Hybrid metaheuristic to solve job sequencing problem

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ABSTRACT

In this article, the job shop scheduling problem was solved, which is considered as an NP-Hard problem, so it is difficult to solve. The proposed algorithm consists in the combination and adaptation of the nearest neighbor metaheuristics and iterated local search (ILS) to find optimal solutions. The proposed algorithm was evaluated by comparing the results with those presented in different instances and sizes of the literature. The results of the proposed method were close in high percentages to the best results and averages obtained by other methods in the literature and in some cases, they were improved. The above represents the feasibility of a new combination of efficient methods in solving task scheduling problems.

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1. INTRODUCTION

Business organizations are immersed in a process of constant competition to maintain and increase market share, offering quality products at a competitive price. This can be achieved through process standardization and cost minimization strategies. The reduction of costs in the production processes is the purpose of one of the classic problems of the industry, known as job shop scheduling problem (JSSP) arises from the interest in studying the advantages offered by the use of a computer with many processors in parallel, despite the fact that a set of tasks can be processed in less time by this type of multiprocessing [1], certain anomalies were identified that affect the time required to process a set of tasks [2], classifying it as a NP-hard type combinatorial optimization problem [3]. This type of problem had already been approached using exhaustive exploration methods, such as linear programming, in the search for optimal solutions [4], however, they did not provide the expected results, due to the computational nature of the problem. The nature of the problem classified with NP – hard. Later, heuristic methods and techniques were used in the exploration of feasible solutions, showing that heuristics did not ensure optimal values, but with the advantage of finding solutions in a feasible computational time for the industry. The complexity of the problem lies in its approach, as described:

The JSSP seeks to program a finite set J of n jobs in a finite group M of m machines. Several operations are considered in each job, in addition, the order of the machines for each job is fixed and predefined [5]. The objective of the JSSP is to find an appropriate sequence, represented in a schedule that schedules the appropriate operation for all jobs that can minimize the makespan or one that minimizes the idle time of the machines [6]. The formulation of the model is as follows:
N: Number of jobs.
M: Number of machines.
Pij: Processing time for job j on m/c, j=1, 2…, N and i=1, 2…M
Dj: Job expiration date j.
SEQ: Processing Sequence Matrix.
NUMT: Number of machines for each job
NUMJ: Number of jobs per machine J.
DISJ: Disjunction matrix.

Decision variables:
Cj: Completion time of work j.
Sji: Start time of job j on machine i.
Fji: Completion time of job j on machine i.
Pij: Processing time for job j on m/c, j=1, 2…, N and i=1, 2…M.

The objective function consists of minimizing the makespan by means of the equation.

\[
\text{Makespan} = \text{MAX}(C_j), \forall j \in N
\] (1)

Subject to restrictions:
Restriction to avoid task overlaps

\[
(S_{hi} - S_{ hj}) \geq P_{ hi} - MY_{ hij}, \forall i,j \in N, \forall h \in M
\] (2)

Constraint to satisfy the precedence of operations

\[
(S_{ hj} - S_{ hi}) \geq P_{ hi} - M(1 - Y_{ hij}), \forall i,j \in N, \forall h \in M
\] (3)

\[
\sum_{h \in M} (S_{\text{SEQ}(j,0,j)} + P_{\text{SEQ}(i,0,i)}) \geq \sum_{h \in M} S_{\text{SEQ}(i,j,h)}, \forall j \in N, \forall i \in M - 1
\] (4)

In this problem, jobs J contain the elements \{J_1, J_2, ... J_n\} and a set of machines M contains \{M_1, M_2, ..., M_m\} [7]. Each job Jn must be done by m machines to finish the job, as seen in Figure 1. Each job consists of a set of operations and the order of operation of the machines is predefined. Each operation uses one of the machines to complete its work during a fixed time interval [8]. Once an operation is processed on a particular machine, it cannot be interrupted before the procedure ends. The sequence of operations for a job must be predetermined and can be different for any job. Each job has a sequence of operations. Each machine can process only one operation during the time interval. The goal of the JSSP is to find an appropriate sequence for all jobs, minimizing makespan or machine downtime [6].

