Robust control of a UPFC system with $H_\infty$ control technique

Maamar Benyamina$^{1}$, Mohamed Bouhamida$^{2}$, Tayeb Allaoui$^{3}$, Rachid Taleb$^{4}$, Mouloud Denai$^{5}$
$^{1,2}$Electrical Engineering Department, Mohamed Boudiaf University of Science and Technology, Algeria
$^{3}$Electrical Engineering Department, Ibn Khaldoun University, Algeria
$^{4}$Electrical Engineering Department, Hassiba Benbouali University, LGEER Laboratory, Algeria
$^{5}$School of Engineering and Technology, University of Hertfordshire, United Kingdom

ABSTRACT

FACTS (Flexible AC Transmission Systems) technology has now been accepted as a potential solution to the stability problem and load flow. The Unified Power Flow Controller (UPFC) is considered to be the most powerful and versatile among all FACTS devices. This paper presents the control of a UPFC system using $H_\infty$ robust control technique. A simulation study using Matlab/Simulink is presented to the performance of this control strategy and the robustness with respect to variations of the system parameters such as the inductance of the transmission line.

1. INTRODUCTION

The Unified Power Flow Controller (UPFC), is among the FACTS devices that have attracted the attention of many researchers because it is capable of simultaneously and independently controlling the flow of active and reactive powers in a network. The UPFC combines a shunt compensation, (Static Compensator or STATCOM), and a series compensation (Static Synchronous Series Compensator or SSSC) and has the ability to control three parameters associated with the transit of powers namely the line voltage, the impedance of the line and the load angle. The UPFC is placed on the transmission line between the source and the load as shown in Figure 1 [1, 2].

Figure 1. Electrical network with UPFC

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It consists of two voltage source converters (VSC) connected through a common DC link. The first converter (A) is connected in series and the second (B) in parallel to the line (Figure 2). Converter A performs the main task of the UPFC by injecting an AC voltage adjustable via transformer T2. The role of converter B is to supply or absorb the active power required by Converter A to the common DC circuit. It can also generate or absorb reactive power.

![Figure 2. UPFC components and structure](image)

This paper presents the control design approach based on control H∞ and a robust controller is formulated in the state space domain. H∞ robust control theory is probably the control theme which has led to the largest number of publications and greater efforts since the mid-80s. The main reason for its popularity is because it represents a very natural way to formulate the problem of robustness. It all started with an article by Zames [3], published in 1981 and followed by other articles [4, 5]. These articles did not concern the robustness problem, but rather the disturbance rejection. It is Kimura [4] who, in 1984, formulated the first robust control problem in terms of H∞. Although his position of the problem is hardly different from that proposed by Doyle and Stein [6, 7] (novelty residing in the explicit use of the H∞ framework that allowed Kimura to solve the synthesis problem). Thanks to the important concept of the standard problem that the work of Francis Doyle and [8] were unified.

The resolution of the standard problem grew very significantly in 1988 with Glover-Doyle algorithm [9], which uses state space representation. In the solutions of the problem, they introduced Ricatti equations [6, 8]. This approach is employed in this paper for solving the H∞ control problem. Robust stability is defined as the ability of a system to remain stable when subjected to perturbation such as modeling errors, measurement errors and external disturbances. Robust performance, on the other hand reflects the ability of the system to maintain the specified performance characteristics (stability, decoupling, time response...) when subjected to disturbances. Indeed, a physical system generally has non-linear characteristics that are not usually included in the model for simplification purposes. So an invariant model cannot accurately represent the reality and for this it is necessary to consider these errors in all control techniques. The H∞ control method is considered to be a very powerful design technique which has attracted many researchers from the electric power community.

2. MODELING OF THE UPFC

The simplified phase circuit of the UPFC is shown in Figure 3. Using Kirchhoff’s laws, we can write:

\[
\begin{align*}
V_{sa} - V_{ca} - V_{ra} &= L_s \frac{d}{dt}(i_a + i'_a) + L \frac{d}{dt}i_a + \frac{1}{C_L} \int i_a dt + R_i \\
V_{sb} - V_{cb} - V_{rb} &= L_s \frac{d}{dt}(i_b + i'_b) + L \frac{d}{dt}i_b + \frac{1}{C_L} \int i_b dt + R_i \\
V_{sc} - V_{cc} - V_{rc} &= L_s \frac{d}{dt}(i_c + i'_c) + L \frac{d}{dt}i_c + \frac{1}{C_L} \int i_c dt + R_i 
\end{align*}
\]

(1)

$L$ is the total inductance of the line and the load.
Figure 3. Equivalent circuit of the UPFC

