

Modified BPNN via Iterated Least Median Squares, Particle Swarm Optimization and Firefly Algorithm

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Abstract

There is doubtlessly manufactured artificial neural system (ANN) is a standout amongst the most acclaimed all-inclusive approximators, and has been executed in numerous fields. This is because of its capacity to naturally take in any example with no earlier suppositions and loss of all inclusive statement. ANNs have contributed fundamentally towards time arrangement expectation field, yet the nearness of exceptions that normally happen in the time arrangement information may dirty the system preparing information. Hypothetically, the most widely recognized calculation to prepare the system is the backpropagation (BP) calculation which depends on the minimization of the common ordinary least squares (OLS) estimator as far as mean squared error (MSE). Be that as it may, this calculation is not absolutely strong within the sight of exceptions and may bring about the bogus forecast of future qualities. Accordingly, in this paper, we actualize another calculation which exploits firefly calculation on the minimal middle of squares (FA-LMedS) estimator for manufactured neural system nonlinear autoregressive (BPNN-NAR) and counterfeit neural system nonlinear autoregressive moving normal (BPNN-NARMA) models to cook the different degrees of remote issue in time arrangement information. In addition, the execution of the proposed powerful estimator with correlation with the first MSE and strong iterative slightest middle squares (ILMedS) and molecule swarm advancement on minimum middle squares (PSO-LMedS) estimators utilizing reenactment information, in light of root mean squared blunder (RMSE) are likewise talked about in this paper. It was found that the robustified backpropagation learning calculation utilizing FA-LMedS beat the first and other powerful estimators of ILMedS and PSO-LMedS. As a conclusion, developmental calculations beat the first MSE mistake capacity in giving hearty preparing of counterfeit neural systems.

Keywords: Anomalies; time series, learning algorithm, robust estimators, evolutionary algorithms.

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1. Introduction

The backpropagation calculation depends on the feedforward multilayer neural system for an arrangement of inputs with determined known orders. The calculation permits multilayer feedforward neural systems to take in info yield mappings from preparing tests [1]. Once every section of the specimen set is displayed to the system, its yield reaction will be analyzed by the system as for the example information design. The yield reaction is then contrasted with the known and sought yield and the blundering quality is figured, where the association weights are balanced. The backpropagation calculation depends on Widrow-Hoff delta learning principle in which the weight change is done through mean square error (MSE) of the yield reaction to the example info [2]. The arrangement of these specimen examples is over and again introduced to the system until the mistake quality is minimized. Despite the fact that ANNs have effectively caught the premium and worry of numerous specialists in numerous fields because of its

widespread capacity as capacity approximator, the notable backpropagation learning calculation which depends on the minimization of the mean square mistake (MSE) cost capacity, is not vigorous within the sight of anomalies that may bring about blunder in information preparing process [3]. MSE is a mistake measure between the genuine and craved yield that is utilized as a part of the mainstream backpropagation learning calculation of multilayered feedforward neural systems (MFNNs) preparing. [3] Concur that this prominent calculation is not totally strong within the sight of exceptions. Indeed, even a solitary anomaly can destroy the whole neural system fit [4].

For all intents and purposes, acquiring great information is the most entangled a portion of estimating [5]. In this way, achieving complete and smooth genuine information are right around zero likelihood. Exceptions are information seriously going amiss from the example set of the dominant part information. It has been accounted for that the event of anomalies reaches about 1% to more than 10% in normal routine information [6] [7]. In view of past studies [8-10] the presence of these exceptions represents an extreme danger to the standard or customary minimum squares investigation.

In time arrangement investigation, the examiners need to depend on information to recognize which point in time are exceptions to gauge the proper remedial moves to be made so that the distorted occasions can be assessed precisely. Hypothesis and practice are greater part worried with direct techniques, such ARMA and ARIMA models [11]. Be that as it may, numerous arrangement show design which cannot be clarified by a linear system which triggers the need for non-direct models, for instance bilinear models [12] and non-straight ARMA models (NARMA) [13].

2. Material and method

In this examination, there were three distinctive reenactment information were utilized. Foundation Noise Data focuses were chosen aimlessly and after that substituted with likelihood δ with a foundation commotion consistently appropriated in the particular race.