![Figure 1. Job shop scheduling problem](image-url)
Different methods have been proposed to solve the JSSP [9–13], using genetic algorithms [14], as well as the shifting bottleneck heuristic [15], which found better results in 20.69% compared to the solution provided by the first-come, first-served priority rule [16], in addition, the bat algorithm [17] is an approximation method that shows better results with respect to the particle swarm heuristic [18–21], when comparing the speed of convergence and the precision. Likewise, it justifies the use of approximation methods in this problem classified as NP-hard because, although exact methods guarantee optimal solutions for small problems, they are not useful for large problems [22]. The method is based on echolocation characteristics of bats, in the search to optimize the problem. Furthermore, the JSSP was solved using the neighborhood search variable method variable neighborhood search (VNS) [23] to find feasible solutions to the problem within a limit of solutions called neighborhood, showing that VNS outperforms results to other metaheuristics with which it was compared, such as genetic algorithms [24] and tabu search method [25].

In addition, the golden ball algorithm metaheuristic was used, adapting it to solve the JSSP, this technique uses soccer concepts, in the search for feasible solutions to the problem, relating concepts such as player with schedule, team with group of schedules, among others, proposing a treatment for the populations generated in which the players are assigned (Schedule) to the teams (group schedules), competing among them and using stoppage criteria. This algorithm proves to be competitive in its results when compared to other algorithms in the literature [26]. The above shows a trend in the search for new methods that provide better performance in solving the problem. In this context, this work proposes an adaptation of the nearest neighbor [27] and iterated local search (ILS) metaheuristics to find feasible solutions to the JSSP, testing the appropriate instances that allow comparing the results of this proposal with other methods from the literature [14], [17], [26], [28].

Some investigations have used the nearest neighbor methods in conjunction with repair techniques to block JSSP problems and with the intention of evaluating, generating a statistical analysis of a large set of generated neighbor solutions to detect critical characteristics of the repair scheme and the space search in general [29]. To solve the multi-objective JSSP, the nearest neighbor method was used to evaluate the density of solutions and guide the search more efficiently [30]. The closest neighbor was used to find not only the closest neighbor of distance 1 but also the closest neighbor of distances n where n>1. To do this, neighbors that are connected by routes of length n were considered [31].

The iterated local search [32] has been widely used to solve problems such as the traveling agent [33], the flow shop scheduling problem [34], inventory routing problem [35], transportation problems [36], the location problem optimal service units [37], vehicle routing problem [38] and the JSSP [39]. The main contribution of the paper is the adaptation of the closest neighbor heuristics with the Iterated Local Search in determination of efficient and feasible solutions to the indicated problem. In section 2 the research methodology is presented, in section 3 is presented the results and discussion, in section 4 the conclusion is presented.

2. RESEARCH METHOD

The present research proposes the adaptation of the nearest neighbor metaheuristics [27] and iterated local search to find feasible solutions to the job shop scheduling problem (JSSP). Unlike previous works in which a local iterated search is carried out in a neighborhood [23], using a variable, this work adapts two metaheuristics, applying the nearest neighbor method to obtain a good feasible solution used as the initial solution of the iterated local search technique, which is later applied to obtain feasible solutions to the JSSP. Diagram alir dari algoritma yang diusulkan as shown in Figure 2.

The pseudocode of the proposed algorithm is as follows:
A. Pseudocode: start
   - Encrypt the instance data.
B. Pseudocode: the closest neighbor.
   - Randomly take an instance operation.
   - Select the next operation, the one that represents the lowest cost.
   - Repeat the previous procedure until obtaining a feasible sequence.
   - Store the solution in a stack of size n.
   - Generate an initial population of size n-1.
   - Add the new population to the stack.
   - Determine the makespan of each element in the stack
C. Pseudocode: Iterated Local Search
   - Define numIter = number of iterations
- For i: 1: numIter
- Generate a new population of n individuals
- Calculate the makespan of each individual in the new population.
- Compare the makespan of each element in the new population with that of each element in the stack.
- If makespan stack element > makespan population element
- Substitute the best in the pile.
- End if
- End For

D. Print to the best sequence with your makespan

Figure 2. Flow diagram of the proposed algorithm

To compare the performance of the algorithm, 30 trials were carried out in each of the tested instances, with the intention of comparing the results obtained by other methods and techniques that are available in the literature for each instance.

