Development of Approximate Prediction Model for 3-DOF Helicopter and Benchmarking with Existing Controllers

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Abstract
        Recent trend of living is getting modernized rapidly by the involvement of automatic systems. Within the aviation industry, automatic systems had become heavily reliable by the end of the nineteen centuries. The systems usually require controllable devices with desired control algorithm known as controller. Controllers can be replaced with, almost every mechanical automation aspect where, safety is a serious issue. But it is not easy to adapt a controller with a specific model at the beginning. It is important to predict the model before a controller works on the model and the controller parameters need to be adapted to get maximum efficiency. A 3-DOF (Three Degrees of Freedom) airframe model is an advanced benchmark model of real 3-DOF helicopter. It has the same uncommon model dynamics with nonlinearities, strong duel motor cross coupling system, uncertain characteristics, disturbances dependent, unmodeled dynamics and many more. The 3-DOF airframe model is a well-known platform for controller performance benchmarking. This research paper shows the development of an approximate prediction model of a Three Degrees of Freedom helicopter model and uses the proposed approximate model to observe the performance of an existent hybrid controller. The hybrid controller is the combination of two different controllers named Quantitative Feedback Theory (QFT) controller and Adaptive controller. To achieve the research objective, the proposed mathematical model of this airframe was used to develop transfer function and simulate with the hybrid controller in MATLAB. The performance of the controller based on the proposed heliframe model of 3-DOF helicopter have also been reported added within this paper.

Keywords: 3-Degrees of Freedom helicopter, Benchmarking, Hybrid controller, Uncertainty characteristics, Unmodelled dynamics, Disturbances, Nonlinearities, Uncertainty of dynamics

1. Introduction
        The 3-DOF Helicopter model is a simplified airframe model, useful to step-up from an intermediate to advance level of model concepts and relevant to the real applications of 3-DOF flight control dynamics, real helicopters or an equivalent similar model. For the model, it is important to understand how: I) to predict an approximate 3-DOF model, ii) design the transfer function of the angles known as elevation, pitch and travel, using the proposed approximate 3-DOF model, iii) Simulate the angles and evaluate its performance using a hybrid combined controller and iv) determine the model performance for future research. Though, development of approximate model for 3-DOF helicopter is not new in the control field, however to describe the model in a way where all the possible problems were considered is a unique approach. Previous research has described the model in their own way where, their concept was similar but with a different aspect.

1.1. Three Degrees of Freedom (3-DOF) Helicopter
        The three degrees of freedom (3-DOF) Helicopter airframe has two main parts, i) the airframe and ii) a metal base. The model has two main parallels named Front and Back propeller which is run by two DC motors as similar to a duel rotor helicopter. A 3-DOF helicopter airframe is shown in Figure 1 developed by Quanser [1]. The model body suspends on the

metal base mounted at the 1:2 point of the long arm and it is free to move within two of degrees movement known as elevation angles and travel angles. A propeller arm is suspended on one end of the body which can be free to one degree of freedom known as pitch angle. Another end of the body arm is joined with readjustable counterweight to reduce the body weight of the other side. The three angles are measured by encoders having high regulation to get angle positions. The front and back propellers of the 3-DOF airframe control the three-axis movement of the heliframe [2-3].

The 3-DOF heliframe principal is to observe the performance of the controllers by controlling the three-degrees angles using the front and back propeller, to make helicrane movement. The 3-DOF model is horizontal when the elevation is zero. If the travel angle moves to the positive direction, the propeller arm rotates in a counter-clockwise direction. Again, if the pitch angle is positive it means, $\rho(t) > 0$, the force of the front propeller is greater than the back propeller. If both propellers produce the same thrust and increase gradually, it moves the body up on the elevation axis. Again, pitch angle also controls the travel angle position of the helicrane [4-8]. The parts of the 3-DOF helicrane consist of the front propeller, back propeller, main heliframe body arm, propeller arm and counterweight as shown in Figure 1 [1].
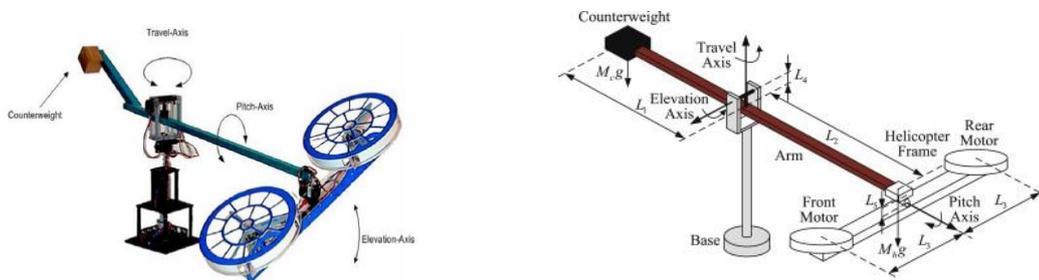