Case 1- With a specific end goal to test our calculation on the 1-D approximation task, the capacity by [8] was considered in this examination, as likewise utilized by past works, for example, [14-17].

$$y = |x|^{-\frac{2}{3}} \quad (1)$$

The facts utilized because that looking after incorporate over $N=400$ focuses to that amount have been constructed by using trying out the autonomous variable within the scope on $[-2, 2]$ with meantime 0.01. Durability.

Case 2-Another 1-D capability to lie approximated was once as regarded of much articles [14-18] characterised as:

$$y = \frac{\sin(x)}{x} \quad (2)$$

The records utilized for it evaluation comprise concerning $N=1500$ focuses that were built by inspecting the self sustaining variable into the scope on $[-7.5, 7.5]$ along interim 0.01. Case 3-The 2nd calculation was once as proposed through [9][14] as do lie characterized as:

$$y = x_1 e^{-x_1^2 - x_2^2} \quad (3)$$

The records focuses have been committed by using tested capability of the honor 16 x 16 framework. Here, the informational collection used to be manufactured through checking out the fair factors, x_1, x_2 $[-2, 2]$ including meantime 0.01. In the investigation flowchart of mass 4, the experiment method execute stand honestly observed.

Here, the current rigid estimators regarding backpropagation neural skeleton had been

completed. To reply the middle goal of the investigation, the possible butter strenuous estimators of nonlinear autoregressive (NAR) yet nonlinear autoregressive transferring regular (NARMA) regarding the neural fabric time course regarding labor were completed the usage of MATLAB.

At this progression, MATLAB scripts or codings were composed parallel to the scientific plan done before. After that, the execution of the proposed robustified neural system models was thought about utilizing recreation information; 1-D and 2-D utilizing the standard execution measure, root mean square mistake (RMSE). At that point the powerful BPNN-NAR and BPNN-NARMA technique were tried on benchmark information. The similar results have attracted those strides.

3. Results and discussions

In view of the Tables 1, 2 and 3, the conventional calculation delivered the best results in light of the littlest RMSE values for the perfect information without exceptions. This result is parallel with the case that the MSE mistake capacity is ideal for the information without anomalies by [3][14][19].

In any case, the circumstance is changed since the information containing misleadingly produced anomalies where the MSE-based strategy totally loses its productivity. This can be demonstrated by the breakdowns of the technique as appeared in Tables 1, 2 and 3. Every single hearty calculation of ILMedS, PSO-LMedS and FA-LMedS perform fundamentally better contrasted with MSE-based cost capacity. Additionally, by simply looking at the two developmental improvement calculations, FA-LMedS has indicated more prominent execution with the most minimal RMSE values in every one of the four cases, as appeared in Tables 1, 2 and 3. This is maybe an immediate aftereffect of the reason that firefly estimation perform better for a greater measure of disturbance [20] and when it joined into a backpropagation neural system, preparing calculation, it unites at a quicker rate with least feedforward neural system plan [21]. As specified by [22], the fireflies' calculation is the exceptional instance of quickened molecule swarm enhancement calculation. Besides, contrasted with FA-LMedS and ILMedS, the PSO-LMedS calculation has a tendency to accomplish more noteworthy RSME as the rate of exceptions increment. In this situation, PSO-LMedS was seen to deliver more noteworthy blunders for the information comprising exceptions more than 60 percent, as appeared in Tables 1, 2 and 3. Here, it is trusted that the PSO-LMedS may perform better on the off chance that we expand the quantity of swarm size and cycle esteem. The modified neural network can be further implemented to solve many other real-life problems, such as image processing [23], water treatment plant [24] and power plant [25].

