59,0.61,0.81)	(0.35,0.48,0.56,0.72)	(0.35,0.53,0.59,0.81)	(0.35,0.48,0.56,0.72)

The fuzzy positive ideal solution (FPIS), $A^* = \{A^*_{T1}, A^*_{T2}\}$ and the fuzzy negative ideal solution (FNIS), $A^- = \{A^-_{T1}, A^-_{T2}\}$ are determined based on the weighted normalized fuzzy decision matrix. The corresponding FPISs and FNISs are presented as follows.

$$A^*_{T1} = ((1,1,1,1),\ (1,1,1,1),\ (1,1,1,1),\ (1,1,1,1),\ (1,1,1,1)),$$
$$A^-_{T1} = ((0.35,0.35,0.35,0.35),(0.35,0.35,0.35,0.35),(0.35,0.35,0.35,0.35),(0.35,0.35,0.35,0.35),$$
$$(0.35,0.35,0.35,0.35)),$$
$$A^*_{T2} = ((0.56,0.72,0.87,1),\ (0.64,0.72,1,1),(0.64,0.72,1,1),(0.56,0.72,0.93,1),\ (0.64,0.72,1,1)),$$
$$A^-_{T2} = ((0.35,0.48,0.56,0.72),(0.35,0.53,0.61,0.81),(0.35,0.48,0.56,0.72),(0.35,0.53,0.59,0.81),$$
$$(0.35,0.48,0.56,0.72)).$$

Using Definition 3, similarity values between each alternative and the two types of FPIS (FNIS) are calculated. Based on these values, the closeness coefficients $CC_{T1}(A_i)$ and $CC_{T2}(A_i)$, $i = 1,2,...,5$ are derived, and the results are displayed in Table 6 and Table 7, respectively. Ranking orders of alternatives obtained based the calculated closeness coefficients with respect to the two types of FPIS and FNIS are also compared with the ranking orders by [1].

Table 6. Closeness Coefficients of Alternatives, $CC_{T1}(A_i)$

Similarity values		Criteria					Closeness Coefficient	Ranking	
		C_1	C_2	C_3	C_4	C_5	$CC_{T1}(A_i)$	Proposed Method	Result using [1]
A_1	$S^*_1(A_1,A^*_{T1})$	0.138	0.195	0.251	0.251	0.251	0.337	4	4
	$S^-_1(A_1,A^-_{T1})$	0.603	0.452	0.360	0.360	0.360			
A_2	$S^*_2(A_2,A^*_{T1})$	0.364	0.503	0.503	0.429	0.503	0.736	1	1
	$S^-_2(A_2,A^-_{T1})$	0.231	0.139	0.139	0.176	0.139			
A_3	$S^*_3(A_3,A^*_{T1})$	0.363	0.350	0.364	0.421	0.350	0.628	2	2
	$S^-_3(A_3,A^-_{T1})$	0.221	0.236	0.221	0.180	0.236			
A_4	$S^*_4(A_4,A^*_{T1})$	0.251	0.182	0.168	0.250	0.285	0.358	3	3
	$S^-_4(A_4,A^-_{T1})$	0.360	0.494	0.521	0.360	0.310			
A_5	$S^*_5(A_5,A^*_{T1})$	0.138	0.182	0.138	0.168	0.138	0.213	5	5
	$S^-_5(A_5,A^-_{T1})$	0.603	0.493	0.603	0.521	0.603			

Table 7. Closeness coefficients of alternatives, $CC_{T2}(A_i)$

Similarity values		Criteria					Closeness Coefficient	Ranking	
		C_1	C_2	C_3	C_4	C_5	$CC_{T2}(A_i)$	Proposed Method	Result using [1]
A_1	$S_1^*(A_1,A_{T2}^*)$	0.365	0.433	0.479	0.553	0.479	0.387	4	4
	$S_1^-(A_1,A_{T2}^-)$	0.902	0.830	0.708	0.756	0.462			
A_2	$S_2^*(A_2,A_{T2}^*)$	0.797	0.932	0.932	0.938	0.932	0.713	1	1
	$S_2^-(A_2,A_{T2}^-)$	0.511	0.364	0.330	0.429	0.191			
A_3	$S_3^*(A_3,A_{T2}^*)$	0.832	0.713	0.777	0.937	0.713	0.643	2	2
	$S_3^-(A_3,A_{T2}^-)$	0.452	0.552	0.452	0.436	0.314			
A_4	$S_4^*(A_4,A_{T2}^*)$	0.553	0.399	0.370	0.553	0.553	0.404	3	3
	$S_4^-(A_4,A_{T2}^-)$	0.708	0.878	0.833	0.756	0.403			
A_5	$S_5^*(A_5,A_{T2}^*)$	0.365	0.399	0.313	0.429	0.313	0.300	5	5
	$S_5^-(A_5,A_{T2}^-)$	0.902	0.878	0.902	0.919	0.658			

Based on Table 6 and Table 7, it is observed that similarity values between the *i*-th alternatives, A_i, $i = 1,2,...,5$ and the FPIS, A_{T2}^* (or FNIS, A_{T2}^-) are relatively higher as compared to similarity values between the *i*-th alternatives and the FPIS, A_{T1}^* (or FNIS, A_{T1}^-), across all criteria. Nevertheless, both types of closeness coefficients, $CC_{T1}(A_i)$ and $CC_{T2}(A_i)$, lead to similar orderings of alternatives in which A_2 is ranked first followed by A_3, A_4, A_1 and A_5. The ranking orders of alternatives are found to be consistent with [1].

5. CONCLUSION

In this paper, we have proposed an Extended FTOPSIS procedure incorporating the similarity measure proposed by [10]. The advantage of the similarity measure [10] which are composed of the geometric distance, the center of gravity, Hausdorf distance and the Dice similarity index lies in its ability in discriminating two similar shape fuzzy numbers effectively with two different locations. The implementation of the Extended FTOPSISin which the closeness coefficients of the decision alternatives are calculated using the similarity measure given by [10] is elucidated with an application in solving a supplier selection problem. Two different rules for determining the positive ideal solution and the negative ideal solution [1] are employed. The result shows that ranking orders of alternatives with respect to any type of ideal solutions being used as benchmarking fuzzy numbers in calculating the similarity degrees are consistent with one another. Comparison with [1] also indicates that ranking of alternatives are preserved as both approaches give similar orderings of alternatives. For future work, different similarity measures may be considered in determining the closeness coefficients in the Extended FTOPSIS and comparative analysis can be made.

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