Figure 1. 3-DOF helicopter model manufactured by Quanser [1]

1.2. Automatic Controller

Automatic controller is the application process of control theory to regulate the processes or systems without direct human involvement [9-11]. A automatic controller connected with a system having measured process of the system by the controller, having a preset value and processes the control output by correcting the errors, because the system need to control by the set point despite of uncertain disturbances [12]. This type of system with close-loop properties is known as negative feedback. The development of mathematical design based controller to use for automatic control theory started rapidly in the 20th century [13].

1.3 Hybrid Controller

The hybrid controller was designed by two individual controllers known as the adaptive controller and robust QFT controller. Before combining the controllers, the controllers are calibrated separately for the helicopter simulation. The Quanser provide LQR controller as default which was maximized for the 3-DOF helicopter model. For the current research the LQR controller was replaced by the hybrid controller, while the old helicopter transfer functions were replaced by proposed transfer functions.
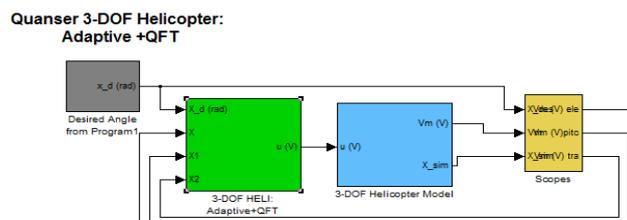

Figure 2. Simulation of the system modulation of 3-DOF helicopter [17]

Figure 2, shows the simulation of the system modulation of 3-DOF helicopter hybrid combined controller and proposed approximate transfer function model block. The four blocks in this simulation also contains, own internal sub-blocks. The first simulation block is the desired angle sub program block, the second block is the hybrid controller block (adaptive +QFT), the third block contains the proposed approximate 3-DOF helicopter model transfer functions and the last block contains all the output measurement scopes [14-19]. The desired angle block is the system block that can be used to alter the three angles named as pitch, elevation and travel. Figure 3 shows the internal controllers' connectivity of the hybrid controller (Adaptive +QFT) block. According to the QFT controller procedure the filters are required before each and every angle [20].
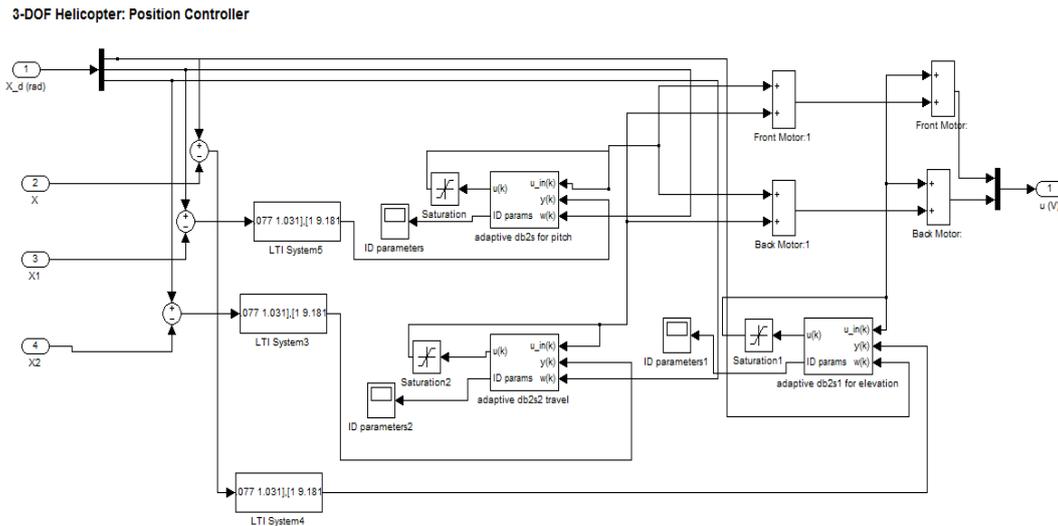

Figure 3. Internal controllers' connectivity of hybrid controller block (Adaptive +QFT) [18]

2. Research Method
In this paper a 3 DOF helicopter mathematical model has been developed. The model dynamics were developed starting from the motor dynamics because the two propellers have the same configuration and efficiency. The mathematical modelling is derived below.