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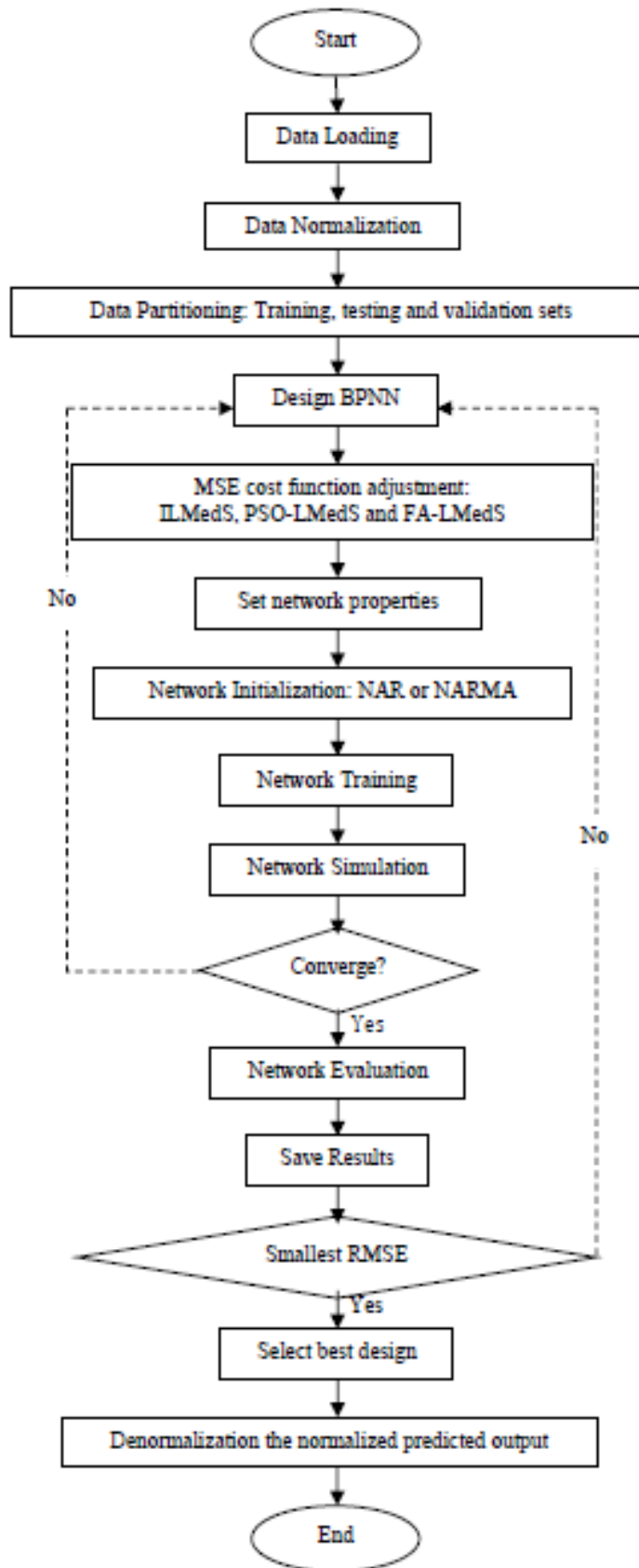