Using Park transformation, this system becomes:

\[
\begin{align*}
V_{ad} - V_{cd} - V_{a} &= L_{a} \frac{d}{dt} \left( i_{q} + i_{a} \right) - \omega L_{a} \left( i_{q} + i_{a} \right) + L \frac{d i_{q}}{dt} - \omega L_{a} i_{q} + V_{cd} + R i_{q} \\
V_{aq} - V_{cq} &= L_{a} \frac{d}{dt} \left( i_{q} + i_{a} \right) + \omega L_{a} \left( i_{q} + i_{a} \right) + L \frac{d i_{q}}{dt} + \omega L_{a} i_{q} + V_{cq} + R i_{q}
\end{align*}
\]

(2)

for the shunt compensator:

\[
\begin{align*}
V_{sd} - V_{pd} &= L_{a} \frac{d}{dt} \left( i_{q} + i_{d} \right) - \omega L_{a} \left( i_{q} + i_{d} \right) + L \frac{d i_{q}}{dt} - \omega L_{a} i_{q} + r_{p} i_{d} \\
V_{sq} - V_{pq} &= L_{a} \frac{d}{dt} \left( i_{q} + i_{d} \right) + \omega L_{a} \left( i_{q} + i_{d} \right) + L \frac{d i_{q}}{dt} + \omega L_{a} i_{q} + r_{p} i_{q}
\end{align*}
\]

(3)

for DC link:

\[
P_{e} = V_{pa} i_{a} + V_{pb} i_{b} + V_{pc} i_{c}
\]

\[
P_{e} = V_{ca} i_{a} + V_{cb} i_{b} + V_{cc} i_{c}
\]

(4)

\[
\frac{1}{2} c \frac{dV_{dc}}{dt} = P_{e} - P_{e}
\]

(5)

Let \( \delta \) and \( \theta \) be the phase shifts between the reference and the converter output voltages \( V_{c} \) and \( V_{p} \), respectively. The d-q components can be expressed as follows [6]:

\[
\begin{align*}
V_{cd} &= k_{1} \cdot V_{dc} \cdot \cos(\delta) \\
V_{cq} &= k_{2} \cdot V_{dc} \cdot \sin(\delta)
\end{align*}
\]

(6)

\[
\begin{align*}
V_{pd} &= k_{1} \cdot V_{dc} \cdot \cos(\theta) \\
V_{pq} &= k_{2} \cdot V_{dc} \cdot \sin(\theta)
\end{align*}
\]

(7)
Assuming that the voltages at the source and receiving end are equal and the influence of the output shunt is neglected, the previous equations become:

\[
\begin{align*}
L_s \frac{di_d}{dt} + L \frac{di_d}{dt} - \omega L_i d - \omega L_i q + u_{cd} + U_i + R_i d &= 0 \\
L_s \frac{di_q}{dt} + L \frac{di_q}{dt} + \omega L_i d + \omega L_i q + u_{cq} + U_i q + R_i q &= 0
\end{align*}
\]

(8)

\[
\begin{align*}
i_d - C_L \frac{du_{cd}}{dt} + \omega C_L u_{cq} &= 0 \\
i_q - C_L \frac{du_{cq}}{dt} - \omega C_L u_{cd} &= 0
\end{align*}
\]

(9)

Rearranging (8) and (9):

\[
\begin{align*}
\frac{di_d}{dt} &= -\frac{R}{L_T} i_d + \omega i_q - \frac{L_T}{L_T} (u_{cd} + U_{cd}) \\
\frac{di_q}{dt} &= -\frac{R}{L_T} i_q - \omega i_d - \frac{L_T}{L_T} (u_{cq} + U_{cq}) \\
C_L \frac{du_{cd}}{dt} &= i_d + \omega C_i u_{cq} \Rightarrow \frac{du_{cd}}{dt} = \frac{1}{C_L} i_d + \omega u_{cq} \\
C_L \frac{du_{cq}}{dt} &= i_q - \omega C_L u_{cd} \Rightarrow \frac{du_{cq}}{dt} = \frac{1}{C_L} i_q - \omega u_{cd}
\end{align*}
\]

(10)

with: \( L_T = L + L_s \).