2.1. Motor Dynamics
According to the general controller derivation process the motor dynamics is separated into two parts: i) electrical equation and ii) mechanical equation. The derivations are given below.

2.1.1. Electrical Equation
For this plant, suppose the DC motor used in the model has the same configuration a possible DC motor circuit is shown in Figure 4.
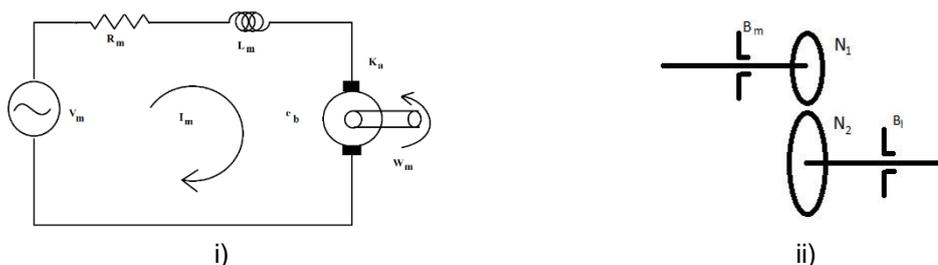

i)   ii)

Figure 4. DC motor close loop circuit and gear wheel design

Suppose, in the DC motor circuit R_m, L_m, Ka, e_b and ω_m are the motor resistance, coil inductance, constant of back e.m.f, back e.m.f. and motor shaft speed respectively. So,

$$e_b = K_a * \omega_m. \tag{1}$$

If current the current I_m , using

$$\text{KVL: } V_m[t] - R_m * I_m[t] - L_m \frac{d\,I_m[t]}{dt} - k_a * \omega_m[t] = 0 \tag{2}$$

If, L_m is too small then R_m. So, L_m can be ignored from the Equation 2,

$$V_m[t] - R_m * I_m[t] - k_a * \omega_m(t) = 0; \tag{3}$$

The relation between the angular velocity and the input voltage can be found by using Laplace transformation.

$$\text{Im [t] = Im [s], } \omega m \text{ [t] = } \theta m[s] \text{ and Vm[t] = Vm [s]}$$
$$Vm[s] = R_m * I_m[s] - k_a * [\theta_m[0 - \theta_m[s] * s] \tag{4}$$

Though, initial time is zero, So,

$$Vm[s] = Rm * Im[s] - ka * s * \theta m[s] \ (\theta m(0) = 0.)$$
$$Im[s] = \frac{V_m([s]) - s * K_a * \theta_m[s]}{R_m[s]} \tag{5}$$

2.1.2 Mechanical Equations
The DC motor has one-degrees movement, the ω_l and τ_m are the shaft speed and torque of motor respectively. Using Newton's theorem, $J*\alpha = \tau$. Here J, α, τ, Bm and Bl are the moment of inertia, angular acceleration, total torque, viscous friction and viscus friction of load on shaft then, $Jml*\omega' m[t] + Bm*\omega m[t] = Km*Im$, [Where, τm=Km*Im], Using Laplace form when initial condition is zero,

$$(Jml*\theta''m)[s] + (Bm*\theta' m)[s] = (Km.Im)[s]$$
$$Im[s] = \frac{\theta_m(s)(J_{ml}*s^2 + B_m*s)}{K_m} \tag{6}$$

2.1.3 Comparing the Mechanical and Electrical Equations
From the mechanical equation 5 in section 2.1.1 and electrical equation 6 in section 2.1.2,

$$\frac{V_m - s*K_a*\theta_m}{R_m} = \frac{\theta_m (J_{ml}*s^2 + B_m*s)}{K_m}$$
$$\frac{\theta_m}{V_m} = \frac{[\frac{K_m}{R_m}]}{s\,(J_{ml}*s + B_m + [\frac{K_m}{R_m}]K_a\,)} \tag{7}$$