Figure 1. Process of the proposed BPNN-NAR and BPNN-NARMA

Table 1. The RMSE scores for test function in case

Input Lags (Ny)	Error Lags (Ne)	Hidden	Swarm Size	Iteration	Percentage of Anomalies (δ)	Algorithm							
						MSE		ILMedS		PSO-LMedS		FA-LMedS	
						NAR	NARMA	NAR	NARMA	NAR	NARMA	NAR	NARMA
2	2	5	5	5	0	0.0165	0.0144	0.0575	0.0687	0.0070	0.0024	0.0006	0.0006
2	2	10	5	5	0	0.0317	0.0232	0.0428	0.0480	0.0094	0.0088	0.0009	0.0009
2	2	20	5	5	0	0.0559	0.0373	0.0190	0.0242	0.0095	0.0035	0.0010	0.0009
2	2	5	5	5	10	2537.2571	2243.7403	0.2686	0.2622	0.0096	0.0088	0.0073	0.0055
2	2	10	5	5	10	7806.4086	6968.4181	0.2729	0.2827	0.0076	0.0085	0.0040	0.0048
2	2	20	5	5	10	3197.1237	2696.4526	0.2304	0.3171	0.0061	0.0071	0.0024	0.0013
2	2	5	5	5	20	2691.9232	2483.4234	0.4367	0.4806	0.0274	0.0128	0.0005	0.0069
2	2	10	5	5	20	2681.6996	1561.1672	0.1847	0.1636	0.0236	0.0153	0.0041	0.0073
2	2	20	5	5	20	3837.5206	3669.8813	0.2282	0.2405	0.0291	0.0286	0.0092	0.0026
2	2	5	5	5	30	3436.0919	4352.2928	0.2321	0.2266	0.0376	0.0294	0.0003	0.0050
2	2	10	5	5	30	3483.9997	3468.8221	0.2042	0.1561	0.0321	0.2293	0.0047	0.0056
2	2	20	5	5	30	8194.4084	9399.368	0.2379	0.2445	0.0530	0.3392	0.0068	0.0071
2	2	5	5	5	40	4842.7778	3768.5752	0.2070	0.2182	0.0271	0.0250	0.0080	0.0024
2	2	10	5	5	40	12529.7165	12785.6565	0.1596	0.1331	0.1574	0.0971	0.0042	0.0018
2	2	20	5	5	40	2616.636	2040.8494	0.1534	0.1704	0.0213	0.0060	0.0023	0.0020
2	2	5	5	5	50	4613.7007	4797.0081	0.1421	0.1756	0.0503	0.0276	0.0028	0.0070
2	2	10	5	5	50	6696.0732	7111.9116	0.1207	0.1631	0.0688	0.0193	0.0071	0.0073
2	2	20	5	5	50	5870.884	7450.8175	0.1590	0.1733	0.0799	0.011	0.0007	0.0028
2	2	5	5	5	60	6304.1263	5309.1228	0.1658	0.1702	10.0524	7.0512	0.0037	0.0034
2	2	10	5	5	60	15210.3341	19075.8857	0.1980	0.1629	3.1016	5.6931	0.0005	0.0039
2	2	20	5	5	60	2440.4538	3412.5697	0.1811	0.1740	6.7627	8.3582	0.0266	0.092
3	3	5	5	5	0	0.2266	0.9105	0.0730	0.0137	0.0598	0.0061	0.0331	0.0381
3	3	10	5	5	0	0.1888	0.5777	0.0946	0.0687	0.0476	0.0737	0.0704	0.0363
3	3	20	5	5	0	0.8679	0.3433	0.0920	0.0176	0.0177	0.0954	0.0756	0.0610
3	3	5	5	5	10	2422.4903	2354.6119	0.1888	0.1870	0.0290	0.0106	0.0631	0.0243
3	3	10	5	5	10	2961.5131	2019.4265	0.1657	0.1843	0.0308	0.0471	0.0733	0.0626
3	3	20	5	5	10	1703.9431	1457.9506	0.2092	0.2251	0.0286	0.0561	0.0702	0.0752
3	3	5	5	5	20	2782.5779	2530.3214	0.2185	0.2076	0.0301	0.0203	0.0629	0.0421
3	3	10	5	5	20	4023.1339	2749.3138	0.2966	0.2736	0.0289	0.0742	0.0740	0.0395
3	3	20	5	5	20	2388.1344	3923.2193	0.1268	0.1406	0.0265	0.0052	0.0390	0.0408
3	3	5	5	5	30	2340.4295	2597.6883	0.1562	0.1727	0.0302	0.0089	0.0995	0.2822
3	3	10	5	5	30	2565.6173	2779.8586	0.2117	0.1821	0.0193	0.0020	0.0981	0.0690
3	3	20	5	5	30	8138.588	10017.3766	0.1410	0.1479	0.0897	0.0312	0.0137	0.0487
3	3	5	5	5	40	3929.4474	4579.1315	0.3575	0.3967	0.0392	0.0628	0.0867	0.0609
3	3	10	5	5	40	3069.3175	2993.2578	0.1428	0.1480	0.0456	0.0206	0.0176	0.0983
3	3	20	5	5	40	3252.8795	3788.1158	0.2190	0.2242	0.0284	0.0468	0.0870	0.0507
3	3	5	5	5	50	4103.3899	3686.9169	0.2666	0.2622	0.0473	0.0135	0.0843	0.0731
3	3	10	5	5	50	3213.4135	3489.5679	0.2729	0.2827	0.0349	0.0253	0.0251	0.0554
3	3	20	5	5	50	2344.8786	2746.5734	0.2304	0.3171	0.0339	0.0704	0.0076	0.0121
3	3	5	5	5	60	4979.5091	4377.9622	0.4367	0.4806	5.0661	3.5519	0.0736	0.0320
3	3	10	5	5	60	4662.0489	47763.665	0.1847	0.1636	4.3144	3.4804	0.0452	0.0295
3	3	20	5	5	60	7100.6318	8801.7551	0.2282	0.2405	7.4872	6.3311	0.0989	0.0926