This system can be written in the state space form as follows:

\[
\begin{align*}
&X = AX + BU \\
&Y = CX + DU
\end{align*}
\]

\[
A = \begin{bmatrix}
-\frac{R}{L_T} & \omega & -\frac{1}{L_T} & 0 \\
0 & -\frac{R}{L_T} & 0 & -\frac{1}{L_T} \\
\frac{1}{C_L} & 0 & 0 & \omega \\
0 & \frac{1}{C_L} & -\omega & 0
\end{bmatrix},
B = \begin{bmatrix}
0 \\
-\frac{1}{L_T} \\
0 \\
0
\end{bmatrix},
C = \begin{bmatrix}
\begin{bmatrix}
u_{rd} & u_{rq}
\end{bmatrix}^T \\
u_{rd} & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

\[
D = \begin{bmatrix} 0 & 0 \end{bmatrix}, U = \begin{bmatrix} V_{cq} \\
V_{cd} \end{bmatrix}, Y = \begin{bmatrix} P \\
Q \end{bmatrix}
\]

and the system transfer functions can be easily obtained from the state space equations:

\[
\begin{bmatrix} P \\
Q \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \end{bmatrix} \begin{bmatrix} V_{cq} \\
V_{cd} \end{bmatrix}
\]

with:

\[
G_{11}(s) = \frac{P}{V_{cq}}, G_{12}(s) = \frac{P}{V_{cd}} \quad G_{21}(s) = \frac{Q}{V_{cq}} \quad G_{22}(s) = \frac{Q}{V_{cd}}
\]
The instantaneous active and reactive power generated and absorbed are defined as follows:

\[
P_{\text{source}} = \frac{3}{2} (V_{sd} i_{sd} + V_{sq} i_{sq}) ; Q_{\text{source}} = \frac{3}{2} (V_{sq} i_{sd} - V_{sd} i_{sq})
\]

and the active and reactive powers absorbed by the load are:

\[
P_{r} = \frac{3}{2} (V_{rd} i_{d} + V_{rq} i_{q}) ; Q_{r} = \frac{3}{2} (V_{rq} i_{d} - V_{rd} i_{q})
\]

with:

\[
i_{sd} = i_{d} + i'_{d} \quad \text{and} \quad i_{sq} = i_{q} + i'_{q}
\]

3. CONTROL OF THE UPFC

The combination of both parallel and serial converters with a DC link provides four quadrants control. Figure 4 show the block diagram of overall control scheme of the UPFC [10-12].

![Block diagram of overall control scheme of the UPFC](image)

3.1. H∞ Control Approach

3.1.1. The H∞ Optimal Control Synthesis

Our system is represented by a transfer matrix \(G(s)\) with a number of disturbance elements associated with the environment of the physical system (interference signals, etc.) and modeling errors (reduced order model, idealization actuators, parametric uncertainties, etc.) as shown in Figure 5 [7].

The aim of compensation \(K(s)\) is to ensure the stability of the closed loop system and a nominal satisfactory behavior. All controller qualities should be preserved as much as possible in the presence of external perturbations \(w\) and modeling uncertainties \(\Delta(s)\), the latter being translated using interference signals \(v '\). At this level, weighting matrices can be introduced on the signals \(v\) and \(w\) to perform a frequency and distribution of their structural effects. Assume first that these weights have been addressed in the \(P(s)\) model as shown in Figure 6.
The goal is to find a dynamic compensator such that the stability of the system is ensured and that some transfer norm from $w$ to $z$ denoted $F_l(P, K)$ characterizing the performance criteria and/or robustness will be minimized. The problem is then:

$$\min \| F_l(P, K) \|$$ \hspace{1cm} (13)

Let $\begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}$ be a partition of matrix $P$, the dimensions of each sub-matrix matches with the number of inputs and outputs. Then $F_l(P, K)$ is expressed as:

$$z = F_l(P, K) w = (P_{11} + P_{12} K(I - P_{22} K)^{-1} P_{21}) w$$ \hspace{1cm} (14)

The problem formulated in (13) is therefore re-written as:

$$\min \| P_{11} + P_{12} K(I - P_{22} K)^{-1} P_{21} \|$$ \hspace{1cm} (15)

The stabilization of the system $P$ by the compensator $K$ is one of the objectives of the compensation. However, $P$ is a multivariable system (multiple inputs and multiple outputs) then, the concept of stability must be specified.

Based on Figure 7, the standard problem is as follows: find the compensator $K(s)$ stabilizes $P(s)$ and minimizes $\| T_{zw} \|_\infty$. $T_{zw}$ represents the transfer matrix between $w$ and $z$. $P(s)$ is the augmented system that allows us to generate $z$, corresponding to the objectives of the command and can be represented in state space form by:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} A & B_1 \\ C_1 & D_{11} \\ C_2 & D_{21} \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} - \begin{bmatrix} B_2 \\ D_{12} \\ D_{22} \end{bmatrix} \begin{bmatrix} u \end{bmatrix}$$ \hspace{1cm} (16)

or by its transfer matrix.

$$P = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}, \begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} u \end{bmatrix}$$ \hspace{1cm} (17)

To achieve these results, our control problem should be transformed into the standard form according to the principle illustrated in Figure 7.