If, η_g and K_g are the Gear efficiency and Gear-ratio. Where, τ_l and τ_m are depending on η_g and K_g Then, $\theta_l = K_g * \theta_m$. The Figure 4 ii), shows the motor shaft and load shaft of a gear. Then,
$$J_{eq} = J.*(N_1/N_2)^2 \text{ and } B_{eq} = B.*(N_1/N_2)^2.$$
If, G_{ml} is the motor load equation then, A_m is the gain, B_{eq} is the viscous damping and J_{eq} is the moment of inertia accordingly.

$$G_{ml} = \frac{\theta_l}{V_m} = \frac{\eta_g * \eta_m * K_g * K_t}{J_{eq}*R_m*s^2 + (B_{eq}*R_m + \eta_g*\eta_m*K_m*K_t*K_g^2)*s} \tag{8}$$

By solving equation no. 8, we found motor transfer function and from the state space solution we also developed the parameter equations and values.

2.2 Equation for the Angles

The 3-DOF heliframe have three different freedom angles so it requires three differential equations for the model dynamics. The angle equations developed by Quanser for each angle are discussed below.

2.2.1 Angle Equation for Pitch

Though, the angle of pitch depends on the propeller thrust difference. If the front propeller produces more thrust, than the back propeller then the frame will pitch upward. while, if the back propeller produces more power than the front one the airframe will pitch to downwards. If "ρ" pitch angle, moment of inertia J_p, F_f and F_b are the front and back propeller force, l_h body length and V_d is the maximum voltage difference then the equation for pitch angle is [21-22],

$$J_p\, \ddot{\rho} = K_f l_h V_d \tag{9}$$

2.2.2 Angle Equation for Travel

The force generated by the pitch also causes Travel angle movement. Considering the 3-DOF airframe, if the pitch angle increases by "ρ" then the frame will fly back and vice versa. If the frame needs to be hold in a certain position, on a certain position the propeller requires a stable force, F_g where F_f and F_b are the two-horizontal component of force of front and back propeller respectively [21-22]. According to Quasar the proposed equation for acceleration of the component of F_f and F_b for when the Heliframe can go forward or backward are,

$$J_t \cdot \dot{\tau} = -\{F_g \sin(\rho) * l_a\} \tag{10}$$

$$J_t \cdot \dot{\tau} = -\{K_p \sin(\rho) * l_a\} \tag{11}$$

Here, $\dot{\tau}$ is the travel angle, J_t and K_p are the moment of inertia and force required to maintain the heliframe in a certain position. If pitch is zero, it means no movement have done.

2.2.3 Angle Equation for Elevation

Considering the heliframe weight "m" and force required to hold up the heliframe is F_m. If the pitch angle is zero, for maintained elevation angle F_m will be the resulting force of the both propeller F_f and F_b. So, $F_m = F_f + F_b$. If the both propellers produce the same force, the resulting force change the elevation angle. After analyzing all parameters, the equating was solved as,

$$J_e \ddot{\varepsilon} = K_f l_a V_s - T_g \tag{12}$$

Where, K_f, J_e, V_f, V_b, l_a & T_g are the force constant, moment of inertia, voltage applied to the front motor and voltage applied to the back motor, heliframe body from the pivot point & the effect of gravitational torque respectively. The counter weight $M_g = 70$gms is used to adjust the F_g (mass differential function).

3.3 Angular Transfer Function of the controller

Three transfer functions that we have developed from the equating no. 9, 10, 11 & 12 for the three angles of 3-DOF helicopter are represented below.

2.3.1 Elevation Controller Transfer Function

If T_g is neglected from the equation 9, $J_e * \ddot{\varepsilon} - K_f * l_a * V_s = 0$ and $V_s = K_{ep}(\varepsilon - \varepsilon_c) + K_{ed}\dot{\varepsilon}$ [if, $V_s = V_f + V_b$]. So, the equation stands as,

$$J_e \ddot{\varepsilon} - (K_f . l_a . K_{ep} . \varepsilon) + (K_f . l_a . K_{ep} . \varepsilon_c) - (K_f . l_a . K_{ed} . \dot{\varepsilon}) = 0 \tag{13}$$