Table 2. The RMSE scores for test function in case 2

Max Lags of inputs (Ny)	Max Lags of errors (Ne)	Hidden	Swarm Size	Iteration	Percentage of Anomalies (δ)	MSE		Algorithm				PSO-LMedS		FA-LMedS	
						NAR	NARMA	ILMedS		NAR	NARMA	NAR	NARMA	NAR	NARMA
								NAR	NARMA						
2	2	5	5	5	0	0.1288	0.1202	0.3209	0.2715	0.0305	0.0290	0.0297	0.0252		
2	2	10	5	5	0	0.1782	0.1824	0.0272	0.0432	0.0718	0.0230	0.0392	0.0982		
2	2	20	5	5	0	0.2366	0.2595	0.0494	0.0480	0.0225	0.0144	0.0456	0.0751		
2	2	5	5	5	10	1592.8770	1801.0386	0.1236	0.1195	0.1631	0.2421	0.0284	0.0803		
2	2	10	5	5	10	2793.9951	2639.77614	0.1136	0.1076	0.2785	0.3694	0.0473	0.0025		
2	2	20	5	5	10	1788.0502	1642.0878	0.1378	0.1389	0.1734	0.2030	0.0349	0.0092		
2	2	5	5	5	20	1640.7081	1575.8881	0.3490	0.2692	0.1657	0.1801	0.0339	0.0031		
2	2	10	5	5	20	1637.5895	1249.4667	0.0446	0.0410	0.1538	0.1564	0.0506	0.0231		
2	2	20	5	5	20	1958.9590	1915.6934	0.0806	0.0623	0.1705	0.2592	0.0431	0.0159		
2	2	5	5	5	30	1853.6698	2086.2149	0.1412	0.1008	0.1941	0.2523	0.0748	0.0684		
2	2	10	5	5	30	1866.5475	1862.4774	0.0836	0.0333	0.1792	0.2159	0.0197	0.0036		
2	2	20	5	5	30	2862.5877	3065.8388	0.1255	0.0923	0.2302	0.3396	0.0298	0.0016		
2	2	5	5	5	40	2200.6312	1941.2818	0.0933	0.1088	0.1647	0.2346	0.0331	0.0074		
2	2	10	5	5	40	3539.7339	3575.7036	0.0379	0.0247	0.3967	0.5684	0.0218	0.0089		
2	2	20	5	5	40	1617.6019	1428.5830	0.0454	0.0391	0.1460	0.1649	0.1915	0.1705		
2	2	5	5	5	50	2147.9526	2190.2073	0.0407	0.0501	0.2242	0.2224	0.2088	0.1941		
2	2	10	5	5	50	2587.6771	2666.8167	0.0224	0.0277	0.2622	0.3515	0.1862	0.1792		
2	2	20	5	5	50	2422.9907	2729.6185	0.0805	0.0558	0.2827	0.3371	0.3085	0.2302		
2	2	5	5	5	60	2510.8019	2304.1533	0.0779	0.0780	31.7163	37.1251	0.1941	0.1647		