3. RESULTS AND DISCUSSION

The method described in the previous section is used to solve instances of the JSSP resolved by other methods in the literature [14], [26], [28], [40]–[42]. With the intention of comparing its results with those obtained by the method proposed in the present investigation, for which 30 tests were carried out for each instance, using a computer with the following characteristics: Intel (R) Core (TM) i5-8265U processor CPU @ 1.60GHz 1.80 GHz, installed RAM 8.00 GB. The results obtained are shown in 2 tables, Table 1, presents the best results of the proposed method compared with the best results obtained by methods in the literature, while Table 2, shows the averages of each instance obtained by the proposed method, comparing them with those of literature [14], [26], [28], [40]–[42].

The results obtained through the proposed algorithm show to be competitive with those of the literature, as seen in Table 1, where the approximation of the results of the proposed algorithm was compared with the best makespan in the literature, in percentage terms, reaching 100% proximity in instances LA01.
LA03 and FT06, with an average percentage of 92.5% of total algorithm approximation for the instances evaluated. The worst results reach an approximation of 76.7, 76.8 and 78.8%, for instances LA17, LA18 and FT20, respectively. On the other hand, the makespan averages of the 30 trials in each instance are compared with the averages of the instances of the different methods in the literature, as presented in Table 2, in which a 100% approximation is found in the FT06 instance, high percentages in instances such as LA02, LA03, LA05 and FT20, with an average of 87.79%. The previous results allow us to identify a high performance of the proposed algorithm, in comparison with methods proposed in the literature, being viable and feasible for its use in solving problems related to the sequencing of activities.

The previous results allow us to identify that the makespan decreases as the number of iterations of the proposed algorithm grows, reducing from 1420 to 850 units in 100 iterations, which represents an improvement of 31.73%. Figure 5 figures the makespan improvements for the LA12 instance, in this Figure an improvement from 2290 to 1400 units is observed in 100 iterations, which represents a reduction of 40.14% in the LA04 instance, which seeks to minimize the makespan in sequencing 20 jobs and 5 machines, reduces the results from 2330 to 1550 in 100 iterations, representing an improvement of 33.43% as shown in Figure 6.

Table 1. Better results found in the literature compared to the results of the proposed method

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Table 2. Instance averages

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The results obtained in some instances are shown below, analyzing the improvements obtained in the first 100 iterations. In Figure 3 it is possible to identify that the makespan decreases as the number of iterations of the proposed algorithm grows, reducing from 1420 to 850 units in 100 iterations, which represents a reduction of 40.14% in the LA04 instance, which seeks to minimize the makespan in sequencing 10 jobs on 5 machines.

The results of the LA12 instance are shown in Figure 4 identifying improvements in each iteration going from 2080 to a makespan of 1420 in 100 iterations, this represents an improvement of 31.73%. Figure 5 shows the makespan improvements for the LA18 instance, in this Figure an improvement from 2290 to 1400 units is observed in 100 iterations, which represents a reduction of 38.86%. The FT20 instance, which seeks to minimize the makespan of the sequencing of 20 jobs and 5 machines, reduces the results from 2330 to 1550 in 100 iterations, representing an improvement of 33.43% as shown in Figure 6.
Hybrid metaheuristic to solve job sequencing problem (Gustavo Erick Anaya-Fuentes)

4. CONCLUSION

The results of the present investigation allow us to conclude that the proposed method is competitive with respect to different methods in the literature, because high percentages of approximation were found in accordance with the results shown in Tables 1 and 2, being better those corresponding to the minimum values found, compared with the average, in addition, it is possible to observe that as the number of iterations in the algorithm grows, the results continue to improve until finding a local optimum or a general optimum without being able to identify whether it is one or another of the above, because this method does not ensure optimization. The best results are presented in small instances, when the problem is less complex, therefore, the metaheuristics still do not ensure the optimization of the problems, but they provide good feasible results. Therefore, it is necessary to continue in the search for techniques and tools that allow greater efficiency in the results. The proposed algorithm could be improved by incorporating operators such as elitism within the process of the so-called stack, as well as optimizing the parameters according to the size of the instance to be evaluated, such as an optimal value in the population size in each iteration and the number of iterations, it is also possible to adapt the algorithm proposed with others from the literature, in the search to improve the results.

REFERENCES
BIOGRAPHIES OF AUTHORS

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