By Laplace transformation,

$$\frac{\varepsilon}{\varepsilon_c} = \frac{\frac{(K_f * l_a * K_{ep})}{J_e}}{s^2 + \frac{(K_f * l_a * K_{ep})}{J_e} + \frac{(K_f * l_a * K_{ed} * s)}{J_e}}$$

$$\frac{\varepsilon}{\varepsilon_c} = \frac{\frac{K_f.la.K_{ep}}{J_e}}{\{s^2 + s\frac{(K_f.la.K_{ed})}{J_e} + \frac{(K_f.la.K_{ep})}{J_e}\}} \tag{14}$$

2.3.2 Pitch Controller Transfer Function

The Pitch angle of the 3-DOF heliframe depends on 'r' which is known as the travel ratio. Using the equation 9, similarly we can develop the Pitch controller transfer function that is,

$$\frac{\rho}{\rho_c} = \frac{\frac{K_f.l_h.K_{pp}}{J_p}}{s^2 + s\frac{K_f.l_h.K_{pd}}{J_p} + \frac{K_f.l_h.K_{pp}}{J_p}} \tag{15}$$

From the equation 15, the $\frac{\rho}{\rho_c}$ is the ratio of pitch angle output to the initial pitch and error compensation is $(\rho - \rho_c)$.

2.3.3 Travel Angle Controller Transfer Function

For the travel angle, using the equations 10 and 11, it is possible to linearize the process and a close loop system is designed to find the desired travel angle τ_c and the travel angle dependent on the pitch angle, so we can write,

$$J_t\dot{\tau} = K_p\rho l_a \qquad \text{and} \qquad \rho = K_{\tau p}(\tau - \tau_c) + K_{\tau i}\int(\tau - \tau_c) \tag{16}$$

By solving the both equations we can find travel angel equation, that is,

$$\frac{\tau}{\tau_c} = \frac{-\frac{s.K_p.l_a.K_{\tau p} + K_p.l_a.K_{\tau i}}{j\tau}}{s^2 - s\frac{K_p.l_a.K_{\tau p}}{j\tau} - \frac{K_p.l_a.K_{\tau i}}{j\tau}} \tag{17}$$

2.4. Final Output Equation for The Controller

To finalize the equations, we need to know the motor parameters which we have found quasar motor specification or data page. From the equation 8, we can get the final motor transfer function and can determine the gains of the system. Solving the equations no. 14, 15 and 17 it is possible to finalize the three-degrees angles transfer functions, as,

$$\rho(t) = \frac{\rho}{\rho_c} = \frac{(0.6004)}{(s^2 + 0.9522716s + 0.6004)} \tag{18}$$

$$\varepsilon(t) = \frac{(3.139167) * s}{\{s^2 + s.(0.958333) - 3.139167\}} \tag{19}$$

$$\tau(t) = \frac{\{s(0.06331) - 0.13275\}}{\{s^2 + s.(0.06331) + 0.13275\}} \tag{20}$$

3. Results and Analysis

The development of the three angles transfer function has been done. Though it is an approximate mathematical expression, but it is required to determine the performance of the approximate model with a proposed hybrid controller which we developed previously.

3.1. Hybrid Controller Performance

The previous developed hybrid controller is a combined controller. It was designed using robust QFT and adaptive controller and can be simulated by the well-known platform MATLAB. From the simulation we have observed the hybrids controller output which can be used for future research.

3.1.1. Travel Angle performance

The Figure no.5 shows the travel angle performance waveform for hybrid controller. For better understanding Table 1 is showing the summarized performance of the controller with the derivation.

Table 1. Travel angle output for Hybrid controller

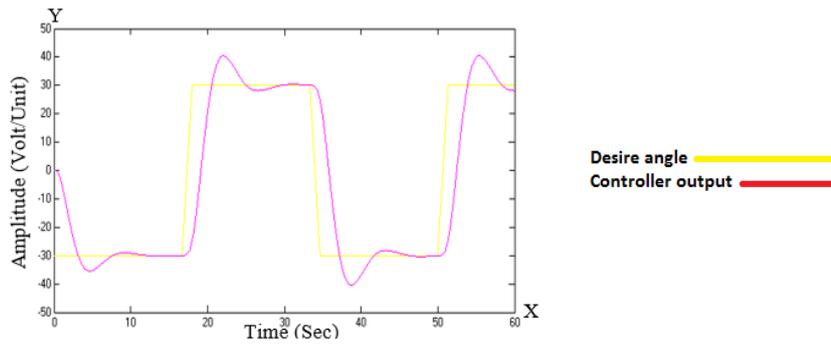| Angle name	Settling time	Overshoot	Steady-state error (Ess)
Hybrid controller travel angle	12.5ms	26.67 %	00

Figure 5. Travel angle performance waveform for hybrid controller

3.1.2. Elevation Angle performance

Figure 6 is the performance waveform of elevation angle output for the hybrid controller and the values are projected in Table no.2.