2	2	10	5	5	60	3900.0428	4367.5949	0.0526	0.0480	4.8062	5.9087	0.3575	0.0987		
2	2	20	5	5	60	1562.1951	1847.3141	0.0371	0.0639	1.6367	2.1119	0.1428	0.1400		
3	3	5	5	5	0	0.2388	0.2379	0.0527	0.0578	0.0445	0.0343	0.2304	0.0171		
3	3	10	5	5	0	0.1973	0.2071	0.0693	0.0500	0.0182	0.0055	0.4367	0.4006		
3	3	20	5	5	0	0.1596	0.1596	0.0890	0.1178	0.1331	0.1847	0.1847	0.1836		
3	3	5	5	5	10	1556.4351	1534.4744	0.0496	0.0694	0.2704	0.2132	0.2282	0.2005		
3	3	10	5	5	10	1720.9047	1421.0652	0.0572	0.0803	0.1756	0.1018	0.2321	0.0266		
3	3	20	5	5	10	1305.3517	1207.4562	0.0714	0.0873	0.1631	0.1497	0.2042	0.0561		
3	3	5	5	5	20	1668.1068	1590.6983	0.1202	0.0940	0.1733	0.2461	0.2379	0.0445		
3	3	10	5	5	20	2005.7751	1658.1054	0.1271	0.1724	0.1702	0.2791	0.2070	0.0182		
3	3	20	5	5	20	1545.3589	1980.7118	0.0234	0.0224	0.1629	0.2295	0.1596	0.0331		
3	3	5	5	5	30	1529.8462	1611.7345	0.0301	0.0413	0.1740	0.1926	0.1534	0.0704		
3	3	10	5	5	30	1601.7544	1667.2908	0.0877	0.0620	0.1390	0.1959	0.1421	0.0756		
3	3	20	5	5	30	2852.8210	3165.0239	0.0241	0.0295	0.2995	0.5362	0.1207	0.0631		
3	3	5	5	5	40	1982.2833	2139.8905	0.3209	0.2715	0.1981	0.3002	0.1590	0.0732		
3	3	10	5	5	40	1751.9467	1730.1034	0.0272	0.0432	0.2137	0.0296	0.1658	0.0702		
3	3	20	5	5	40	18035.7409	19463.0826	0.0494	0.0480	0.1687	0.0633	0.1980	0.0629		
3	3	5	5	5	50	20256.8258	19201.3461	0.1236	0.1195	0.2176	0.0983	0.1611	0.0740		
3	3	10	5	5	50	17925.996	18680.385	0.1136	0.1076	0.1870	0.0228	0.1667	0.0390		
3	3	20	5	5	50	1531.2996	1657.2788	0.1378	0.1389	0.1843	0.0363	0.3165	0.0995		
3	3	5	5	5	60	2231.4813	2092.3580	0.3490	0.2692	22.5109	20.7358	0.2139	0.0981		
3	3	10	5	5	60	2159.1776	2185.4899	0.0446	0.0410	2.07621	2.4675	0.1730	0.0137		
3	3	20	5	5	60	2664.7010	2966.7752	0.0806	0.0623	2.7367	2.5661	0.1946	0.0887		