The objectives of the control are defined as follows:

a) Disturbance rejection and tracking error:

Minimize $\| W_0(s). T(s) \|_\infty$.

b) Noise attenuation $m(s)$, return to maximizing margin multiplicative stability (multiplicative uncertainty output)

Minimize $\| W_0(s). T(s) \|_\infty$.

c) Maximize the margin of stability additive, i.e. limiting the amplitude of the control signal.

Minimize $\| W_0(s). K(s). S(s) \|_\infty$. 

---

Figure 5. The canonical robust control problem

Figure 6. A simplified representation of the control structure
This implies the minimization of:
\[
\begin{bmatrix}
W_y(s)S(s) \\
W_T(s)T(s) \\
W_u(s)K(s)S(s)
\end{bmatrix} \to \infty
\] (18)

The standard problem can be represented by the equivalent diagram shown in Figure 8.

Figure 7. Classical feedback control system

Figure 8. Closed-loop system with modeling error

which gives:
\[
\begin{bmatrix}
z \\
y
\end{bmatrix} =
\begin{bmatrix}
W_y & -W_yG \\
0 & W_TG \\
0 & W_u \\
I & -G
\end{bmatrix}
\begin{bmatrix}
w \\
u
\end{bmatrix}
\] (19)

replacing \( u \) by \( Ky \):
\[
T_{ZW} =
\begin{bmatrix}
W_yS \\
W_TT \\
W_uK\Sigma
\end{bmatrix}
\] (20)

The solution of this problem is obtained using the function \( H_{\infty} \) or \( H_{\infty-opt} \) from the Matlab Robust Control Toolbox called Mixed Sensitivity Problem. Other representations of the augmented system to several inputs \( (w_1, w_2, w_3) \), and one output \( z \) can be addressed.

The synthesis of \( H_\infty \) control may be summarized by:

a) Translate the objectives \( \| \cdot \|_{\infty} \).

b) Select the weighting functions in terms of their frequency response.

c) Set up the matrix \( P(s) \) of the equivalent standard problem.

d) Solve the optimization problem.

e) Test the performance of the closed-loop system.

3.1.2. Solution of the Standard Problem and Selection of the Weighting Functions

Doyle and others [7, 13-17] have solved this problem by performing standard programs to give the controller status of state space from the system and weighting functions. The assumptions used in the resolution of this problem are:

1) \( (A, B_2) \) stabilizable and \( (A, C_2) \) detectable.
2) Rank \( (D_{12}) = m_2 = \dim (u) \) and rank \( (D_{21}) = P_2 = \dim (y) \).
3) $D_{12} = \begin{bmatrix} 0 \\ I \end{bmatrix}$, $D_{21} = [0, I]$

4) Rank $\begin{bmatrix} A - j\omega B_2 \\ C_1 \\ D_{22} \end{bmatrix} = n + m_2$, $\forall \omega \in [0, \infty]$.

5) Rank $\begin{bmatrix} A - j\omega B_1 \\ C_2 \\ D_{21} \end{bmatrix} = n + P_2$, $\forall \omega \in [0, \infty]$.

a) Assumption (I) is necessary and sufficient for the existence of a solution.
b) Assumptions (II), (IV) and (V) ensure that the problem is well posed, in other words, the compensator is appropriate.
c) Assumptions (IV) and (V) indicate that $P_{12}(s)$ and $P_{21}(s)$ have no zeroes on the imaginary axis, they can be avoided [7].
d) Assumption (III) simplifies the solution.
e) If assumption (II) is satisfied, it can used to check system (III). This operation is performed by the $H_{inf}$ program Matlab.
f) The only problem that may arise is that of the validation of assumption (II).

3.2. Validation of the Assumption (II)

If, by executing the instruction Hinf (Robust Control Toolbox of Matlab), the error signal «matrix $D_{12}$ is not in full column rank», this means that the hypothesis (II) is not verified, and therefore the transfer functions $P_{12}(s)$ and $P_{21}(s)$ have zeros at infinity [9].

In order to ensure the closed loop stability and simultaneously achieve the desired control performance under process variations or in the presence of other disturbances weighting functions dependent of the frequency in the process are introduced as shown in Figure 9.