Table 2. Elevation angle output waveform for the hybrid controller.

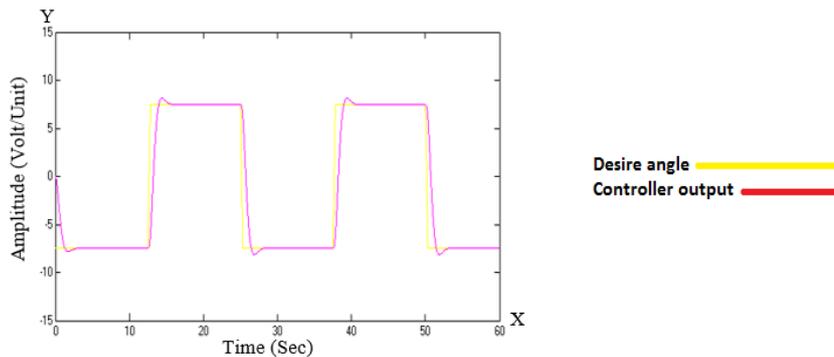| Angle name	Settling time	Overshoot	Steady-state error(Ess)
Hybrid controller elevation angle	5ms	10 %	00

Figure 6. Elevation angle output waveform for the hybrid controller.

3.1.3. Pitch Angle performance

Figure 7 shows the pitch angle output waveform for the hybrid controller and the performance is projected in table no.3.

Table 3. Pitch angle results for the hybrid controller.

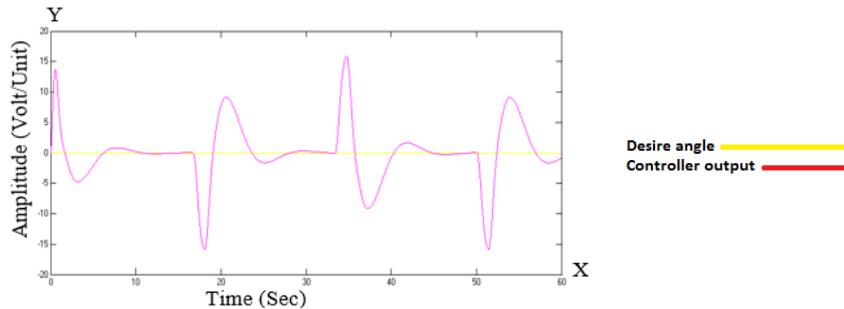| Angle name	Settling time	Overshoot	Steady-state error(Ess)
Hybrid controller Pitch angle	10.8ms	0 %	00

Figure 7. pitch angle output waveform for the hybrid controller

4. Conclusion

Several performance tests were done by tuning the controller. The results of the hybrid controller, in some places showed better performance than previous experiments, but it cannot state that this approximate model is better than previous models until a comparison has been carried out. From all the individual test, the model is working like the previous model developed by Quasar. The same hybrid controller is performing on both models which employ that this model is also a developed model of 3-DOF heliframe. This proposed model has considered the efficiency of the motor system and determined the controller gains from the equations. Thus, disturbance and extra load on controller was mitigated. On the travel angle movement, the overshoot is high, it is done to keep system stability, otherwise the system can be volatile because of mass moment of inertia. The Hybrid controller was re-simulated again and again after changing each and every parameter changes. While designing the approximate mathematical expression for the heli fly model, we used common methods of controller expression used to develop common electrical circuit controller and mechanical automotive hardware controller concept. These concepts are not new in engineering but combining both fields produce a new approximate model for this research. This research was conducted on a simulation platform, but it can be suggested that this model can be a better mathematical replacement model of the 3-DOF heliframe for the researcher who can simulate their controller easily and where direct hardware application is a cost sensitive issue.

5. Future work

Further research will be conducted on a new type of controller design and carry out simulations simulate with this approximate model. It can be possible to simulate more existing controllers with this current model and prepare a benchmark performance indicator that can be useful for future researchers.

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