Table 3. The RMSE scores for test function in case 3

Max Lags of inputs (Ny)	Max Lags of errors (Ne)	Hidden	Swarm Size	Iteration	Percentage of Anomalies (δ)	Algorithm							
						MSE		ILMedS		PSO-LMedS		FA-LMedS	
						NAR	NARMA	NAR	NARMA	NAR	NARMA	NAR	NARMA
2	2	5	5	5	0	0.0226	0.0252	0.1801	0.1933	0.0267	0.1636	0.0371	0.0224
2	2	10	5	5	0	0.0436	0.0497	0.1564	0.1718	0.0434	0.1184	0.0307	0.0244
2	2	20	5	5	0	0.1112	0.1186	0.2592	0.2266	0.1081	0.1258	0.0569	0.0672
2	2	5	5	5	10	4359.7566	8641.6380	0.2523	0.2763	0.0365	0.1912	0.057	0.0636
2	2	10	5	5	10	1079.2603	1365.1487	0.2159	0.2381	0.1014	0.1184	0.0525	0.0466
2	2	20	5	5	10	3988.2749	4121.7564	0.3396	0.2974	0.0388	0.1970	0.1150	0.0153
2	2	5	5	5	20	3717.8782	3244.3472	0.2346	0.2201	0.0373	0.1933	0.0756	0.0550
2	2	10	5	5	20	3073.1547	2446.1292	0.5664	0.5211	0.0295	0.1718	0.2471	0.0209
2	2	20	5	5	20	5694.1254	6723.3495	0.1649	0.2080	0.0513	0.1266	0.0424	0.0272
2	2	5	5	5	30	5730.2224	6368.4176	0.2224	0.2191	0.0763	0.1763	0.0517	0.0494
2	2	10	5	5	30	5256.0255	4664.3883	0.3515	0.3456	0.0566	0.1381	0.1019	0.0236
2	2	20	5	5	30	11503.8636	11533.4735	0.3371	0.3280	0.0884	0.1974	0.0969	0.0136
2	2	5	5	5	40	7562.8682	5504.5465	0.3712	0.3728	0.0484	0.1201	0.1372	0.0378
2	2	10	5	5	40	2471.0467	3209.0136	0.5908	0.5189	0.2715	0.1211	0.1727	0.0490
2	2	20	5	5	40	4243.4113	2721.2212	0.2111	0.2025	0.0432	0.1080	0.0336	0.0046
2	2	5	5	5	50	5179.6874	4948.2608	0.2839	0.2496	0.0480	0.1191	0.0866	0.0075
2	2	10	5	5	50	10196.5565	12361.8442	0.3758	0.3175	0.1195	0.1456	0.0269	0.0201
2	2	20	5	5	50	9697.4329	11364.5493	0.2892	0.1827	0.1076	0.1280	0.0248	0.0174
2	2	5	5	5	60	13726.6642	13780.5056	0.3543	0.3038	13.8992	17.2684	0.0156	0.0136
2	2	10	5	5	60	1727.7562	3490.8264	0.3055	0.3298	2.6921	1.1894	0.0366	0.0290
2	2	20	5	5	60	3368.2453	4460.2079	0.1947	0.1574	4.1081	1.0266	0.0435	0.0376
3	3	5	5	5	0	0.0937	0.1255	0.2461	0.2362	0.0923	0.1138	0.1278	0.1174
3	3	10	5	5	0	0.0749	0.0933	0.2791	0.2757	0.1088	0.0298	0.0204	0.0113
3	3	20	5	5	0	0.0381	0.0379	0.2295	0.2192	0.0247	0.0574	0.0479	0.0102
3	3	5	5	5	10	3636.3138	4549.1711	0.1926	0.2528	0.0391	0.0977	0.0711	0.0287
3	3	10	5	5	10	6106.0489	4076.2713	0.1959	0.1776	0.0501	0.0239	0.0745	0.0399
3	3	20	5	5	10	2437.9512	2242.0911	0.5362	0.4409	0.0277	0.0664	0.1014	0.0085
3	3	5	5	5	20	6265.6872	6059.5287	0.3002	0.3139	0.0568	0.0362	0.0388	0.0097
3	3	10	5	5	20	7522.1978	7795.0577	0.2296	0.2405	0.0760	0.0757	0.0373	0.0028
3	3	20	5	5	20	4214.6130	5269.1885	0.2633	0.2238	0.0488	0.0192	0.0295	0.0053
3	3	5	5	5	30	3956.8955	3710.3527	0.2983	0.3432	0.0639	0.0528	0.0513	0.0086
3	3	10	5	5	30	4083.2797	3841.2332	0.2228	0.2635	0.0315	0.1776	0.0763	0.0303
3	3	20	5	5	30	2822.7018	2875.1272	0.2393	0.2834	0.1944	0.1409	0.0566	0.0292
3	3	5	5	5	40	6906.748	9012.3312	0.2673	0.2954	0.0985	0.1139	0.0884	0.0391
3	3	10	5	5	40	4870.0912	5275.4700	0.3467	0.3066	0.0578	0.2405	0.0484	0.0050
3	3	20	5	5	40	6093.7314	6934.9149	0.3566	0.4152	0.0500	0.2238	0.2715	0.0970
3	3	5	5	5	50	9831.8046	8903.4344	0.1529	0.1498	0.1178	0.3432	0.0432	0.0059
3	3	10	5	5	50	5076.8150	4965.6695	0.1735	0.2032	0.0694	0.0635	0.0480	0.0275
3	3	20	5	5	50	7313.6695	5727.4593	0.2962	0.2491	0.0803	0.0834	0.1195	0.0193
3	3	5	5	5	60	55470.641	7144.9387	0.1553	0.1719	8.7385	9.5403	0.1076	0.0114
3	3	10	5	5	60	11215.7927	12026.7402	0.1801	0.1933	9.4028	6.8874	0.1389	0.0704
3	3	20	5	5	60	13201.5013	12719.1829	0.1564	0.1718	1.7246	1.5251	0.2692	0.0156