![Figure 9. Control system with modeling error](image)

The controller optimization is defined by the following equation:

$$\left\| W_T(s).T(s) \right\|_{\infty} < \gamma \quad \forall \omega \in [0, \infty]$$

(21)

These weighting functions define the frequency characteristics of the signals of the system as well as their amplitudes. Putting $|W_T(j\omega)| \geq 1$ for a certain frequency range, the gain reduction of the complementary sensitivity function $T(j\omega)$ can be achieved beyond this range.

It is now assumed that also seeks a good performance (characterized by $S(j\omega)$) residing in the disturbance rejection. This performance can be achieved by finding the controller $K(s)$ by solving the following equation:

$$\left\| W_S(j\omega).S(j\omega) \right\|_{\infty} \leq \gamma_{\text{const}} \quad \forall \omega \in [0, \infty]$$

(22)

This equation is equivalent to:

$$\left\| W_S(j\omega).S(j\omega) \right\|_{\infty} < \gamma_{\text{const}} \quad \forall \omega \in [0, \infty]$$

(23)

$\gamma_{\text{const}}$ is an arbitrary constant which is not necessarily equal to $\gamma$ (22).
Performance objectives and the robust stability can be simultaneously achieved by seeking a controller $K(s)$ satisfying the following inequality:

$$
\begin{align*}
\left[ \frac{W_s(j\omega)S(j\omega)}{W_T(j\omega)T(j\omega)} \right] < \gamma 
\end{align*}
$$

(24)

The choice of the weighting functions is as follows: Depending on the required performance, $W_s(j\omega)$ is large at low frequency and smaller at high frequency, representing a low pass filter. In our case, it is a diagonal matrix $W_s(j\omega) = w_s I$, where $w_s$ represents the weighting function selected such that:

$$
\begin{align*}
\left| S(j\omega) \right| \left< \frac{1}{w_s(j\omega)} \right| \quad \text{or} \quad \sigma(S(j\omega)) \left< \frac{1}{W_s(j\omega)} \right|
\end{align*}
$$

for multivariable systems.

Generally, the uncertainties and dynamics are neglected high frequencies, $W_T(j\omega)$ must be represented by a high pass filter to ensure robustness for high frequencies ($\omega > \omega_0$) and acceptable performance for low frequencies ($\omega < \omega_0$).

Our choice of weighting coefficients is completely connected to the parameter $\gamma$ as explained above (following the objectives set such that the bandwidth in this case equal 70rd/s [6]), after trial and error, a value of $\gamma = 1.92$ was selected and the transfers functions of the weighting coefficients are:

$$
\begin{align*}
W_s &= \frac{10s+1}{10(1/70s+1)} \\
W_T &= \frac{278}{0.01s+0.01}
\end{align*}
$$

These weighting coefficients $W_s$ and $W_T$ define the controller $K(s)$. The responses of the active and reactive powers are illustrated in Figure 10. From these results, it can be noted that the powers track their respective references perfectly, reflecting the right choice of parameters of our $K(s)$. In addition, through the weights $W_T$ and $W_s$ defined, the interaction between the powers is completely eliminated and hence the controller was able to decouple the system and ensure good performance and achieve the control objective [18-26].

![Figure 10](image)

Figure 10. Response of the active power (a) and reactive power (b) with $K(s)$ control

### 3.3 Robustness Test

#### 3.3.1 Increase of the Inductance of the Line by 20%:

We have introduced changes to one of the line parameters (inductance) to analyze the robustness of our controller. Note that the increase of 20% of this parameter, does not affect the characteristic as shown in Figure 11 (a), because the active power follows perfectly the set point which explains the qualities of our controller $K(s) H\infty$. 

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Robust control of a UPFC system with $H\infty$ control technique (Maamar Benyamina)
3.3.2. Reduction of the Inductance of the Line by 20%:

Figures 12 (a) and (b) represent respectively the curves of the active and reactive powers corrected after the reduction of the inductance of the line by 20%, these characteristics prove once more that our regulator retains its qualities, in particular its robustness.

We can then conclude that the various changes made did not affect the qualities of the controller, which implies that the controller $K(s)$ defined by the $H\infty$ approach is perfect for our system.

4. CONCLUSION

In this work, we focused to show the control of the UPFC by the $H\infty$ control. The control $H\infty$ could ensure the desired performance, this implies that the control $H\infty$ is robust and also implies a good synthesis of coefficients and weighting functions thereof. We can therefore probably say that the $H\infty$ approach is, in the field of robust control, the theme that continues to give rise to a large number of publications following very great efforts. All this is not due to chance, because it represents a very natural way of formulating the problem of robustness.

REFERENCES

Robust control of a UPFC system with $H_\infty$ control technique (Maammar Benyamina)


